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Three-dimensional surface model based on edge collapse improvement model data compression algorithm

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ABSTRACT

Along with the computer technology and the rapid development of the three dimensional imaging software and hardware technology, the three digital technology is more and more get people's attention and application. But limited graphics graphics processing power and the spread of the media has been affected by the network bandwidth, rely on increasing hardware input, not enough to solve the problem, must adopt corresponding algorithm of 3 d surface model data is compressed, can fundamentally solve this defect. Although some common compression algorithms and tools can be used to reduce the three-dimensional geometric data volume, but the efficiency is not ideal, or for some specific application doesn't fit, 3 d model with previous audio (1 d), image (2 d), video (2 d + time) of the three different media types, 3 d mesh model also show any bending, parametric characteristics of complex and the lack of a continuous nature, and define the rules of various properties on the surface of the model are sampling, which makes the classical orthogonal analysis tool 3 d geometry signals cannot be directly used for processing. Therefore, looking for a new algorithm to efficiently express 3 d model is needed. Proposed in this paper Garland compression algorithm based on surface model, this paper proposes a surface data compression algorithm based on edge collapse, and the algorithm based on quadratic error metrics calculate the edge collapse cost, and guidelines to determine the order of edge collapse, on the basis of test methods to judge the legitimacy of the shrinkage is half space, the boundary of the surface by the algorithm and model with internal synchronous compression in the region and in the case of high compression ratio can not only ensure the geometrical characteristic of the original model, and also can effectively guarantee the quality of the compression model.

KEYWORDS

Edge collapse; Compression algorithm; Three-dimensional surface; Quadric error; Halfspace test.

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INTRODUCTION

3 d laser scanning system, also known as 3 d laser imaging system, is mainly composed of 3 d laser scanner and system software, its purpose is to obtain the fast, easy and accurate space threedimensional coordinates, three dimensional model is established, in order to further analysis of the model, data processing and application. 3 d model as the fourth kind of multimedia data type in industrial manufacturing, construction, medical, simulation, and entertainment, and many other fields also plays an increasingly important role. The rapid development of Internet makes the application of three-dimensional model is more and more widely, but the network bandwidth limit a serious impediment to the spread of the media, although the PC graphics display card data processing ability and the network bandwidth has made great progress, but with amazing details rich complex threedimensional network model compared to the amount of data is still limited, so the efficient encoding must be used to solve the problem of the data storage and network transmission. In virtual reality, threedimensional geographic information system, interactive visualization, and other fields, usually in irregular triangles (triangulated irregular network, TIN) as describe the basic elements of space object surface features, and these triangles is through the connection is located in the 3 d feature points on the surface of the object. Currently from the point of the research field, surface modeling technology from traditional research of surface representation, surface intersection and surface, extend to the surface deformation, surface reconstruction and curved surface of surface simplification, transformation and surface potential difference, curved surface reconstruction needs to collect a large amount of data, built the model of general is very complex.

Many experts and scholars at home and abroad have the 3 d surface model data compression method carried out extensive and in-depth research, including the representative are: Schroeder and surface model data compression algorithm based on vertex deleting^[1]. Garland prominent surface model data compression method based on edge collapse^[2]; Hamann's surface model data compression method based on triangle contraction^[3]; Turk is put forward based on the network to redraw the surface of the model data compression method^[4]; Hoppe, puts forward the model based on the surface of the whole grid optimization criteria data compression method^[5]. The research results obtained the very good application, for the development of the technology provides a good theoretical basis. In addition, information engineering university of surveying and mapping institute of nurse xu min and yong-sheng zhang in the 3 d model data compression technology analysis, classification of 3 d model data compression algorithm is introduced, and combining with the classification of different situation from a nationally representative sample of several compression algorithms are analyzed, and puts forward several aspects to the development of the technology summary and outlook of^[6]; CAD & peng-jie wang, zhi-geng pan and CG, zhejiang university state key laboratory of yong-kui liu in the 3 d model based on point compression technology research progress, summed up in the compression process of the model, and gives the classification principle of some model compression algorithm, and then by the resolution algorithm and incremental algorithm as the main line a little detail model compression, the latest research progress of analysis and comparison the corresponding key technology^[7]; Geographic information engineering department of southeast university xian-hua CAI and Zheng Tiandong in studies of digital elevation model data compression method of DEM data compression, DEM terrain compression error () concept, on the basis of this proposed a kind of to DEM terrain compression error influence for the selection of qualified data compression method, and using the method can provide DEM digital integrated with a kind of technical means, reduce the DEM data storage quantity, improve the DEM^[8] subsequent processing speed; China ocean university information engineering college of computer science at Jia Jianfeng Ma Ping and Qingdao university of science and technology a harp island college in the 3 d surface texture image compression in image compression based on wavelet analysis, this paper introduces wavelet analysis and research on the two-dimensional wavelet analysis in the application of image compression, this paper introduces the theoretical framework of 3 d surface texture, the wavelet analysis is carried out on the basis of the framework in the application of 3 d surface texture compression research and development in the field of image compression, wavelet analysis and the effectiveness of the method is proved by experiment and data^[9]. Grid model of compression is currently the main topology compression method, based on the research and analysis the principle of edge collapse, and to apply it to the 3 d surface model in data compression, provides a theoretical basis for the implementation of the algorithm.

EDGE COLLAPSE ALGORITHM PRINCIPLE

Variable definition and theorem

As Figure 1shows, surface model sampling point set $V_{sub} = \{v_0, v_1, v_1^n, v_2^n, ..., v_8^n\}$, is composed TIN sampling point itself that is a partial triangulation network of TIN composed of V_{sub} . In order to easy to describe, it makes following definitions:

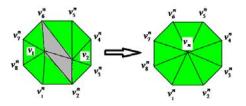


Figure 1 : Edge collapse operation schematic diagram

Definition 1 A connected, node interlinked entity surface TIN model is a limited triangle sequence; the sequence meets following five conditions:

- In surface model, every edge is shared by two triangles at most:
- Surface model vertex can be shared by any multiple triangles;
- Every triangle at least share a vertex with other triangles;

If triangle T_i one vertex is simultaneously shared by another triangle T_j , and then the vertex is also the vertex of triangle T_j ;

Triangle is complemented and overlapped.

Definition 2 For connected, node interlinked surface model, define edge E_i correlated triangle template P_i as all and edge E_i at least one vertex shared triangle T_i set.

Definition 3 Call triangle P_i vertex set (not including contracted two vertexes v_1, v_2) as boundary vertex set. By boundary vertex set paired order (clockwise or anticlockwise) connection composed polygon is called boundary polygon.

Definition 4 Edge collapse is the process that surface combines requested two vertexes into one vertex and deletes degenerated triangle during surface data compression process.

Figure 1 provided edge collapse operation process: simplify edge (v_1, v_2) into a new vertex v_n , and let each vertex that connects to the edge terminal vertex to connect with new vertex, and meanwhile delete all degenerated edges and surfaces.

There are two strategies for new vertex selection after edge collapsing: one is optimization selection method, that is to calculate space position v_n , let collapsing caused error cost to be minimum, it can look v_n in two points connection line, and can also look for v_n in total space; Another is sub set selection method, that is to select edge vertexes v_1 and v_2 , one of them that let collapsing cost to be minimum is new vertex position. The second algorithm not only has advantages in complexity aspect but also is beneficial to form into progressive mesh by comparing to the first algorithm.

Quadric error measurement

Whether one edge can be collapsed or not is decided by its cost size, and cost value is got form edge surrounding information through error measuring. Use quadric error measurement to measures edge collapse cost, allocate every vertex an error measurement matrix, and calculate after edge collapsing generated new vertex error matrix and error value by the matrix, constantly collapse current model's minimum error cost edge to simplify model, till it meets model simplification requests.

Set v is one point in two-dimensional space, coordinate is $[v_x, v_y, v_z, 1]^T$; p represents one plane in space, its equation is as formula (1) shows:

$$ax+by+cz+d=0\tag{1}$$

In formula (1), each coefficient meets $a^2 + b^2 + c^2 = 1$, then the plane is recorded as $p = [a, b, c, d]^r$, point v to plane p distance square is as formula (2) shows:

$$d^{2}(v) = (p^{T}v)^{2} = v^{T}(pp^{T})v = v^{T}K_{p}v$$
(2)

In formula (2), K_p represents 4×4 matrix as formula (3) shows:

$$K_{p} = pp^{T} = \begin{bmatrix} a^{2} & ab & ac & ad \\ ab & b^{2} & bc & bd \\ ac & bc & c^{2} & cd \\ ad & bd & cd & d^{2} \end{bmatrix}$$
(3)

For a vertex v, its corresponding triangle plane set is planes(v), define the vertex quadric error measure is it to these triangle planes distance squares sum, measure is as formula (4) shows:

$$\Delta'(v) = \sum_{p \in planes(v)} d^2(v) = \sum_{p \in planes(v)} v^T(K_p) = v^T\left(\sum_{p \in planes(v)} K_p\right)$$

$$Q'(v) = \sum K_p \qquad Q'(v)$$
(4)

To make formula (4) expressing easy and simple, let in $\sum_{p \in planes} p_{p \in planes}$, and use Q'(v) to represent vertex v quadric error measure matrix, it is four orders matrix that is used to measure edge collapsing costs. When edge (v_i, v_j) collapses to new vertex \overline{v} , then collapse cost is as formula (5) shows:

$$\Delta'(\overline{v}) = \overline{v}^T (Q_i' + Q_j') \overline{v}$$
⁽⁵⁾

In formula (5), $Q'_i + Q'_j$ represents new vertex \overline{v} quadric error measure, intuitionally, point \overline{v} and edge (v_i, v_j) correlated triangle plane distance gets further, its cost will also get larger, so is the practice.

New vertex position defining theorem

For assigned edge collapse as Figure 1shows: $(v_1, v_2) \rightarrow v_n$, it needs to define v_n position according to certain rules, in order to compress model features, selection of new vertex v_n is crucial.

Due to $\Delta'(v_n)$ is a quadratic equation, looking for $\Delta'(v_n) = \min$ condition becomes solving new vertex's equivalence problem, and because v_n is solution of formula (6) equation, it can utilize the equation as constraint condition to solve $\Delta'(v_n)$ minimum value.

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$$\frac{\partial \Delta'(v_n)}{\partial x} = \frac{\partial \Delta'(v_n)}{\partial y} = \frac{\partial \Delta'(v_n)}{\partial z} = 0$$
(6)

Formula (6) applies matrix attribute can be expressed as formula (7) form:

$$\begin{bmatrix} q_{1,1} & q_{1,2} & q_{1,3} & q_{1,4} \\ q_{2,1} & q_{2,2} & q_{2,3} & q_{2,4} \\ q_{1,1} & q_{3,2} & q_{3,3} & q_{3,4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
(7)

By formula (7), it can get formula (8):

$$\begin{bmatrix} q_{1,1} & q_{1,2} & q_{1,3} \\ q_{2,1} & q_{2,2} & q_{2,3} \\ q_{1,1} & q_{3,2} & q_{3,3} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = 0$$
(8)

Let Q' to represent formula (8) three-order matrix, $v_n = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^T$, $L = \begin{bmatrix} q_{1,4} & q_{2,4} & q_{3,4} \end{bmatrix}$, it can get formula (9) expression by formula (7):

$$Q'v_n + L = 0 \tag{9}$$

If matrix Q' is reversible, then can get new vertex as formula (10) shows:

$$v_n = -Q'^{-1}L \tag{10}$$

If matrix Q' isn't reversible, then can take the second best and select $\frac{v_1, v_2, \frac{v_1 + v_2}{2}}{2} \operatorname{let} \Delta'(v_n)$ to take its minimum value, as formula (11) shows:

$$\Delta'(v_n)_{\min} = \min\left\{\Delta'(v_1), \Delta'(v_2), \Delta'\left(\frac{v_1 + v_2}{2}\right)\right\}$$
(11)

In order to effective implement error transmission, it needs to calculate new vertex area error matrix, and further define its quadric error as formula (5)calculation rule, define $\Delta'(v_n)$ as to be collapsed edge v_1v_2 cost of edge collapsing, and according to this, rank all edges that to be collapsed to define contraction order.

Half-space test

In model compression process, in order to effective avoid compression result TIN surface patches' space self intersection, it should adopt proper algorithms to carry out judgment test on edge collapsing operation legality. For specific edge collapsing operation $(v_1, v_2) \rightarrow v_n$, adopt half-space test method to do judgment test on legality, its test flow is as following shows:

(1) For assigned edge collapse $(v_1, v_2) \rightarrow v_n$, according to section 2.3 computational method to work out v_n , and according to vertex (v_1, v_2) surrounding vertexes and new vertex v_n to reconstruct triangle sequence T;

(2) According to reconstructed triangle sequence T, solve equation of passing new vertex v_n average plane \overline{p} , expression is as formula (12) shows:

$$Ax + By + Cz + D = 0 \tag{12}$$

Individual variable coefficient needs to meet formula (13):

$$A^2 + B^2 + C^2 = 0 \tag{13}$$

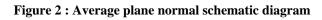
Average plane \overline{P} passes through new vertex V_n , and its normal vector n can utilize formula (14) to solve:

$$\vec{n} = \frac{\vec{N}}{\left|\vec{N}\right|}$$
(14)

In formula (14), \hat{N} is as formula (15) shows:

$$\vec{N} = \frac{\sum \vec{n}_i A_i}{\sum A_i}$$
(15)

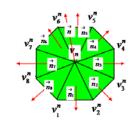
In formula (15), n_i, A_i respectively represents each item diamond triangle patch normal vector and normal area. Average plane normal vector is as Figure 2 shows:



(4) In average plane \overline{P} , establish a two-dimensional partial orthogonal coordinate system, the coordinate system's system coordinate origin is v_n , in average plane \overline{P} look for two units' orthogonal vectors $\mathbf{b}_1, \mathbf{b}_2$, and use $\mathbf{b}_1, \mathbf{b}_2$ directions as the coordinate system two coordinate axis directions, then solve normal vector \mathbf{n} orthogonal unit vector \mathbf{a} is as formula (16) shows:

$$\mathbf{a} = \begin{cases} \frac{\left(-(\mathbf{n}_{y} + \mathbf{n}_{z}), \mathbf{n}_{x}, \mathbf{n}_{x}\right)}{\mathbf{n}_{x}} & , \mathbf{n}_{x} \neq 0\\ \frac{\left(\mathbf{n}_{y}, -(\mathbf{n}_{x} + \mathbf{n}_{z}), \mathbf{n}_{y}\right)}{\mathbf{n}_{y}} & , \mathbf{n}_{y} \neq 0\\ \frac{\left(\mathbf{n}_{z}, \mathbf{n}_{z}, -(\mathbf{n}_{x} + \mathbf{n}_{y})\right)}{\mathbf{n}_{z}} & , \mathbf{n}_{z} \neq 0 \end{cases}$$

Unit vector \mathbf{b}_1 is expressed as formula (17):



(16)

$$\mathbf{b}_1 = \frac{\mathbf{a}}{\|\mathbf{a}\|}, \|\mathbf{a}\| = \sqrt{\mathbf{a} \cdot \mathbf{a}} \tag{17}$$

Unit vector \mathbf{b}_2 is as formula: $\mathbf{b}_2 = \mathbf{n} \times \mathbf{b}_1$ (4) Calculate (v_1, v_2) corresponding boundary vertex to average plane \overline{p} directed distance $d_j (j = 0, 1, ..., n_i)$, distance computation method is as formula (18) shows:

$$d_{j} = dist(y_{j}, \overline{p}) = \frac{Ax_{j} + By_{j} + Cz_{j} + D}{\sqrt{A^{2} + B^{2} + C^{2}}}$$
(18)

And due to formula (13), d_j value and formula (18) numerator value are the same, and it changes into formula (19):

$$d_j = dist(y_j, \overline{p}) = Ax_j + By_j + Cz_j + D$$
(19)

(5) Project (v_1, v_2) corresponding boundary vertex to plane \overline{p} , and calculate v_1, v_2 coordinates in defined coordinate system, the location distribution is as Figure 3 shows.

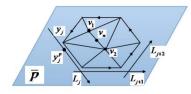


Figure 3 : Half-space test

If project all boundary vertexes into average plane \overline{P} , then projection coordinate computational method is as formula (20) shows:

$$y_j^p = y_j - d_j \bullet \mathbf{n} \tag{20}$$

In formula (20), y_j^p superscript represents projected, transform y_j^p into partial coordinate system; transformation equation is as formula (21) shows:

$$(\boldsymbol{u}_{j},\boldsymbol{v}_{j}) = (\boldsymbol{d}_{j} \bullet \boldsymbol{b}_{1}, \boldsymbol{d}_{j} \bullet \boldsymbol{b}_{2})$$
(21)

(6) Calculate to be collapsed edge v_1v_2 corresponding boundary polygon's each edge linear equation L_j in partial coordinate system, as formula (22) shows:

$$L_{j}(u,v) = -\Delta' v_{j}(u-u_{j}) + \Delta' u_{j}(v-v_{j}) = 0$$
(22)

In formula (22), $\Delta' u_j, \Delta' v_j$ is expressed as formula (23) shows:

$$\begin{cases} \Delta' u_{j} = u_{(j+1) \mod(n_{j}+1)} - u_{j} \\ \Delta' v_{j} = v_{(j+1) \mod(n_{j}+1)} - v_{j} \end{cases}$$
(23)

(7) If v_n meets formula (24) then it shows v_n lies in L_j positive half-space, it can affirm edge collapsing operation legality, otherwise it cannot collapse the edge.

$$L_j(0,0) > 0, \ j = 0,1,\dots,n_j$$
 (24)

IMPROVE ALGORITHM

Improved algorithm principle

In computer graphics, three-dimensional grid model most common attributes are color, texture and normal, in order to let model after compressing has good similarity as initial model, it should ensure model geometric information and meanwhile reserve these attributes, due to point to plane distance considers edge collapsing operation effects on vertex surrounding region attributes value changes, it can relative correct describe partial attributes error, and meanwhile is simpler and faster than distance calculation between points and surfaces or surfaces to surfaces. Therefore, adopt distance from point to plane as attribute error measure; apply quadric error measure into attribute error calculation.

Grid model every vertex except for space coordinate, it also has values to describe its attributes, in grid model triangle plane, attributes values are got according to geometric location's interpolation. Therefore, triangle plane attributes values are continuous, and two attributes values distance is measured by Euclidean distance.

Color attribute can use trivector $[r, g, b]^T$ to express, from which $(0 \le r, g, b \le 1)$, all color vectors are composed RGB color space, as Figure 4shows, in RGB color space, point to plane distance square similarly can use quadric error Q(v) to calculate, when new vertex after edge collapsing adopts sub set selection method, no need to recalculate new vertex space position and attribute value, when calculate error, no need to consider space coordinate and attribute value correlations, only need to respectively establish geometry quadric error measure and attribute quadric error measure, and calculate geometry and attribute errors.

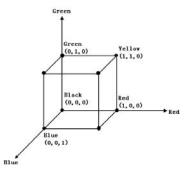


Figure 4 : RGB color space

In algorithm description, it adopts triangle grid model with color attribute, grid models with other attributes can similarly deduced. Triangle grid model every vertex v has two vectors $v_g[x, y, z]^r$ and $v_c[r, g, b]^r (0 \le r, g, b \le 1)$ to represent geometry and color information, establish geometry quadric error measure Q_{fg} and color attribute quadric error Q_{fc} each vertex quadric error measure is equal to its adjacent triangle surface quadric error measure sum for every triangle surface, as formula (25) shows:

$$\begin{cases} Q_{vg} = \sum_{(v \in f)} Q_{fg} \\ Q_{vc} = \sum_{(v \in f)} Q_{fc} \end{cases}$$
(25)

When collapse edge $(v_i, v_j), j \neq i$ to vertex v, total quadric error measure is as formula (26) shows:

$$\begin{cases} Q_g = Q_{vig} + Q_{vjg} \\ Q_c = Q_{vic} + Q_{vjc} \end{cases}$$
(26)

Therefore edge collapsing caused geometric error $E_g = Q_g(v_g)$, color attribute error $E_c = Q_c(v_c)$, then total edge collapsing cost is as formula (27) shows:

$$Cost = E_g + \alpha E_c \tag{27}$$

In formula (26), α represents color attribute error influence coefficient in total cost, it can adjust according to practical demands.

When calculate edge collapsing cost, it needs to focus on two aspects' problems:one is vertex set coordinate value range is $(-\infty, +\infty)$, and color component value range is [0,1], so it should make preprocessing with vertex geometric position data, let the two range to be the same, so as to ensure E_g and E_c effects on *Cost* are equivalent; the other is it will appear triangle surface three vertexes color vectors are the same, in such case three color vectors cannot compose plane but degenerate into a point, therefore it should calculate and let color vectors quadric error to met formula (28):

$$Q_{fc}(v_c) = (v_c - v_{ic})^2$$
(28)

In formula (27), v_{ic} represents the triangle surface any one color vector.

Algorithm flow

Edge collapsing algorithm basic thought is taking edge as deleted basic geometric element in every simplification operation, and increasing new vertex, all points that connect with deleted edges should connect with the new vertex, and let model to still keep as triangle grid, after carrying out multiple times selective edge collapsing, model can be simplified into requested extent. Algorithm flow is as Figure 5 shows:

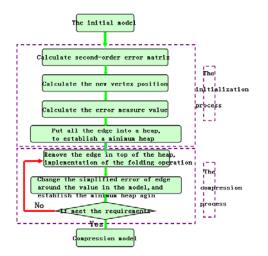


Figure 5 : Edge collapsing algorithm flow

Algorithm basic steps:

Step1: Read initial grid model data, and make preprocessing with it, mark boundary edge and feature edge;

Step2: Calculate initial grid every triangle surface geometric and color attributes quadric error measure values;

Step3: Calculate every vertex geometry and color attributes quadric error measure values;

Step4: Calculate every edge collapsing cost and new vertex after collapsing, and carry out legality judgment test on edge collapsing;

Step5: All edges that meet legality judgment compose a candidate edge set, rank the set edges according to edge collapsing cost;

Step6: Select minimum collapsing cost edge from candidate set to execute collapsing operation, and update all relative information;

Step7: If candidate set is empty or triangles numbers already arrive at users' demands, then end, otherwise transfer to Step6.

Algorithm performance analysis

For storage space perspective, the algorithm with respect to Garland multiple dimensional quadric error measure algorithm, it needs fewer coefficients to save quadric error measure, and no need to save new vertex information after edge collapsing operating every time, storage Q required coefficients are as TABLE 1 and TABLE 2 show:

Model type	Garland multiple-dimensional quadric error measure algorithm		
	Vertex expression	Order number	Q coefficient
Only contain geometric information	$[x, y, z]^T$	3×3	$\binom{5}{2} = 10$
Geometric information and color	$[x, y, z, r, g, b]^T$	6×6	$\binom{8}{2} = 28$
Geometric information and normal	$[x, y, z, a, b, c]^T$	6×6	$\binom{8}{2} = 28$

TABLE 2 : Improved edge collapse algorithm

Model type	Improved edge collapse algorithm		
	Vertex expression	Order number	Q coefficient
Only contain geometric information	$\begin{bmatrix} x, y, z \end{bmatrix}^T$	3×3	$\binom{5}{2} = 10$
Geometric information and color	$[x, y, z]^T, [r, g, b]^T$	3×3	$2 \times \binom{5}{2} = 20$
Geometric information and normal	$[x, y, z]^T, [a, b, c]^T$	3×3	$2 \times \binom{5}{2} = 20$

By TABLE 1 and TABLE 2, it is clear that improved algorithm is superior to Garland multipledimensional quadric error measure algorithm in the perspective of storage.

CONCLUSION

3 d surface model is reasonable and effective compression is necessary, the merits of the compression algorithm of space target surface modeling development has deep influence; Proposed in this paper the three-dimensional surface compression algorithm based on edge collapse, first of all,

based on the quadratic error measure criterion to determine the order of the folded edge, then using the half space test algorithm to judge the legitimacy of the edge collapse, and in order to achieve the surface model of the inner regions of the border areas and synchronous compression; In the study of image compression algorithm was presented at the same time, need to know the image attributes, including geometric properties, color attribute and normal properties, compression for the 3 d image processing can be better and faster to achieve the needs of users; through the analysis of the algorithm performance is improved algorithm is superior to other algorithms in terms of coefficient of storage.

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