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Thermal instability in anisotropic porous medium saturated by a nanofluid-A realistic approach

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ABSTRACT

Thermal instability in a horizontal layer of nanofluid saturated by anisotropic porous medium is investigated for realistic boundary conditions. The flux of volume fraction of nanoparticles is taken to be zero on the isothermal boundaries. The modified Darcy equation that includes the time derivative term used to model the momentum equation. A linear stability analysis based upon normal mode technique is used to study the onset of instabilities of nanofluid saturated by anisotropic porous medium. Rayliegh number on the onset of stationary convection has been derived using Galerkin method and graphs have been plotted for case of stationary convection to study the effects of the thermal anisotropy parameter, mechanical anisotropy parameter, Lewis number, modified diffusivity ratio, porosity and nanoparticles Rayleigh number on stationary convection. Oscillatory convection has been ruled under certain condtion.

KEYWORDS

Thermal instability;
Nanofluid;
Anisotropic porous medium;
Rayleigh number;
Galerkin method.

INTRODUCTION

The subject of thermal instability in porous medium has been studied extensively in recent years. There are many real world applications of thermal instability in porous medium for instance, in geophysics, food processing, oil reservoir modeling, petroleum industry, biomechanics, building of thermal insulations, nuclear reactors and many other areas. Theoretical and experimental results on the stability of cellular convection of a fluid layer in nonporous medium have been given by Chandrasekhar (1961). Lapwood[16] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding[25]. A good account of convection problems in a porous medium is given by Vafai and Hadim[24] and Nield and Bejan[17]. In geothermal system with a ground structure composed of many strata of different permeabilities, the overall horizontal permeability may be up to ten times as large as the vertical component. Processes such as sedimentation, compaction, frost action, and reorientation of the solid matrix are responsible for the creation of anisotropic natural porous media. Anisotropy can also be a characteristic of artificial porous material like pelleting used in chemical engineering process and fiber materials used in insulating purposes.

Nanofluid is the mixture of base fluid such as water or ethylene glycol and other coolants, oil and other lubricants, bio-fluids, polymer solutions etc. with a very small amount of nanoparticles or nanofibres such as metals or metallic oxides (Cu, CuO, Al_2O_3), metal carbides (SiC), nitrides (AlN, SiN) or metals (Al, Cu) etc. having dimensions from 1 to 100 nm. It was Choi[12] who first proposed this term "nanofluid." The convection of nanofluids based on model of Buongiorno[2] has been studied by Nield and Kuznetsov[18], Nield and Kuznetsov[13,14,19-21], Kuznetsov and Nield[13-15,19,20], Chand and Rana[5-7,10], Chand et al.[3,4,8,9], Chand[3,4] and Rana et al.[10,23] while Agarwal et al.[1] studied the effect of anisotropy on the onset of convection in a porous layer of nanofluid. All these study based upon Buongiorno model, which incorporates the effects of Brownian motion and thermophoresis. The choice of the boundary conditions imposed in these studies on nanoparticles volume fraction is somewhat arbitrary, it could be argued that zero-flux for nanoparticles volume fraction is more realistic. Recently Nield and Kuznetsov[22] studied the thermal instability of nanofluid in a porous medium by taking normal component of the nanoparticle flux zero at boundary which is more physically realistic. Zero-flux for nanoparticles mean one could control the value of the nanoparticle fraction at the boundary in the same way as the temperture there could be controlled.

In this paper an attempt has been made to study the thermal instability in a horizontal layer of nanofluid in an anisotropic porous medium by imposing nanoparticles flux zero at boundaries.

MATHEMATICAL FORMULATIONS OF THE PROBLEM

Consider an infinite horizontal layer of nanofluid of thickness 'd' bounded by planes z = 0 and z = d, heated from below in an anisotropic porous medium of medium permeability K and porosity ε as shown in Figure 1. Fluid layer is acted upon by gravity force **g** (0, 0, -g). The normal component of the nanoparticles flux has to vanish at an impermeable boundary and the temperature T is taken to be T_0 at z = 0 and T_1 at z = d, ($T_0 > T_1$). The reference scale for temperature and nanoparticles fraction is taken to be T_1 and φ_0 respectively. For simplicity, Darcy's Law is assumed to be hold and the Oberbeck-Boussinesq is employed. The mathematical equations describing the physical model are based upon the following assumptions.

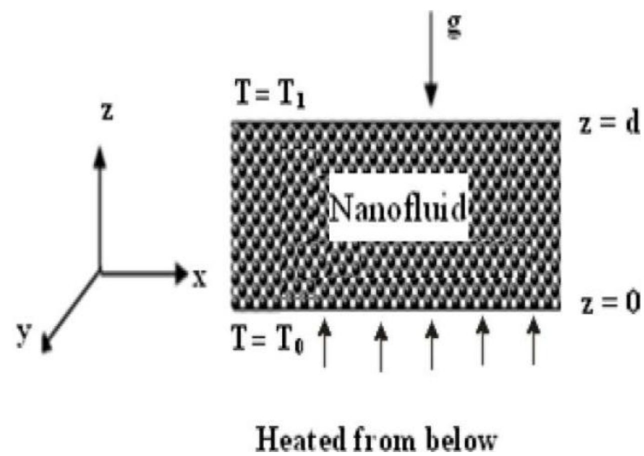

Figure 1 : Geometrical configuration of the problem

Assumptions

1) Nanoparticles are considered spherical in shape,
2) No chemical reaction in a horizontal layer of fluid,
3) Size of nanoparticles are small as compared to pore matrix,
4) Nanoparticles are spherical;
5) The porous medium is assumed to be possessing isotropy in the horizontal isotropy,
6) The fluid phase and nano particles are in thermal equilibrium state,
7) Radiation heat transfer between the sides of wall is negligible when compared with other modes of the heat transfer,
8) Nanoparticles are being suspended in the nanofluid using either surfactant or surface charge technology, preventing the agglomeration and deposition of these on the porous matrix.

Governing equations

According to the works of Chandrasekhar (1961), Nield and Kuznetsov[22] and Agarwal et al.[1], the governing equations in anisotropic porous medium are

$$\nabla \cdot \mathbf{q} = 0, \quad (1)$$

$$0 = -\nabla p + \{\varphi\rho_p + (1-\varphi)\rho(1-\alpha(T-T_0))\}\mathbf{g} - \mu K.\mathbf{q}, \quad (2)$$

wh ere **q** (u, v, w) is the Darcy velocity vector, ρ the density of nanofluid, ρ_p density of nanoparticles, p the hydrostatic pressure, φ the volume fraction of the nanoparticles, α is the coefficient of thermal expansion, μ is viscosity and $\mathbf{K}\left(= K_x^{-1}\left(\hat{i}\hat{i}+\hat{j}\hat{j}\right)+K_z^{-1}\left(\hat{k}\hat{k}\right)\right)$ the anisotropic permeability tensor; where K_x, denotes the permeability in x- direction and K_z permeability in z- direction.

The energy equation for nanofluid is given by

$$(\rho c)_m \frac{\partial T}{\partial t}+(\rho c)_f\, q\cdot\nabla T = k_m \nabla^2 T + \varepsilon(\rho c)_p\left(D_B \nabla\varphi\cdot\nabla T + \frac{D_T}{T_1}\nabla T\cdot\nabla T\right) \quad (3)$$

where $(\rho c)_m$ is heat capacity of fluid in porous medium, $(\rho c)_p$ is heat capacity of nanoparticles and k_m is thermal conductivity.

The equation of continuity for the nanoparticles is

$$\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}q\cdot\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T, \quad (4)$$

where D_B is the Brownian diffusion coefficient, given by Einstein-Stokes equation, D_T is the thermoporetic diffusion coefficient of the nanoparticles and ε is porosity in horizontal plane.

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. Thus boundary conditions[22] are

$$w=0,\quad T=T_0,\quad D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \ \text{at}\ \ z=0 \ \text{and}$$

$$w=0,\quad T=T_0,\quad D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \ \text{at}\ \ z=d. \quad (5)$$

We introduce non-dimensional variables as

$$(x',y',z')=\left(\frac{x,y,z}{d}\right),\ (u',v',w')=\left(\frac{u,v,w}{\kappa_{Tz}}\right)d,\ t'=\frac{t\kappa_{Tz}}{\sigma d^2},$$

$$p'=\frac{p\kappa_{Tz}}{\mu K_z},\ \varphi'=\frac{(\varphi-\varphi_0)}{\varphi_0},\ T'=\frac{(T-T_1)}{(T_0-T_1)},$$

where $\kappa_T=\frac{k_m}{(\rho c)_p}$ is effective thermal diffusivity of the fluid.

There after dropping the dashes (') for simplicity.

Equations (1) - (5) in non-dimensional form can be written as

$$\nabla\cdot q = 0, \quad (6)$$

$$0 = -\nabla p - q_a - Rm\hat{e}_z + RaT\hat{e}_z - Rn\varphi\hat{e}_z \quad (7)$$

$$\frac{\partial T}{\partial t}+q.\nabla T=\left(\eta\nabla_h^2+\frac{\partial^2}{\partial z^2}\right)T+\frac{N_B}{Le}\nabla\varphi.\nabla T+\frac{N_A N_B}{Le}\nabla T.\nabla T, \quad (8)$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}q.\nabla\varphi=\frac{1}{Le}\nabla^2\varphi+\frac{N_A}{Le}\nabla^2 T, \quad (9)$$

where $Le=\frac{\kappa_T}{D_B}$ is Lewis number; $q_a=\left(\frac{1}{\xi}u,\frac{1}{\xi}v,w\right)$ is anisotropic modified velocity vector; $\xi=\frac{K_x}{K_z}$ is mechanical anisotropy parameter; $\eta=\frac{\kappa_{Tx}}{\kappa_{Tz}}$ is thermal anisotropy parameter; $Ra=\frac{\rho g\alpha d K_z(T_0-T_1)}{\mu\kappa_{Tz}}$ is Rayleigh Number; $Rm=\frac{(\rho_p\varphi_0+\rho(1-\varphi_0))gdK_z}{\mu\kappa_{Tz}}$ is density Rayleigh number; $Rn=\frac{(\rho_p-\rho)\varphi_0 g k_1 d}{\mu\kappa_{Tz}}$ is nanoparticles Rayleigh number; $N_A=\frac{D_T(T_0-T_1)}{D_B T_1\varphi_0}$ is modified diffusivity ratio, $N_B=\frac{\varepsilon(\rho c)_p\varphi_0}{(\rho c)_f}$ is modified particle-density increment.

The dimensionless boundary conditions are

$$w=0,\quad T=1,\quad \frac{\partial\varphi}{\partial z}+N_A\frac{\partial T}{\partial z}=0\ \text{at}\ \ z=0 \quad \text{and}$$

$$w=0,\quad T=0,\quad \frac{\partial\varphi}{\partial z}+N_A\frac{\partial T}{\partial z}=0\ \text{at}\ \ z=1. \quad (10)$$

Basic solutios

The basic state was assumed to be quiescent and is given by

$$u=v=w=0,\quad p=p(z),\quad T=T_b(z)\quad \varphi=\varphi_b(z).$$

Equations (6) – (9) using boundary condition (10) give

$$T_b=1-z,\ \ \varphi_b=\varphi_0+N_A z, \quad (11)$$

where φ_0 is reference value for nanoparticles volume fraction.

The basic is identical with solutions obtained by Nield and Kuznetsov[22]while basic solution for the nanoparticles volume fraction is different than Agarwal et al.[1].

Perturbation solutions

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are of the forms

$$q(u,v,w)=0+q'(u,v,w), T=T_{b}+T', \varphi=\varphi_b+\varphi', p=p_b+p', \quad \text{with } T_b=1-z, \varphi_b=\varphi_0+N_A Z. \tag{12}$$

There after dropping the dashes (') for simplicity.

Using the equation (11) in the equations (6) - (9), we obtain the linearized perturbation (neglecting the product of the prime quantities) equations as

$$\nabla\cdot\mathbf{q}=0, \tag{13}$$

$$0=-\nabla p-\mathbf{q}_a+RaT\hat{e}_z-Rn\varphi\hat{e}_z, \tag{14}$$

$$\frac{\partial T}{\partial t}-w=\left(\eta\nabla_h^2+\frac{\partial^2}{\partial z^2}\right)T+\frac{N_B}{Le}\left(N_A\frac{\partial T}{\partial z}-\frac{\partial\varphi}{\partial z}\right)-\frac{2N_AN_B}{Le}\frac{\partial T}{\partial z}, \tag{15}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}w=\frac{1}{Le}\nabla^2\varphi+\frac{N_A}{Le}\nabla^2 T \tag{16}$$

It will be noted that the parameter R_m is not involved in these and subsequent equations. It is just a measure of the basic static pressure gradient.

Eliminating 'p' from equation (14), we have

$$\left(\nabla_H^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w=Ra\nabla_H^2T-Rn\nabla_H^2\varphi, \tag{17}$$

where ∇_H^2, is two-dimensional Laplacian operator on horizontal plane.

NORMAL MODES AND STABILITY ANALYSIS

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w,T,\varphi]=[W(z),\Theta(z),\Phi(z)]\exp(ik_x x+ik_y y+nt), \tag{18}$$

where k_x, k_y are wave numbers in x and y directions respectively and n is growth rate of disturbances.

Using equation (18), equations (17), (15) – (16) become

$$\left(\frac{1}{\xi}D^2-a^2\right)W+a^2Ra\Theta-a^2Rn\Phi=0, \tag{19}$$

$$W+\left(D^2-\eta a^2-n+\frac{N_AN_B}{Le}D-\frac{2N_AN_B}{Le}D\right)\Theta-\frac{N_B}{Le}D\Phi=0, \tag{20}$$

$$\frac{W}{\varepsilon}-\frac{N_A}{Le}(D^2-a^2)\Theta-\left(\frac{1}{Le}(D^2-a^2)-\frac{n}{\sigma}\right)\Phi=0. \tag{21}$$

Bboundary conditions are

$$w=0,\quad T=0,\quad \frac{\partial\varphi}{\partial z}+N_A\frac{\partial T}{\partial z}=0 \text{ at } z=0,1. \tag{22}$$

Where $D=\frac{d}{dz}$ and $a=\sqrt{k_x^2+k_y^2}$ is dimensionless horizontal resultant wave number.

METHOD OF SOLUTION

The Galerkin weighted residuals method is used to obtain an approximate solution to the system of equations (19) – (21) with boundary conditions (22). In this method, the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W=\sum_{p=1}^{n}A_pW_p,\ \Theta=\sum_{p=1}^{n}B_p\Theta_p,\ \Phi=\sum_{p=1}^{n}C_p\Phi_p. \tag{23}$$

Where A_p, B_p and C_p are unknown coefficients, p =1, 2, 3,...N and the base functions W_p, Θ_p and Φ_p are assumed in the following form

$$W_p=\Theta_p=z^p-z^{p+1},\ \Phi_1=N_A(z^2-z) \quad \text{and} \quad \Phi_p=\frac{1}{2}N_Az^2, p=2,3,4...... \tag{24}$$

such that W_p, Θ_p and Φ_p satisfy the corresponding boundary conditions. Using expression for W, Θ and Φ in equations (19) – (21) and multiplying first equation by W_p second equation by Θ_p and third by Φ_p and integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations in 3N unknown A_p, B_p and C_p; p =1,2,3,...N. For existing of non trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number Ra.

STABILITY ANALYSIS

We confine ourselves to the one- term Galerkin approximation. Thus eigenvalue equation is given by

$$a^2Ra\left((a^2+10)+\frac{nLe}{\sigma}\right)+a^2Rn\left(N_A(a^2+10)+\frac{Le}{\varepsilon}(\eta a^2+10+n)\right)-\left(a^2+\frac{10}{\xi}\right)\left((a^2+10)+\frac{nLe}{\sigma}\right)(\eta a^2+10+n)=0. \tag{25}$$

For neutral stability the real parts of the n is zero. Hence on putting n = iω, (where ω is real and dimensionless quantity) in equation (25), we have

$$a^2Ra\left(\left(a^2+10\right)+\frac{i\omega Le}{\sigma}\right)+a^2Rn\left(N_A\left(a^2+10\right)+\frac{Le}{\varepsilon}\left(\eta a^2+10+i\omega\right)\right)$$
$$-\left(a^2+\frac{10}{\xi}\right)\left(\left(a^2+10\right)+\frac{i\omega Le}{\sigma}\right)\left(\eta a^2+10+i\omega\right)=0. \quad (26)$$

Stationary convection

For stationary convection $\omega = 0$ with one- term Galerkin approximation, equation (25) reduces to

$$Ra=\frac{1}{a^2}\left(a^2+\frac{10}{\xi}\right)\left(\eta a^2+10\right)-\frac{\left(a^2+10\right)N_A+\left(\eta a^2+10\right)\frac{Le}{\varepsilon}}{\left(a^2+10\right)}Rn \quad (27)$$

It is clear from equation (27) that stationary Rayleigh number Ra is a function of dimensionless wave number a, thermal anisotropy parameter η, mechanical anisotropy parameter ξ, Lewis number Le, modified diffusivity ratio N_A, porosity ε and nanoparticles Rayleigh number Rn independent of modified particle -density increment parameter N_B. Thus instability is purely phenomena due to buoyancy coupled with conservation of nanoparticles. Thus average contribution of nanoparticles flux in the thermal energy equation is zero.

The critical cell size at the onset of instability is obtained from the condition $\left(\frac{\partial Ra}{\partial a}\right)_{a=a_c}=0,$ which gives

$$\left(\eta a_c^4-\frac{100}{\xi}\right)\left(a^2+10\right)^2-\left(10\eta-10\right)a_c^2\frac{Le}{\varepsilon}Rn=0. \quad (28)$$

Thus critical wave number a_c depends upon thermal anisotropy parameter η, mechanical anisotropy parameter ξ, Lewis number Le, modified diffusivity ratio N_A, porosity ε and nanoparticles Rayleigh number Rn and independent of modified particle -density increment parameter N_B. The interweaving behaviors' of Brownian motion and thermoporesis of nanoparticles evidently does not change the critical size of the Bénard cell at the onset of instability. As such, the critical size is not a function of any thermo physical properties of nanofluid.

In the absence nanoparticles [Rn = Le = N_A = 0] i.e. for ordinary fluid, critical wave number is given by $a_c=\sqrt{\frac{10}{\sqrt{\eta\xi}}}$ and the corresponding critical Rayleigh number is given by $Ra_c=10\left(\frac{\sqrt{\eta}}{\xi}+1+\frac{\eta}{\xi}+\frac{\sqrt{\eta}}{\sqrt{\xi}}\right).$

For isotropic porous medium i.e. if $\xi=\eta=1$, the critical wave number $a=\sqrt{10}$ and corresponding critical Rayleigh number $Ra_c=40$.

Thus in the absence of nanoparticles [Rn = Le = N_A = 0] for the case of isotropic porous medium [$\xi=\eta=1$] the critical Rayleigh number is given by Ra_c = 40, which slightly greater than critical Rayleigh number $Ra_c = 4\pi^2$, result obtained by Lapwood (1948) for regular fluid.

Oscillatory convection

For oscillatory convection we have $\omega \# 0$, thus on equating the real and imaginary parts of equation (26), we have

$$a^2Ra\left(a^2+10\right)+a^2Rn\left(\left(a^2+10\right)N_A+\left(\eta a^2+10\right)\frac{Le}{\varepsilon}\right)=$$
$$\left(a^2+\frac{10}{\xi}\right)\left(\left(\eta a^2+10\right)\left(a^2+10\right)-\frac{\omega^2Le}{\sigma}\right) \quad ,(29)$$

and

$$\omega\left(a^2Ra\frac{Le}{\sigma}+a^2Rn\frac{Le}{\varepsilon}-\left(a^2+\frac{10}{\xi}\right)\left(\left(a^2+10\right)+\left(\eta a^2+10\right)\frac{Le}{\sigma}\right)\right)=0. \quad (30)$$

In order for ω to be real it is necessary that

$$a^2Ra\left(a^2+10\right)+a^2Rn\left(\left(a^2+10\right)N_A+\left(\eta a^2+10\right)\frac{Le}{\varepsilon}\right)\leq$$
$$\left(a^2+\frac{10}{\xi}\right)\left(\left(\eta a^2+10\right)\left(a^2+10\right)\right) \quad . \quad (31)$$

As we have noted that for typical nanofluid Le is of order 10^2-10^3 and N_A is not greater than 10, Rn are of order 10-10^2 and ε, ξ and η are of the order 10^{-3} - 10^{-1}. Under these approximations inequality (31) does not hold if

$$a^2Ra\left(a^2+10\right)+a^2Rn\left(\left(a^2+10\right)N_A+\left(\eta a^2+10\right)\frac{Le}{\varepsilon}\right)\geq$$
$$\left(a^2+\frac{10}{\xi}\right)\left(\left(\eta a^2+10\right)\left(a^2+10\right)\right) \quad \text{and}$$

hence ω is not real.

Thus oscillatory convection has been ruled out if

$$a^2Ra\left(a^2+10\right)+a^2Rn\left(\left(a^2+10\right)N_A+\left(\eta a^2+10\right)\frac{Le}{\varepsilon}\right)\geq.$$

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$$\left(a^2+\frac{10}{\xi}\right)\left((\eta a^2+10)(a^2+10)\right)$$

To study the influence of thermal anisotropy parameter η, mechanical anisotropy parameter ξ, Lewis number Le, modified diffusivity ratio N_A and porosity ε on the stationary convection for the case of bottom-heavy distribution of nanoparticles [negative value of Rn], we examine the behaviour of $\frac{\partial Ra}{\partial \eta}$, $\frac{\partial Ra}{\partial \xi}$, $\frac{\partial Ra}{\partial Le}$, $\frac{\partial Ra}{\partial N_A}$ and $\frac{\partial Ra}{\partial \varepsilon}$ analytically.

From equation (27), we have

$$\mathbf{Ra}=\frac{1}{a^2}\left(a^2+\frac{10}{\xi}\right)(\eta a^2+10)-\frac{(a^2+10)N_A+(\eta a^2+10)\frac{Le}{\varepsilon}}{(a^2+10)}\mathbf{Rn}.$$

1) $\frac{\partial Ra}{\partial \eta}=\left(a^2+\frac{10}{\xi}\right)-\frac{a^2}{(a^2+10)}\frac{Le}{\varepsilon}Rn,$ As we have noted that for typical nanofluid Le is of order 10^2-10^3, ε, ξ and η are of the order 10^{-3} - 10^{-1} and Rn <0 [for a bottom-heavy distribution of nanoparticles]. Under these approximations $\frac{\partial Ra}{\partial \eta}>0$, thus thermal anisotropy parameter η has stabilizing effect on the stationary convection.

2) $\frac{\partial Ra}{\partial \xi}<0$, thus mechanical anisotropy parameter ξ destabilize the stationary convection.

3) $\frac{\partial Ra}{\partial \varepsilon}<0$, thus porosity destabilizes the stationary convection.

4) $\frac{\partial Ra}{\partial Le}>0$, thus Lewis number stabilize the stationary convection.

5) $\frac{\partial Ra}{\partial N_A}>0$, which mean that modified diffusivity ratio stabilize the stationary convection.

RESULTS AND DISCUSSION

The thermal instability in a horizontal layer of nanofluid is investigated. The expression for the stationary Rayleigh number is given by equation (27). The graphical representation of the effects of anisotropic parameters on stationary convections for a bottom-heavy distribution of nanoparticles [negative value of Rn] ares given in Figures 2- 6.

Figure 2 indicates the effect of thermal anisotropy parameter η, on the stationary convection for the fixed value of Le = 500, ξ = 0.6, ε = 0.4, N_A = 5, Rn = - 1 and it is found that the critical Rayleigh number increases with increase in the value of thermal anisotropy parameter η, indicating that the effect of thermal anisotropy parameter η is to inhibit the onset of convection.

Figure 3 indicates the effect of mechanical anisotropy parameter ξ on the stationary convection for the fixed value of Le = 500, η = 0.8, ε = 0.4, N_A = 5, Rn = - 1 and it is found that the stationary Rayleigh number decreases with increase in the value of mechanical anisotropy parameter ξ, thus mechanical anisotropy parameter ξ is to advance the onset of stationary convection.

Figure 4 shows the variation of Rayleigh number with wave number for the fixed value of Le = 500, η = 0.7, ξ = 0.4, N_A = 5, Rn = - 1 and different value of the porosity and it is found that the Rayleigh number decreases with increase in the value of porosity thus porosity destabilize the stationary convection. This is good agrement of the result obtained by Chand and Rana[5].

Figure 5 shows the variation of stationary Rayleigh number Ra with wave number a for for the fixed value

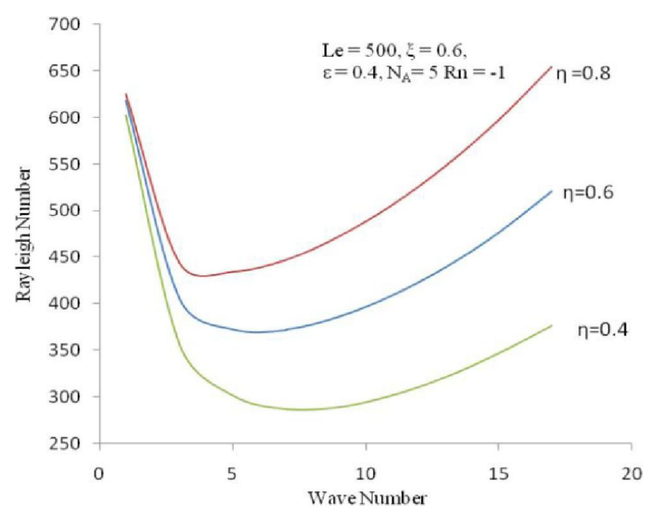

Figure 2 : Variation of stationary Rayleigh number with wave number for different values of thermal anisotropy parameter η

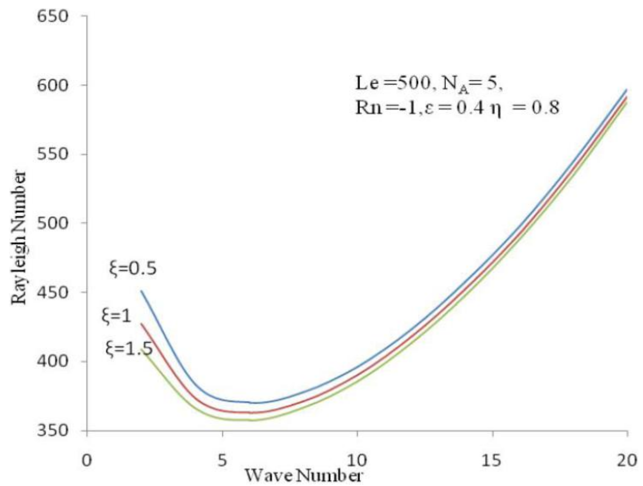

Figure 3 : Variation of stationary Rayleigh number with wave number for different values of mechanical anisotropy parameter ξ

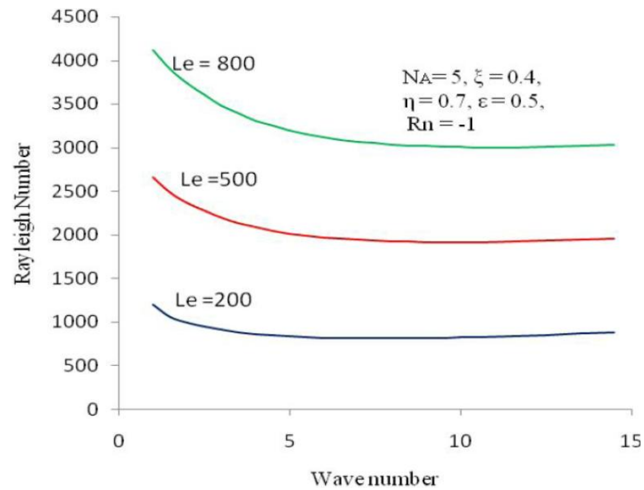

Figure 5 : Variation of stationary Rayleigh number with wave number for different values of Lewis number

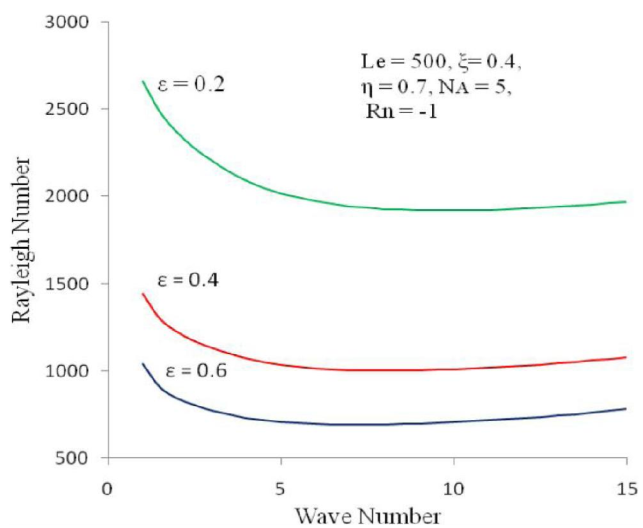

Figure 4 : Variation of stationary Rayleigh number with wave number for different values of porosity ε

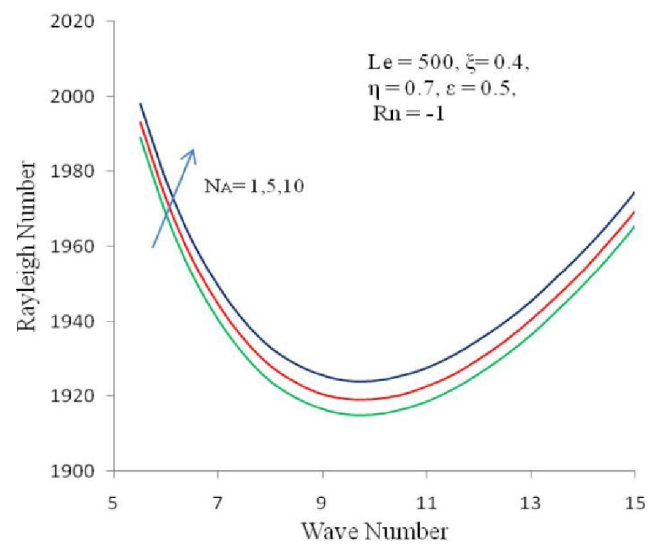

Figure 6 : Variation of stationary Rayleigh number with wave number for different values of modified diffusivity ratio NA

of η = 0.7, ξ = 0.4, ε = 0.5, N_A= 5, Rn = - 1 and different values of Lewis number. It is found that the Rayleigh number Ra increases as values of Lewis number inceases. Thus, Lewis number stabilizes the stationary convection. This is good agrement of the result obtained by Chand and Rana (2012a).

Figure 6 shows the variation of stationary Rayleigh number Ra with wave number a for fixed value of η = 0.7, ξ = 0.4, ε = 0.5, Le = 500, Rn = - 1 and different values of modified diffusivity ratio N_A. It is found that the Rayleigh number Ra increases as values of modified diffusivity ratio inceases. Thus, modified diffusivity ratio stabilizes the stationary convection. This is good agrement of the result obtained by Chand and Rana[5].

NOMENCLATURE

a	wave number
d	depth of fluid layer
D_B	diffusion coefficient
D_T	thermophoretic diffusion coefficient
g	gravity force
K	Permeability tensor
K_x,	permeability in x- direction
K_z	permeability in z- direction
Le	Lewis number
n	growth rate of disturbances
N_A	modified diffusivity ratio
N_B	modified particle -density increment

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p pressure
q Darcy velocity vector
$\mathbf{q}_a$ anisotropic modified velocity vector
Ra thermal Rayleigh Darcy number
Ra_c critical Rayleigh Darcy number
Rm density Rayleigh number
Rn concentration Rayleigh number
t time
T temperature
(u, v, w) velocity components
(x, y, z) space co-ordinates

Greek symbols

α thermal expansion coefficient
μ viscosity
ε porosity along horizontal plane
σ heat capacity ratio
ρ density of the nanofluid
ρ_p density of nano particles and
$(\rho c)_m$ heat capacity in porous medium
$(\rho c)_p$ heat capacity of nano particles
φ volume fraction of the nanoparticles
ξ, mechanical anisotropy parameter
η thermal anisotropy parameter
κ_T thermal diffusivity tensor
ω frequency of oscillation
∇_H two-dimensional Laplacian operator

Superscripts

' non-dimensional variables

Subscripts

p particle
f fluid
b basic solution
h horizontal plane

CONCLUSIONS

Thermal instability in a horizontal layer of nanofluid an in anisotropic porous medium is investigated. The influences of anisotropic parameters and other parameters on the stationary have been investigated both analytically and graphically.

The main conclusions are:

1) Basic solution for the nanoparticles volume fraction is changed with zero-flux of volume fraction of nanoparticles
2) Presence of nanoparticles decreases the stability of system.
3) Oscillatory convection has been ruled out if

$$a^2Ra\left(a^2+10\right)+a^2Rn\left(\left(a^2+10\right)N_A+\left(\eta a^2+10\right)\frac{Le}{\varepsilon}\right)\geq$$
$$\left(a^2+\frac{10}{\xi}\right)\left(\left(\eta a^2+10\right)\left(a^2+10\right)\right)$$

4) The presence of the nanoparticles lowers the value of the critical Rayleigh number by usually by substantial amount. Thus nanofluid is less stable as compared to regular fluid.
5) Thermal anisotropy parameter, Lewis number and modified diffusivity ratio stabilize the stationary convection for the case of bottom-heavy distribution of nanoparticles [negative value of Rn].
6) Porosity and mechanical anisotropy parameter destabilizes the stationary convection for the case of bottom-heavy distribution of nanoparticles [negative value of Rn].

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