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Theoretical Power Per Unit Volume And Unbaffled Stirred Tanks

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ABSTRACT

Present paper shows the importance of theoretical power per unit volume concept in scale up studies in stirred tanks. Developed by Rao(1999), theoretical power per unit volume parameter, X is now a basis for scale up studies in stirred tank. The usual similitude either with Reynolds number or Froude number is singularly not valid to scale up the parameters of the stirred tanks. Interestingly X contains both Reynolds and Froude number. Two major numbers enough to describe any fluid flow phenomena. Present work goes deeper in the observation of X and based on the observations, it is said that X describes the dissipation rate of turbulent kinetic energy of the smallest eddies in turbulent flow. Representations on scaling ability of X have been shown by scaling the actual power consumption in square and circular surface aerators.

KEYWORDS

Froude number;
Reynolds number;
Surface aerator;
Theoretical power per unit volume;
Wastewater treatment.

INTRODUCTION

The purposes of aeration and agitation in biological reactors are, firstly to supply microorganisms with oxygen, and secondly, to mix fermentation broth in such a way that a uniform suspension of microbes is achieved and the mass-transfer of the metabolic product accelerated. Aeration and agitation can be achieved both with bubble and mechanical agitation. Mechanical agitation, especially surface aerators, is largely used in biological wastewater treatment processes. Although the technique has been in use for several decades, intensive research is being conducted in the directions of finding the most adequate scale-up criteria.

Entrainment of gas from a gas-liquid surface is known as surface aeration. Stirred reactors designed for this type of gas-liquid contact are called surface aerators. For effective aeration, the impeller is located near the free liquid surface. In surface aerators, water (polluted water) in a given shape of vessels is agitated by rotating the impeller or rotor. The physical and chemical processes taking place in the aeration tank are complex and closely coupled to the underlying transport processes, in particular-the flow field. Therefore, a detailed understanding of the hydrodynamics of aeration tank(velocity field, turbulence, stress field etc.) is useful for optimum design.

Aeration is usually the single largest cost in a wastewater treatment system comprising as much as 50to90 percent of the total energy requirements of a wastewater treatment plant[23]. Therefore the task beforehand is to find the optimal or efficient criterion for scaling up the parameters involved in the experiments of stirred tank.

"Scale-up" is an inherent part of process development. Scale-up has been successfully achieved when yields and productivities, previously demonstrated on a small scale, have been produced in larger capacity units. Two basic approaches to scale-up are available; first, extrapolation of model experiments based on the principles of similitude and, second, mathematical analysis of the complete(or controlling) mechanism[10,13]. While the second of these has unlimited potential value, it also has serious limitations in practice. Often the relationships are too involved to permit rigorous definition or the resultant mathematical expressions are too complex for economical solution, even with computing equipment. Electrical or mechanical analogs may sometimes be used to overcome the second of these limitations, but their application to microbiological processes is still a long way off. This leaves us with scale-up from model experiments by the principles of similitude as the main technique in practice. To scale-up by similitude, one must first establish some functional relationship between the various dimensionless parameters which can be used to characterize the system. There are several strategies frequently used to scale up the stirred tanks phenomena, i.e. constant reactor geometry, constant volumetric oxygen transfer coefficient, constant maximum shear, constant power per unit volume, constant mixing time, constant Reynolds number etc. Among these strategies, constant reactor geometry is a kind of basic requirements for scale up the operation and is called geometric similarity. After maintaining geometric similarity, scale up of the operational variables such as oxygen transfer coefficient, power consumption, rotor speed etc. are required for optimal scale up of the process.

Recently, theoretical power per unit volume proposed by Rao[15] is becoming the main criteria for scaling up the dynamic parameter of the stirred tanks. Present work tries to show that the theoretical power per unit volume is indeed a basic kind of scale up strategy and also explores its theoretical background and importance in scaling up the power consumption in stirred tanks.

Theoretical power per unit volume, X

Many investigators[1,7,22] have suggested that power per unit volume is a useful concept to simulate oxygen transfer in geometrically similar systems. It is also suggested that if the "power per unit volume" remains the same in geometrically similar tanks of different sizes, the dynamic parameters also remain the same in all the tanks.

Power is required to agitate the water body in the tank by rotating the rotor or impeller blades. According to the basic hydrodynamic principles, the power P is generally expressed as $P \approx Qh_f$, where Q is the discharge of water being pumped by the rotation of the impeller blades of dimensions b and l and the head loss h_f due to rotational movement of water in the tank. The characteristic tangential velocity of water v may be considered as proportional to ND. Because of geometric similarity, $b \approx D$ and $l \approx D$, then the cross-sectional area of the blades $bl \approx D^2$. Therefore the water discharge due to the rotor action can be expressed as $Q \approx ND^3$. The head loss h_f may be considered to be proportional to the velocity head $v^2/2g$, where $v \approx ND$, and g is the gravitational constant. Therefore the power $P \approx N^3D^5$. From the geometric similarity as, the volume of water in the tank $V \approx D^3$. Therefore, the power per unit volume, P/V, is proportional N^3D^2. The dimensional parameter N^3D^2 is converted to a non-dimensional parameter, X, by expressing it in terms of $F = N^2D/g$, and $R = ND^2/\nu$ as

$$X = N^3D^2/(g^{4/3}\nu^{1/3}) = F^{4/3}R^{1/3} \quad (1)$$

Here, $X = F^{4/3}R^{1/3}$ is considered as the parameter representing the theoretical power per unit volume[15].

The intensity of turbulence and wave action on the water are the major sources normally associated with surface aeration. Turbulence and viscous effects are generally described by the Reynolds number(R), where the surface wave action is described by the

Froude number(F). As shown in Equation(1), X is a combination of both R and F. Generally unbaffled tanks are being employed for surface aerators, because unbaffled tanks give rise to higher fluid-particle mass transfer rates for a given power consumption[5], which is the paramount importance in designing aeration system. In the free surface unbaffled tanks, existence of central vortex plays an important role in enhancing the mass transfer process [11,9]. As far as the vortex system is concerned, it is required to incorporate the Froude number in the analysis[19]. For unbaffled tanks, the vortex phenomenon increases with the Reynolds number. The influence of Froude number becomes important in this case[8]. Extensive works have been published in the literature to correlate the stirred tank's dynamic parameter with Reynolds number(R). However, these correlations do not fully represent the phenomena associated with the unbaffled tank. In the case of unbaffled system, there is a necessity of incorporating the Froude number(F).

Significance of X Si

The mean flow in stirred reactors can be characterized by power consumption in stirred tanks and by the velocity and turbulence properties in the vicinity of the impeller[26]. Concern with the local rates of energy dissipation is based on frequently made observation that power per unit volume is a very useful criterion of agitation. Kolmogoroff theory of local isotropy[6] provides an explanation of this observation in terms of the intensity of the small-scale eddies in the turbulent flow, since the theory predicts that those eddies are isotropic and their intensity in a turbulent flow depends only on the local rate of energy dissipation.

The determination of the local dissipation rate of turbulent kinetic energy(ε is important. However, the direct measurement of ε is very difficult, since it needs to capture precisely the smallest turbulent structures[20](Saarentine and Piirto, 2000). In the past, several methods were developed to estimate the dissipation rate of turbulent kinetic energy:

- Kinetic energy balance term averaged over a control volume[4,25,28]
- Integration of dissipation spectrum[14,5]
- Dimensional analysis[28]

The dimensional argument is based on the fact, that all the kinetic energy which is transferred from the large-scale eddies must be dissipated by the small-scale eddies. The dissipation rate is given(Hinze, 1959).

$$\varepsilon = Au'^3/L \quad (2)$$

Where, A is a constant, u' is a turbulent velocity characteristic and L is a length-scale characteristic. The length scale, L, can be taken as proportional to the impeller diameter, D:(a) L=D[3] (b) L=D/2[21] and (c) L=D /10[12].

However, this dimensional analysis assumes that the turbulence is fully developed. The larger free eddies formed in the usual turbulence production by an impeller are not isotropic, however, though as the energy is transferred to smaller and smaller eddies, the geometric orientation becomes lost, and the very small eddies produced are of random orientation, i.e., they are isotropic. This range of turbulent eddy spectrum is independent of the details of the agitation and depends only on the power supplied and on viscous dissipation.

Now in Equation(2)u can be represented as ND, where N is the rotation of impeller of diameter, D.

$$\varepsilon \approx (ND)^3/D \quad (3)$$

$$\varepsilon \approx N^3D^2 = X \quad (4)$$

(ε also can be non-dimensionalized as the same way as X

Based on the Equation 4, we come to see that the parameter, X is a very powerful parameter dealing with agitation and it can be used in scaling up the stirred tanks. In fact Rao[15], Rao et al.[16], Rao and Kumar[17] and Rao and Kumar[18] have shown this parameter's usefulness in scaling up the mass transfer and power number of the stirred tanks.

Scaling up the parameter

Experiments were carried out with an objective to show the scaling capability of X on power consumption in a stirred tank. Power draw is a very important variable and very difficult to scale up for geometrically similar unbaffled systems[20]. It is defined as the amount of energy necessary in a period of time, in order to generate the movement of the fluid within a container by means of mechanical or pneu-

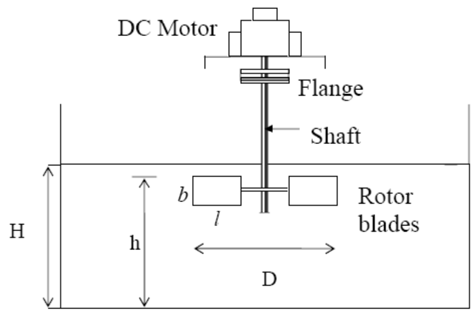

Figure 1: Schematic diagram of a surface aerator

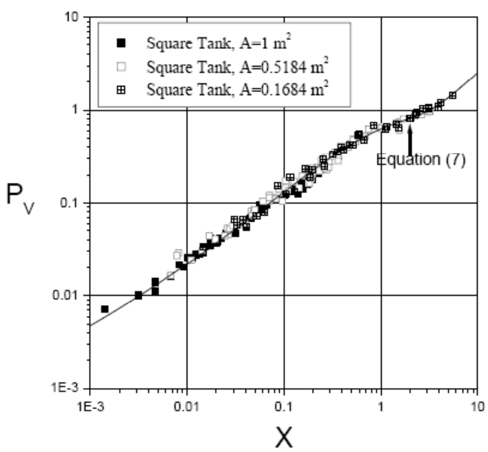

Figure 2: Actual power consumption with X in square surface aerators

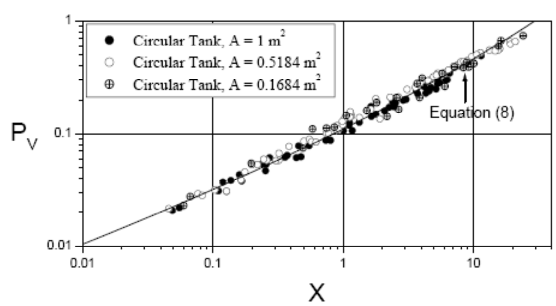

Figure 3: Actual power consumption with X in circular surface aerators

matic agitation. The costs associated with power draw contribute significantly to the overall operation costs of industrial plants.

Two shapes(square and circular) of unbaffled aeration tanks(as shown in figure 1) of three different cross-sectional sizes($0.1684m^2$, $0.5184m^2$ and $1m^2$) were taken for the experiments. The rotor is fitted with six flat blades Rushton turbines. The geometric similarity conditions(Equation 5) given by udaya et al. [22] was maintained in the aerators.

$$\sqrt{A}/D=2.88;\ H/D=1.0;\ l/D=0.3;\ b/D=0.24;\ h/H=0.94;\ n=6 \qquad (5)$$

Where, A is the cross-sectional area of the tank, H is the depth of water in the tank, D is the diameter of the rotor and the distance between the top of the blades and the horizontal floor of the tank is h. n is the number of blades fitted to the rotor and b, l are their linear dimensions. The rotor is connected to DC motor with speed regulator to run the rotor at desired rotational speeds.

The power available at the shaft was calculated as follows(Cook and Carr, 1947):

$$P=I_2V_2-I_1V_1-R_a(I_2^2-I_1^2) \qquad (6)$$

Where, I_1 and I_2 are currents measures in amperes under no load and loading conditions respectively, similarly the respective voltages in volts are V_1 and V_2. Armature resistance R_a is measured in ohms. So the experimental data in terms of P_V (Non-dimensional form of $P/V=P/=P/(V\gamma(g\nu)^{1/3}$, where V is the volume of water, g is the gravitational acceleration and ν is kinematic viscosity of the water) have been plotted with X for square and circular shape surface aerators in figures 1 and 2 respectively.

It is of interest to note that all the experimental points of actual(effective) power per unit volume when plotted with X show a unique relationship between them. The relationship can be best described by the following equations:

$$P_V=0.213X+0.12\sqrt{X}+0.79e^{-x} \qquad (7)$$

(Square Surface Aerators)

$$P_V=0.001+0.0926\sqrt{X}+0.017e^{-x} \qquad (8)$$

(Circular Surface Aerators)

CONCLUSION

The parameter, X has a very fundamental basis for its use in scale up studies in agitation. This parameter X can be used in scaling up the basic parameters(Oxygen transfer and power consumption) in stirred tanks.

Full Paper

Nomenclature

A	cross-sectional area of an aeration tank(L^2)
B	width of rotor blade(L)
D	diameter of rotor/impeller(L)
F	N^2D/g, Froude number
h f	head loss due to rotational movement of water in tank (L)
H	depth of water in an aeration tank(L)
h	distance between the top of the blades and the horizontal of the tank(L)
I_1, I_2	input current at no load and loading conditions respectively
l	length of blade(L)
L	characteristics length
N	rotation speed of rotor(rpm)
P	theoretical power required to rotor rotation(W)
Q	discharge of water being pumped by rotor rotation(L^3/T)
R	ND^2/ν, Reynolds number
R_a	armature resistance of DC motor
u'	characteristics velocity
V_1, V_2	input voltage at no load and loading conditions respectively
V	volume of liquid in tank(L^3)
v	characteristic tangential velocity, proportional to ND(L/T)
X	power per unit volume parameter;
ν	kinematic viscosity of water (L^2/T)
ε	dissipation rate of turbulent kinetic energy.

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