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THERMAL STRESSES OF THREE DIMENSIONAL THERMOELASTIC PROBLEM OF A THIN RECTANGULAR PLATE

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ABSTRACT

In this paper, an attempt has been made to solve three dimensional thermoelastic problem to determine temperature distribution, unknown temperature gradient, displacement and thermal stresses at the edge y = b of a thin rectangular plate occupying the space $D : -\alpha \leq \chi \leq \alpha;\ 0 \leq \gamma \leq b;\ -h \leq z \leq h$, with the known boundary and initial conditions by applying integral transform techniques. The solutions are obtained in the form of infinite series and are depicted graphically.

Key words: Thin rectangular plate, Three dimensional thermoelastic problem, Integral transform.

INTRODUCTION

Tanigawa and Komatsubara (1997), Vihak et al., (1998) and Adams and Bert (1999) have studied the direct problem of thermoelasticity in a rectangular plate under thermal shock. Khobragade and Wankhede (2003) have studied the inverse steady state thermoelastic problem to determine the temperature displacement function and thermal stresses at the boundary of a thin rectangular plate. They have used the finite Fourier sine transform technique.

In the present paper, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement and thermal stress at the edge y = b of a thin rectangular plate occupying the region $D : -\alpha \leq \chi \leq \alpha;\ 0 \leq \gamma \leq b;\ -h \leq z \leq h$, with known boundary conditions. Here Marchi – Fasulo transform and Laplace transform techniques have been used to find the solution of the problem.

Statement of the problem

Consider a thin rectangular plate occupying the space $D : -\alpha \leq \chi \leq \alpha;\ 0 \leq \gamma \leq b;\ -h \leq z \leq h$. The displacement components u_x, u_y and u_z in the X, Y, Z direction respectively are in the integral form as

$$u_x = \int_{-a}^{a} \frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2} + \alpha T\right) dx \qquad \ldots(1)$$

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$$u_y = \int_0^b \frac{1}{E}\left(\frac{\partial^2 \cup}{\partial z^2} + \frac{\partial^2 \cup}{\partial x^2} - \nu \frac{\partial^2 \cup}{\partial y^2} + \alpha T \right) dy \qquad \ldots(2)$$

$$u_z = \int_{-h}^{h} \frac{1}{E}\left(\frac{\partial^2 \cup}{\partial x^2} + \frac{\partial^2 \cup}{\partial y^2} - \nu \frac{\partial^2 \cup}{\partial z^2} + \alpha T \right) dz \qquad \ldots(3)$$

where E, ν and α are the Young's modulus, poisons ratio and the linear coefficient of thermal expansion of the material of the plate respectively and $\cup(x, y, z, t)$ is the Airy's stress function which satisfy the differential equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)^2 \cup(x, y, z, t) = -\alpha E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) T(x, y, z, t) \qquad \ldots(4)$$

Where $T(x, y, z, t)$ denotes the temperature of thin rectangular plate satisfying the following differential equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \qquad \ldots(5)$$

Where k is thermal diffusivity of the material subject to initial conditions

$$T(\chi, \gamma, z, 0) = 0 \qquad \ldots(6)$$

And the boundary conditions are

$$\left[T(x, y, z, t) + k_1 \frac{\partial T(x, y, z, t)}{\partial x} \right]_{x=a} = F_1(y, z, t) \qquad \ldots(7)$$

$$\left[T(x, y, z, t) + k_2 \frac{\partial T(x, y, z, t)}{\partial x} \right]_{x=-a} = F_2(y, z, t) \qquad \ldots(8)$$

$$\left[T(x, y, z, t) \right]_{y=b} = G(x, z, t) \quad \text{(Unknown)} \qquad \ldots(9)$$

$$\left[T(x, y, z, t) + C \frac{\partial T(x, y, z, t)}{\partial y} \right]_{y=0} = g(x, z, t) \qquad \ldots(10)$$

$$\left[T(x, y, z, t) + k_3 \frac{\partial T(x, y, z, t)}{\partial z} \right]_{z=h} = F_3(x, y, t) \qquad \ldots(11)$$

$$\left[T(x, y, z, t) + k_4 \frac{\partial T(x, y, z, t)}{\partial z} \right]_{z=-h} = F_4(x, y, t) \qquad \ldots(12)$$

The interior condition is

$$\left[T(x, y, z, t) + c \frac{\partial T(x, y, z, t)}{\partial z} \right]_{y=\xi} = f(x, z, t) \quad \text{(Known)} \qquad \ldots(13)$$

The stresses components in terms of $\bigcup(x, y, z, t)$ are given by

$$\sigma_{xx} = \left(\frac{\partial^2 \bigcup}{\partial y^2} + \frac{\partial^2 \bigcup}{\partial z^2}\right) \quad \ldots(14)$$

$$\sigma_{yy} = \left(\frac{\partial^2 \bigcup}{\partial z^2} + \frac{\partial^2 \bigcup}{\partial x^2}\right) \quad \ldots(15)$$

$$\sigma_{zz} = \left(\frac{\partial^2 \bigcup}{\partial x^2} + \frac{\partial^2 \bigcup}{\partial y^2}\right) \quad \ldots(16)$$

The equations (1) to (16) constitute the mathematical formulation of the problem under consideration.

Solution of the problem

By applying finite Marchi – Fasulo transform and Laplace transform to the equations (5) to (13), and then taking their inversion, we obtain

$$T(x,y,z,t) = \frac{k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\phi_1(y)\,\tau_1(t) - \phi_2(y)\,\tau_2(t)\right]$$

$$+ \frac{2k\pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{\psi_1(y)\,\tau_3(t) - \psi_2(y)\,\tau_4(t)}{\left[1 + (c\varsigma\pi/\xi)^2\right]}\right]$$

$$- \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right] A_3(m,n,y,t) \quad \ldots(17)$$

$$G(x,z,t) = \frac{k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\phi_1(b)\,\tau_1(t) - \phi_2(b)\,\tau_2(t)\right]$$

$$+ \frac{2k\pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{\psi_1(b)\,\tau_3(t) - \psi_2(b)\,\tau_4(t)}{\left[1 + (c\varsigma\pi/\xi)^2\right]}\right]$$

$$- \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right] A_3(m,n,b,t) \quad \ldots(18)$$

Where,

$$\phi_1(y) = \frac{\sinh\,(y/c) - \cosh\,(y/c)}{\sinh(\xi/c)} \qquad \phi_2(y) = \frac{\sinh\left(\frac{y-\xi}{c}\right) - \cosh\left(\frac{y-\xi}{c}\right)}{\sinh(\xi/c)}$$

$$\psi_1(y) = \sin\left(\frac{\varsigma\pi}{\xi}\right)y - \left(\frac{c\varsigma\pi}{\xi}\right)\cos\left(\frac{\varsigma\pi}{\xi}\right)y\,, \quad \psi_2(y) = \sin\left(\frac{\varsigma\pi}{\xi}\right)(y-\xi) - \left(\frac{c\varsigma\pi}{\xi}\right)\cos\left(\frac{\varsigma\pi}{\xi}\right)(y-\xi)$$

$$\tau_1(t) = \int_0^t [\bar{\bar{f}}(m,n,t-u) - A_1(m,n,t-u)]\, e^{ku\left[\frac{1-c^2q^2}{c^2}\right]} du$$

$$\tau_2(t) = \int_0^t \bar{\bar{g}}(m,n,t-u) - A_2(m,n,t-u) e^{ku\left[\frac{1-c^2q^2}{c^2}\right]} du$$

$$\tau_3(t) = \int_0^t [\bar{\bar{f}}(m,n,t-u) - A_1(m,n,t-u)]\, e^{-ku\left[q^2+\left(\frac{\varsigma\pi}{\xi}\right)^2\right]} du$$

$$\tau_4(t) = \int_0^t [\bar{\bar{g}}(m,n,t-u) - A_2(m,n,t-u) e^{-ku\left[q^2+\left(\frac{\varsigma\pi^2}{\xi}\right)^2\right]}\ du$$

$$A_1(m,n,t) = \left[\left(\chi + c\frac{d\chi}{dz}\right)_{z=\xi}\right],\quad A_2(m,n,t) = \left[\left(\chi + c\frac{d\chi}{dz}\right)_{z=0}\right],\ A_3(m,n,z,t) = L^{-1}[\chi]$$

Here $\bar{\bar{f}}\ (m,n,t)$ and $\bar{\bar{g}}(m,n,t)$ denote the Marchi – Fasulo transforms of $\bar{f}(m,z,t)$ and $\bar{g}(m,z,t)$ respectively. $\bar{f}(m,z,t)$ $\bar{g}(m,z,t)$ denote the finite Marchi – Fasulo transform of $f(x,z,,t)$ and $g(x,z,t)$ respectively.

$$\bar{\bar{f}}(m,n,t) = \int_{-h}^{h} \bar{f}(m,z,t)\, P_n(z)\, dz\ ,\ \bar{\bar{g}}(m,n,t) = \int_{-h}^{h} \bar{g}(m,z,t)\, P_n(z)\, dz\ ,\ \lambda_n = \int_{-h}^{h} P_n^2(z)\, dz$$

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$$

$$Q_n = a_n(\alpha_3 + \alpha_4)\cos(a_n h) + (\beta_3 - \beta_4)\sin(a_n h)$$

$$W_n = (\beta_3 + \beta_4)\cos(a_n h) + (\alpha_4 - \alpha_3)\, a_n \sin(a_n h)$$

Equation (17) is the desired solution of the given problem with $\beta_3 = \beta_4 = 1,\ \alpha_3 = k_3,\ \alpha_4 = k_4$.

Determination of airys stress function

Substituting the values of $T(x, y, z, t)$ from equation (17) in equation (4) one obtains

$$U(x,y,z,t) = \frac{\alpha Ek}{c^2}\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{[\phi_1(z)\,\tau_1(t) - \phi_2(z)\,\tau_2(t)]}{a_m^2 + a_n^2 - 1/c^2}\right]$$

$$+\frac{2\alpha Ek\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{1}{1+(c\varsigma\pi/\xi)^2}\right]$$

$$\times\left[\frac{\psi_1(z)\,\tau_3(t) - \psi_2(z)\,\tau_4(t)}{a_m^2 + a_n^2 + (\varsigma\pi/\xi)^2}\right] - \alpha E\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{A_3(m,n,z,t)}{a_m^2 + a_n^2 - l_0}\right] \quad \ldots(19)$$

Determination of displacement components

Substituting the values (19) in the equation (1) to (3) one obtains

$$u_x = \frac{\alpha k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{(k_1 + k_2) \sin 2a_m a}{\lambda_m} \right] \left[\frac{P_n(z)}{\lambda_n} \right] \left[\frac{(1+\nu) a_m^2}{a_m^2 + a_n^2 - 1/c^2} \right]$$

$$\times [\phi_1(y)\, \tau_1(t) - \phi_2(y)\, \tau_2(t)]$$

$$+ \frac{2\alpha k \pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{(k_1 + k_2) \sin 2a_m a}{\lambda_m} \right] \left[\frac{\varsigma}{\cos \varsigma \pi} \right] \left[\frac{P_n(z)}{\lambda_n} \right] \left[\frac{(1+\nu) a_m^2}{a_m^2 + a_n^2 + (\varsigma \pi / \xi)^2} \right]$$

$$\times \left[\frac{\psi_1(y) \tau_3(t) - \psi_2(y)\, \tau_4(t)}{1 + (c \varsigma \pi / \xi)^2} \right]$$

$$- \alpha \sum_{m,n=1}^{\infty} \left[\frac{(k_1 + k_2) \sin 2a_m a}{\lambda_m} \right] \left[\frac{P_n(z)}{\lambda_n} \right] \left[\frac{(1+\nu)\, a_m^2}{a_m^2 + a_n^2 - l_0} \right] \mathrm{A}_3(m, n, y, t) \qquad \ldots(20)$$

$$u_y = \frac{\alpha k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(z)}{\lambda_n} \right] \left[\frac{-(1+\nu)/c^2}{a_m^2 + a_n^2 - 1/c^2} \right] [\phi'_1(b)\, \tau_1(t) - \phi_2'(b)\, \tau_2(t)]$$

$$+ \frac{2\alpha k \pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(z)}{\lambda_n} \right] \left[\frac{\varsigma}{\cos(\varsigma \pi)} \right] \left[\frac{(1+\nu)\, (\varsigma \pi / \xi)^2}{a_m^2 + a_n^2 + (\varsigma \pi / \xi)^2} \right] \left[\frac{1}{(1 + (c \varsigma \pi / \xi)^2} \right]$$

$$\times [\psi'_1(b)\, \tau_3(t) - \psi'_2(b)\, \tau_4(t)]$$

$$- \alpha \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(z)}{\lambda_n} \right] \left[\frac{-(1+\nu)\, l_0}{a_m^2 + a_n^2 - l_0} \right] A'_3(m, n, b, t) \qquad \ldots(21)$$

$$u_z = \frac{\alpha k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{(k_3 + k_4) \sin 2a_n b}{\lambda_n} \right] \left[\frac{(1+\nu)\, a_n^2}{a_m^2 + a_n^2 - 1/c^2} \right]$$

$$\times [\phi_1(y)\, \tau_1(t) - \phi_2(y)\, \tau_2(t)]$$

$$+ \frac{2\alpha k \pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{(k_3 + k_4) \sin 2a_n b}{\lambda_n} \right] \left[\frac{\varsigma}{\cos \varsigma \pi} \right]$$

$$\left[\frac{(1+v)a_n^2}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right]\left[\frac{\psi_1(y)\tau_3(t)-\psi_2(y)\,\tau_4(t)}{1+(c\varsigma\pi/\xi)^2}\right]$$

$$-\alpha\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{(k_3+k_4)\sin 2a_n b}{\lambda_n}\right]\left[\frac{(1+v)\,a_n^2}{a_m^2+a_n^2-l_0}\right]A_3(m,n,y,t) \qquad \text{…(22)}$$

Where $\phi'_1(b)=\dfrac{\cosh(b/c)-\sinh(b/c)-1}{1/c\sinh(\xi/c)}$

$$\phi'_2(b)=\frac{\cosh((b-\xi)/c)-\sinh((b-\xi)/c)-\cosh(b/c)-\sinh(b/c)}{(1/c)\sinh(\xi/c)}$$

$$\psi'_1(b)=\frac{-\cos(\varsigma\pi/\xi)b-(c\varsigma\pi/\xi)\sin(\varsigma\pi/\xi)b+1}{(\varsigma\pi/\xi)}$$

$$\psi'_2(b)=\frac{-\cos(\varsigma\pi/\xi)(b-\xi)-(c\varsigma\pi/\xi)\sin(\varsigma\pi/\xi)(b-\xi)+\cos\varsigma\pi}{(\varsigma\pi/\xi)}$$

$$A'_3(m,n,h,t)=\int_0^h A_3(m,n,z,t)dz$$

Determination of stress function

Substituting values of (19) in equations (14) to (16) one obtains

$$\sigma_{xx}=\frac{\alpha Ek}{c^2}\sum_{m,n,=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-a_n^2+1/c^2}{a_m^2+a_n^2-1/c^2}\right][\phi_1(y)\,\tau_1(t)-\phi_2(y)\,\tau_2(t)]$$

$$+\frac{2\alpha Ek\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{-a_n^2-(\varsigma\pi/\xi)^2}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right]$$

$$\times\frac{[\psi_1(y)\,\tau_3(t)-\psi_2(y)\,\tau_4(t)]}{[1+(c\varsigma\pi/\xi)^2]}$$

$$-\alpha E\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{a_m^2-(k_0{}^2\pi^2/\xi^2)}{a_m^2+a_n^2-l_0}\right]A_3(m,n,y,t) \qquad \text{…(23)}$$

$$\sigma_{yy}=\left(\frac{\alpha Ek}{c^2}\right)\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-a_m^2-a_n^2}{a_m^2+a_n^2-1/c^2}\right][\phi_1(y)\,\tau_1(t)-\phi_2(y)\,\tau_2(t)]$$

$$+\frac{2\alpha Ek\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{-a_m^2-a_n^2}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right]$$

$$\times\left[\frac{\psi_1(y)\,\tau_3(t)-\psi_2(y)\,\tau_4(t)}{\left[1+(c\varsigma\pi/\xi)^2\right]}\right]$$

$$-\alpha E\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-a_m^2-a_n^2}{a_m^2+a_n^2-l_0}\right]A_3(m,n,y,t) \qquad \ldots(24)$$

$$\sigma_{zz}=\left(\frac{\alpha Ek}{c^2}\right)\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{1/c^2-a_m^2}{a_m^2+a_n^2-1/c^2}\right]\left[\phi_1(y)\,\tau_1(t)-\phi_2(y)\,\tau_2(t)\right]$$

$$+\frac{2\alpha Ek\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{-(\varsigma\pi/\xi)^2-a_m^2}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right]$$

$$\times\frac{\left[\psi_1(y)\,\tau_3(t)-\psi_2(y)\,\tau_4(t)\right]}{\left[1+(c\varsigma\pi/\xi)^2\right]}$$

$$-\alpha E\sum_{m,n,=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{l_0-a_m^2}{a_m^2+a_n^2-l_0}\right]A_3(m,n,y,t) \qquad \ldots(25)$$

Special case and numerical results

Set $f(x,z,t)=(1-e^{-t})(x+a)^2(x-a)^2(z+h)^2(z-h)^2$,

$g(x,z,t)=(1-e^{-t})(x+a)^2(x-a)^2(z+h)^2(z-h)^2e^b$,

$\delta=\dfrac{8(k_1+k_2)k\pi}{h^2}$, $a=1.5$, $k=0.86$, , $b=3,\ h=2,\ t=1$ sec in the equation (17) to obtain

$$\frac{T(x,y,z,t)}{\delta}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{\eta=1}^{\infty}(-1)^{(\eta+1/2)}\left(\eta+\frac{1}{2}\right)\left(\frac{P_n(x)}{\mu_n}\right)\left(\frac{P_m(z)}{\lambda_m}\right)\left(\frac{1}{1-q^2}\right)$$

$$\times\left[\frac{a_n\cos^2(a_n)-\cos(a_n)\sin(a_n)}{a_n{}^2}\right]\times\left[\Phi(y)e-\Psi(y)\right]$$

$$\times\int_0^t(1-e^{-t'})\,e^{-0.86\left(q^2+\left(\eta+\frac{1}{2}\right)^2\pi^2\right)(t-t')}\,dt' \qquad \ldots(26)$$

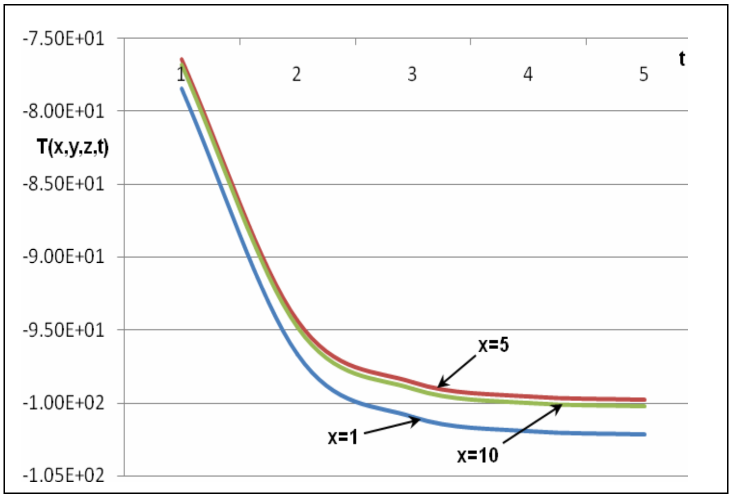

Fig. 1: T(x,y,z,t) versus t for different values of x

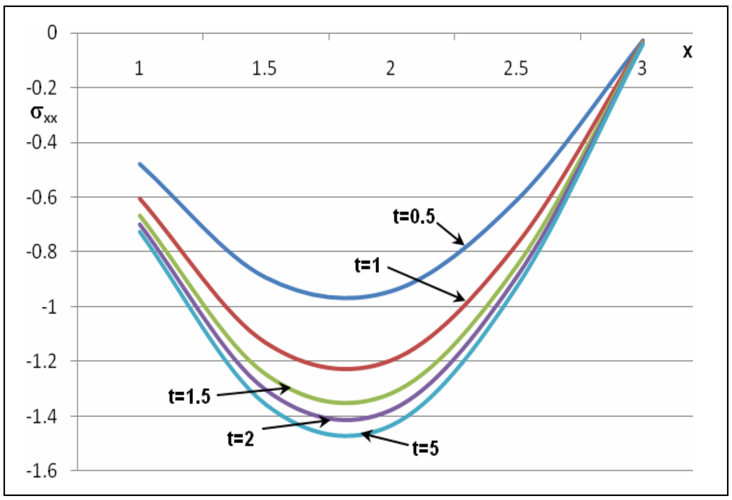

Fig. 2: σ_{xx} versus x for different values of t

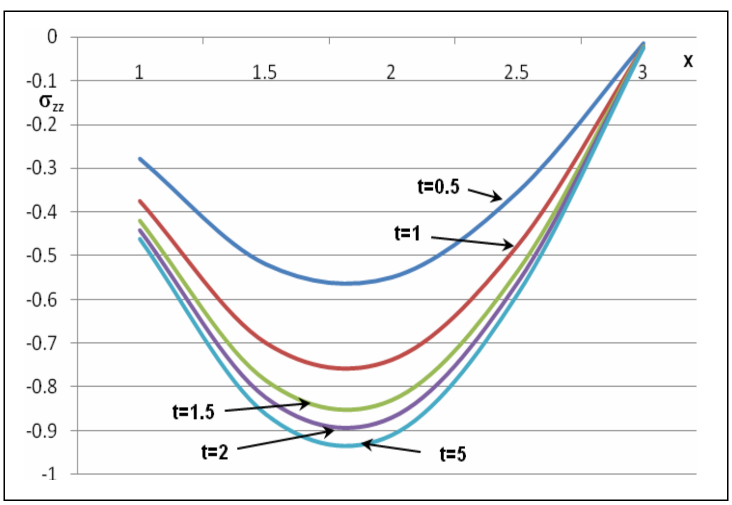

Fig. 3: σ_{zz} versus x for different values of t

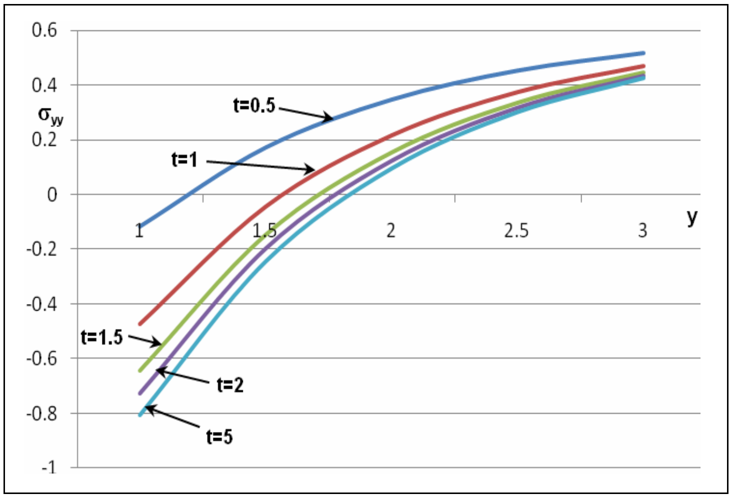

Fig. 4: σ_{yy} versus y for different values of t

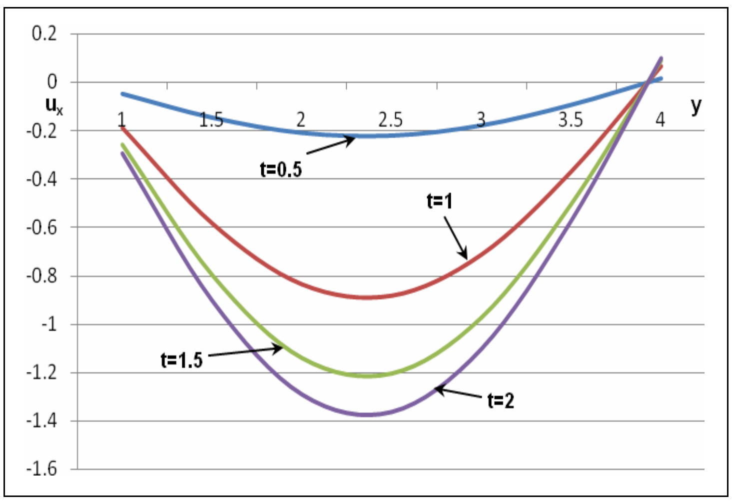

Fig. 5: u_x versus y for different values of t

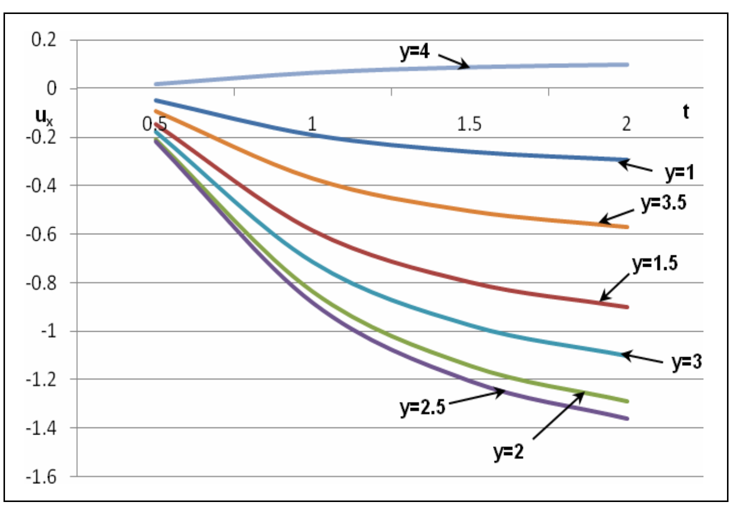

Fig. 6: u_x versus t for different values of y

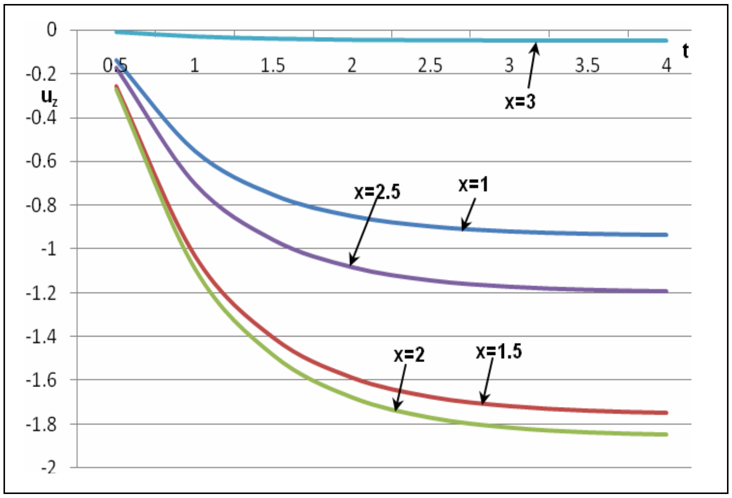

Fig. 7: u_z versus t for different values of x

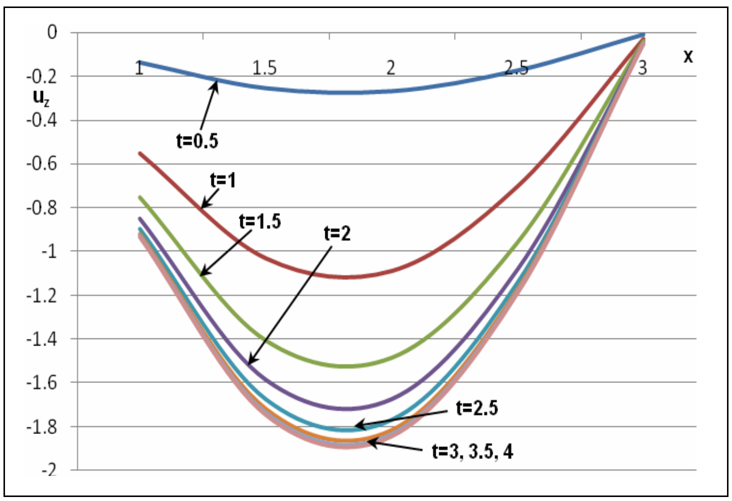

Fig. 8: u_z versus x for different values of t

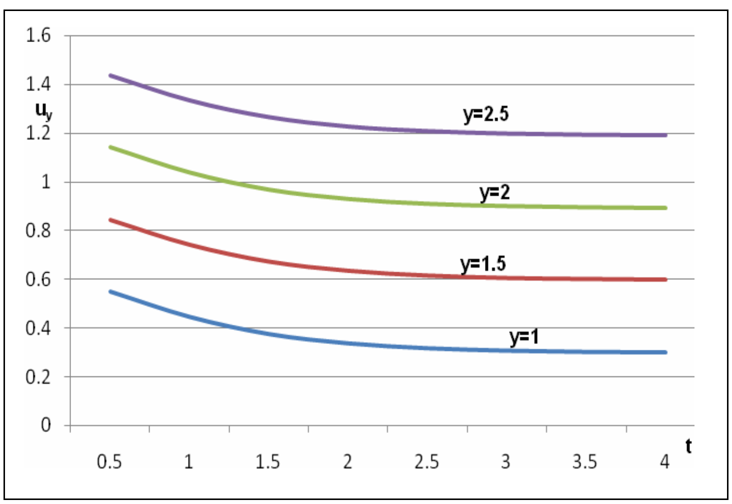

Fig. 9: u_y versus t for different values of y

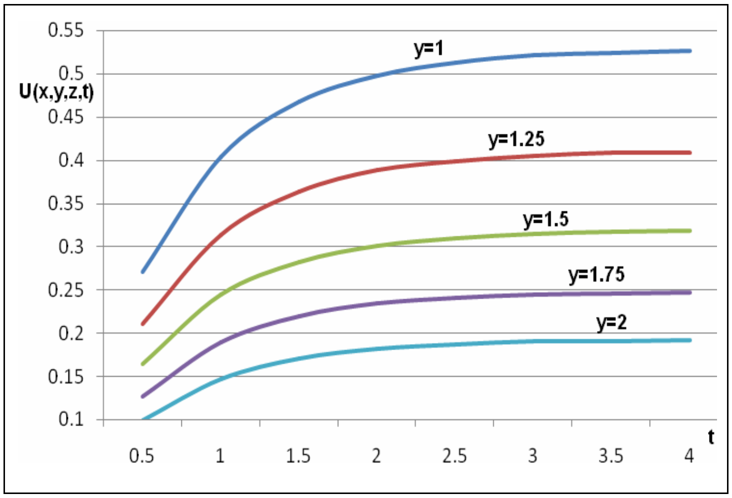

Fig. 10: U(x,y,z,t) versus t for different values of y

CONCLUSION

The temperature distribution, unknown temperature gradient, displacements, and thermal stresses on the edge y = b of a thin rectangular plate have been obtained, when the boundary conditions are known with the aid of finite Marchi-Fasulo transform and Laplace transform techniques.

The expressions are represented graphically. The results are obtained in the form of infinite series. It is observed that as x increases the temperature gradually decreases. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

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Appendix

The finite Marchi-Fasulo integral transform of f(z), -h < z < h is defined to be $\overline{F}(n) = \int_{-h}^{h} f(z) P_n(z)\,dz$ then at each point of (-h, h) at which f(z) is continuous,

$$f(z) = \sum_{n=1}^{\infty} \frac{\overline{F}(n)}{\lambda_n} P_n(z)$$

where $P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + (\alpha_2 - \alpha_1) a_n \sin(a_n h)$$

$$\lambda_n = \int_{-h}^{h} P_n^{\,2}(z)\,dz = h\left[Q_n^{\,2} + W_n^{\,2}\right] + \frac{\sin(2a_n h)}{2a_n}\left[Q_n^{\,2} - W_n^{\,2}\right]$$

The eigen values a_n are the solutions of the equation

$$\left[\alpha_1 a\cos(ah)+\beta_1\sin(ah)\right]\times\left[\beta_2\cos(ah)+\alpha_2 a\sin(ah)\right]$$

$$=\left[\alpha_2 a\cos(ah)-\beta_2\sin(ah)\right]\times\left[\beta_1\cos(ah)-\alpha_1 a\sin(ah)\right]$$

α_1, α_2, β_1 and β_2 are constants.

Moreover the integral transform has the following property:

$$\int_{-h}^{h}\frac{\partial^2 f(z)}{\partial z^2}P_n(z)dz=\frac{P_n(h)}{\alpha_1}\left[\beta_1 f(z)+\alpha_1\frac{\partial f(z)}{\partial z}\right]_{z=h}-\frac{P_n(-h)}{\alpha_2}\left[\beta_2 f(z)+\alpha_2\frac{\partial f(z)}{\partial z}\right]_{z=-h}$$

$$-a_n^{\ 2}\overline{F}(n).$$

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