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The principal accumulation value of simple and compound interest

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ABSTRACT

The calculation of the principal accumulation value is the foundation of all financial calculations, and plays critical role in all financial activity such as investment, and financing. The theorem for calculations of the principal accumulation value of simple and compound interest is proposed. When a term is less than a period, the accumulated value of simple interest is more than that of compound interest; When a term is equal to a period, the accumulated value of simple interest is the same as that of compound interest; When a term is more than a period, the accumulated value of simple interest is less than that of compound interest, and the growth rate of the accumulated value of simple interest is less than that of compound interest. The theorem enriches the theory of interest. Finally, I strictly prove the theorem with Rolle’s Theorem.

KEYWORDS

Principal accumulation value; Compound interest; Simple interest; Rolle’s theorem; Growth rate.

INTRODUCTION

The principal accumulation value is a fundamental concept of the theory of interest and financial theory, in which the time value of money is embodied. The time value of money is the relationship between time and money. It is to say that money in hand today is worth more than money that is expected to be received in the future if there are no inflationary and deflationary trends in an economic system. The reason is straightforward: A dollar that you receive today can be invested such that you will have more than a dollar at some future time. This leads to the saying that we often use to summarize the concept of time value: "A dollar today is worth more than a dollar tomorrow." This concept is used to choose among alternative investment proposals. Understanding the concepts is crucial to understanding all sorts of solutions to financial problems in personal finance, investments, banking, insurance, etc. Khan[1] raises the issue of time preference and the time value of money and their relevance not only to discounting but also to wage, rent. The time value of money is especially appreciated. Hudson[2], Robert and Murdick[3] have provided the formulae to enable the time value of money to be reflected in valuation practice. One of the biggest obstacles to correctly solving time value of money problems is identifying the cash flows and their timing. Every time value of money problem has five variables: Present value, future value, and number of periods, interest rate, and a payment amount. The interest rate is the growth rate of your money over the life of the investment. It is usually the only percentage value that is given. However, some problems will have different interest rates for different time frames. For example, problems involving retirement planning will often give pre-retirement and post-retirement interest rates. Frequently, when you are being asked to solve for the interest rate, you will be asked to find the compound average annual growth rate. On the other hand, Snowden[4], Xi[5] and Li[6] have given the computation issues of compound interest. Compound interest has important applications in human society based on the benefit of storage and investment in economics[7]. The key to compound interest research primarily lies in the organism's choice of interaction characteristics on different temporal and spatial scales[8]. Three formulas for compound interest exist in terms of different periods of time: continuous compound interest; com-pound interest of k periods which has received more attention in evolutionary ecology; compound interest of a unit period. Modeling the term structure of interest rates (TSIR) has been the object of many studies and the aim of attention for economists and financial institutions. Applications and analysis of some of TSIR models can be found in the references[9-11].

However, there are few studies of the principal accumulation value when taking into account simple interest, and the difference between the principal accumulation of simple interest and that of compound interest is not given. In this paper, the theorem for the principal accumulations calculated based on simple interest and compound interest is presented and proved.

In what follows, Firstly, the essential notation and some definitions will be introduced. Secondly, the example for the principal accumulations calculated under simple and compound interest is given. Thirdly, the theorem for the principal accumulations calculated under simple and compound interest is presented and proved. Finally, the conclusion is given.

Preliminaries

In this section, we introduce some definitions and notations.

In this paper, interest rates will be denoted symbolically by i. To simplify the formulas and mathematical calculations, when i is used it will be converted to decimal form even though it may still be referred to as a percentage. The initially deposited amount which earns the interest will be called the principal amount and will be denoted P. Accumulation function $a(t)$ and amount function $A(t)$ are defined respectively as follows.

Define 1. Let t be the number of investment years ($t \geq 0$), where $a(0)=1$, the value at time t, denoted by $a(t)$ will be called the accumulation function. In the case of a positive rate of return, as in the case of interest, the accumulation function is a continuously increasing function.

Define 2. Let P be the initial principal invested ($P>0$), where $A(0)=P$, the accumulated value that amount P grows to in t years, denoted $A(t)$, is defined as the amount function. In the case of a positive rate of return, the accumulation function also is a continuously increasing function. Interest earned is the difference between the accumulated value at the end of a period and the accumulated value at the beginning of the period.

Therefore, the relationship between $a(t)$ and $A(t)$ is

$$A(t)=Pa(t). \tag{1}$$

In this paper, we focus on the accumulation function.

Accumulate a single investment at a constant rate of interest under the operation of simple and compound interest.

If an amount $P=1$ is deposited in an account which pays simple interest at the rate of i per annum and the account is closed after t years, there being no intervening payments to or from the account, then the amount paid to the investor when the account is closed will be

$$a_1(t)=1+ti \tag{2}$$

This payment consists of a return of the initial deposit $P=1$, together with interest of amount ti. The essential feature of simple interest, as expressed algebraically by expression (2), is that interest, once credited to an account, does not itself earn further interest. This leads to inconsistencies which are avoided by the application of compound interest theory.

The essential feature of compound interest is that interest itself earns interest. The operation of compound interest may be described as follows. Consider a savings account, which pays compound interest at rate i per annum, into which is placed an initial deposit $P=1$. (We assume that there are no further payments to or from the account.) If the account is closed after one year, the investor will receive $1+i$. More generally, let $a_2(t)$ be the amount which will be received by the investor if the account is closed after t years. Thus, $a_2(1)=1+i$. By definition, the amount received by the investor on closing the account at the end of any year is equal to the amount which would have been received, if the account had been closed one year previously, plus further interest of i times this amount. Thus the interest credited to the account up to the start of the final year itself earns interest (at rate i per annum) over the final year. So that Thus, if the investor closes the account after t years, the amount received will be

$$a_2(t)=(1+i)^t. \quad \textbf{(3)}$$

Compound interest is the typical computation applied in most time value applications.

In general, we have implicitly assumed that t is an integer. However, the normal commercial practice in relation to fractional periods of a year is to pay interest on a pro rata basis, so that the expressions in this paper may be considered as applying for all non-negative values of t.

Examples for accumulation value of simple and compound interest

In order to introduce our theorem in next section, let us give an example for simple interest and compound interest.

TABLE 1 : Simple and compound interest principal accumulation value comparison

t (years)	**0.0**	**0.1**	**o.2**	**0.3**	**0.4**
$a_1(t)$ ($)	1.0000	1.0700	1.1400	1.2100	1.2800
$a_2(t)$ ($)	1.0000	1.0545	1.1120	1.1726	1.2365
t (years)	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
$a_1(t)$ ($)	1.3500	1.4200	1.4900	1.5600	1.6300
$a_2(t)$ ($)	1.3038	1.3749	1.4498	1.5288	1.6121
t (years)	**1.0**	**1.2**	**1.4**	**1.6**	**2.0**
$a_1(t)$ ($)	1.7000	1.8400	1.9800	2.1200	2.4000
$a_2(t)$ ($)	1.7000	1.8903	2.1020	2.3373	2.8900

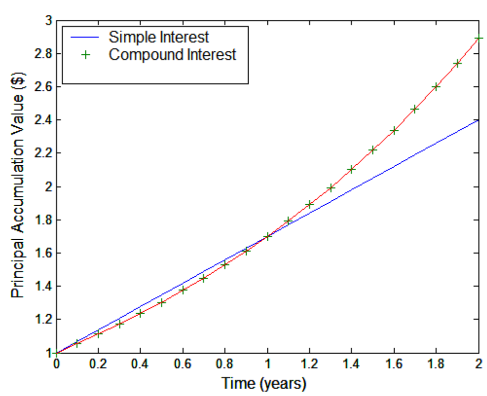

Figure 1 : Simple and compound interest principal accumulation value comparison

Let the annual interest rate $i=0.7$ to illustrate the difference effects of the principal accumulation values calculated with simple interest and compound interest, respectively, the principal amount $P=1$\$. The principal accumulation values, which are calculated using expression (2) and expression (3) respectively, are given in TABLE 1 or in Figure 1.

TABLE 1 or Figure 1 shows that the accumulation functions of simple and compound interest are strictly monotone increasing on $[0,+\infty)$. Furthermore, Figure 1 shows that the graph of the accumulation functions of simple interest is a proportional line; however, the graph of accumulation functions of compound interest is the upward sloping one. The theorem in the section can also be clearly seen from comparison with the graphs.

The theorem for the principal accumulation value of simple and compound interest

Form the example above, the accumulation function theorem can be derived as follows.

Theorem3. Assume that the time variable t is a continuous one on the interval $[0,+\infty)$. We can denote the simple and compound interest accumulation function by the following forms respectively

$$a_1(t) = 1 + ti \tag{4}$$

and

$$a_2(t) = (1+i)^t, \tag{5}$$

where i denotes the interest rate. Then we have

(a)If $t=0$ or $t=1$, then

$$a_1(t) = a_2(t) \; ; \tag{6}$$

(b) If $0 < t < 1$, then

$$a_1(t) > a_2(t) \; ; \tag{7}$$

(c) If $t > 1$, then

$$a_1(t) < a_2(t) \text{ and } a_1'(t) < a_2'(t). \tag{8}$$

Proof. Let $f(t) = a_1(t) - a_2(t)$, i.e.

$$f(t) = 1 + it - (1+i)^t \; (t \geq 0). \tag{9}$$

Using the derivation rules, we have

$$f'(t) = i - (1+i)^t \ln(1+i) \tag{10}$$

and

$$f''(t) = -(1+i)^t [\ln(1+i)]^2 < 0. \tag{11}$$

Therefore, $f'(t)$ is strictly monotone decreasing on the interval $[0,+\infty)$.

Firstly, we shall prove (a). Since

$$a_1(0) = a_2(0) = 1,$$

and

$$a_1(1) = a_2(1) = 1 + i,$$

The statements of (a) follow at once from the two expressions above.

Secondly, we shall prove (b). Obviously, $f(t)$ is differentiable in the interval $(0,1)$. Also, $f(t)$ is continuous on the interval $[0,1]$, and $f(0) = f(1)$. By Rolle's Theorem we have that there exist at least one point $\xi \in (0,1)$ such that $f'(\xi) = 0$. Note that $f'(t)$ is strictly monotone decreasing in the interval $(0,1)$, so that we have that there exist one and only one point $\xi \in (0,1)$ such that $f'(\xi) = 0$.

When $0 < t < \xi$, we have

$$f'(t) > f'(\xi) = 0 . \tag{12}$$

Hence, we have that $f(t)$ is strictly monotone increasing on the interval $[0,\xi]$, then

$$f(\xi) > f(t) > f(0) = 0 . \tag{13}$$

So, we have

$$a_1(t) > a_2(t) .$$

On the other hand, when $\xi < t < 1$, obviously, we have

$$f'(t) < f'(\xi) = 0 . \tag{14}$$

Hence, we have that $f(t)$ is strictly monotone decreasing on the interval $[\xi,1]$, then

$$f(\xi) > f(t) > f(1) = 0 . \tag{15}$$

So, we have

$$a_1(t) > a_2(t) .$$

Therefore, this proves (b).

In the end, we shall prove (c). Since $f'(t)$ is strictly monotone decreasing on the interval $(0,+\infty)$, when $t > 1$, we have

$$f'(t) < f'(1) < f'(\xi) = 0 , \tag{16}$$

So, we have

$$a_1'(t) < a_2'(t) .$$

Obviously, $f(t)$ is strictly monotone decreasing on the interval $[1,+\infty)$, when $t > 1$, we have

$$f(t) < f(1) = 0 . \tag{17}$$

Namely,

$$a_1(t) < a_2(t)$$

Therefore, these prove (c).

CONCLUSIONS

In this paper, the accumulation function theorem can be derived from the practical example, and proved. When a term is less than a period, the accumulated value of simple interest is more than that of compound interest; When a term is equal to a period, the accumulated value of simple interest is the same as that of compound interest; When a term is more than a period, the accumulated value of simple interest is less than that of compound interest. The theorem enriches the theory of interest and financial theory.

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