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## The optimal direct yaw-moment control of the fourwheel steering tractor-semitrailer

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### ABSTRACT

Considering the Gim tire model, the nonlinear four-wheel steering tractor-semitrailer dynamic model was set up to analyse the lateral stability of the tractor-semitrailer. An optimal direct yaw-moment controller of the four-wheel steering tractor-semitrailer was developed based on model following technique and the optimal control theory. Simulation on the nonlinear four-wheel steering tractor-semitrailer dynamic model in Matlab/Simulink software environment was described. The Simulation results show that the four-wheel steering direct yaw-moment control tractor-semitrailer have the most appropriate handling stability compared with the front wheel steering tractor-semitrailer, the front wheel steering direct yaw-moment control tractor-semitrailer, and the four-wheel steering tractor-semitrailer.

## **KEYWORDS**

Four-wheel steering; Handling stability; Direct yaw-moment control; Gim tire model; Optimal control.

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#### **INTRODUCTION**

Tractor-semitrailer, has become one of the most important road transport, because of high transport efficiency, low fuel consumption, and door-to-door transportation. Tractor-semitrailer which includes a tractor and a semitrailer has different handling and stability properties compared with the passenger cars. The lateral instability (trailer swing, jack-knifing, rollover) of the tractor-semitrailer has caused more and more accidents. For the passenger car, DYC and 4WS can effectively improve handling and stability performances<sup>[1-4]</sup>. In recent years, more and more researchers focus on DYC and 4WS of the tractor-semitrailer, and have considerable achievements<sup>[5-8]</sup>. In the future, DYC and 4WS will be widely used in the tractor-semitrailer, because DYC and 4WS can make a driver handle the tractor-semitrailer more stable. However, for the tractor-semitrailer, most studies independently verified the validity of four-wheel steering or direct yaw moment control based on the linear model, and the study of four-wheel steering direct yaw moment control based on the nonlinear model is little.

In this paper, a nonlinear tractor-semitrailer dynamic model was built based on Gim tire model, and four-wheel steering direct yaw moment controller was designed.

#### NONLINEA 4WS TRACTOR-SEMITRAILER DYNAMIC MODEL

#### Vehicle model



Figure 1 : Schematic diagram of 4WS tractor-semitrailer

The single-body 4WS tractor-semitrailer model is shown in Figure 1. The motions can be described respectively as follow:

$$m_1 u(\beta_1 + r_1) = F_{yf} + F_{yr1} + F_y$$
(1)

$$I_{Z1}\dot{r}_{1} = a_{1}F_{yf} - b_{1}F_{yr1} - c_{1}F_{y}$$
<sup>(2)</sup>

$$m_2 u(\dot{\beta}_2 + r_2) = F_{yr2} - F_y \tag{3}$$

$$I_{Z2}\dot{r}_{2} = -b_{2}F_{yr2} - c_{2}F_{y}$$
(4)

Where,  $m_1$  and  $m_2$  stand for the mass of tractor and semitrailer respectively;  $r_1$  and  $r_2$  stand for the yaw rate of tractor and semitrailer respectively;  $\beta_1$  and  $\beta_2$  stand for the slip angle of tractor and semitrailer respectively;  $I_{z1}$  and  $I_{z2}$  are the moments of inertia of tractor and the semitrailer, respectively;  $F_{yf}$ ,  $F_{yr1}$  and  $F_{yr2}$  are the lateral forces produced by each

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of the front and rear tires, respectively; For the tractor,  $a_1$  and  $b_1$  and  $c_1$  are the distances from the center of gravity to the front and rear axle and hinge point, respectively; For the semitrailer,  $b_2$  and  $c_2$  are the distances from the center of gravity to the rear axle and hinge point.

#### Gim tire model

The Gim tire model was used here. The tire forces can be formulated as follow<sup>[9]</sup>:

$$\begin{cases} F_{x} = C_{s}s_{s}l_{n}^{2} + \mu_{x}F_{z}(1 - 3l_{n}^{2} + 2l_{n}^{3}) s_{s} < s_{sc} \\ F_{x} = \mu_{x}F_{z} \qquad \qquad s_{s} \ge s_{sc} \end{cases}$$
(5)

$$\begin{cases} F_{y} = C_{\alpha}s_{a}l_{n}^{2} + \mu_{y}F_{z}(1 - 3l_{n}^{2} + 2l_{n}^{3}) & s_{\alpha} < s_{ac} \\ F_{y} = \mu_{y}F_{z} & s_{\alpha} \ge s_{ac} \end{cases}$$
(6)

Where,  $\mu_x$  and  $\mu_y$  are the longitudinal and transverse adhesion coefficients respectively;  $l_n$  is the dimensionless value of the tire ground wire length;  $C_s$  and  $C_{\alpha}$  are the tire longitudinal and transverse stiffness respectively;  $s_s$  and  $s_{\alpha}$  are the vertical and lateral slip rate respectively;  $s_{sc}$  and  $s_{\alpha c}$  are the tire lateral and longitudinal critical slip ratio respectively;  $F_z$  is wheel normal load.

#### **4WS Design**

To achieve zero slip angle of steady steering, front wheel and rear wheel angle of the tractor can be defined as follow<sup>[3]</sup>:

$$\xi = \frac{\delta_{r1}}{\delta_f} = \frac{-b_1 + [\frac{ma_1}{c_{r1}(a_1 + b_1)}]u^2}{a_1 + [\frac{mb_1}{c_f(a_1 + b_1)}]u^2}$$
(7)

#### **REFERENCE MODEL**

Considering the linear tire and taking the front steering angle  $\delta_f$  and corrective yaw moment M as the inputs, the tractor slip angle  $\beta_1$ , tractor yaw rate  $r_1$ , semitrailer yaw rate  $r_2$  and tractor and semitrailer center line angle  $\theta$  as the state variables, combining the equations (1)-(4), the state response can be written as:

$$A_{ac}\dot{X}_{ac} = B_{ac}X_{ac} + C_{ac1}U_1 + C_{ac2}U_2$$
(8)

Where,

$$X_{ac} = \begin{bmatrix} \beta_{1} & r_{1} & r_{2} & \theta \end{bmatrix}^{T}, U_{1} = \begin{bmatrix} \delta_{f} \end{bmatrix}, U_{2} = \begin{bmatrix} M \end{bmatrix}, C_{ac1} = \begin{bmatrix} -C_{f} & -C_{f}a_{1} & 0 & 0 \end{bmatrix}^{T}, C_{ac2} = \begin{bmatrix} 0 & 1 & I_{22} / I_{21} & 0 \end{bmatrix}^{T}$$

$$A_{ac} = \begin{bmatrix} (m_{1} + m_{2})u & -m_{2}c_{1} & -m_{2}c_{2} & 0 \\ -m_{2}uc_{1} & Iz_{1} + m_{2}c_{1}^{2} & m_{2}c_{1}c_{2} & 0 \\ -m_{2}uc_{2} & m_{2}c_{1}c_{2} & Iz_{2} + m_{2}c_{2}^{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B_{ac} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & C_{r2} \\ b_{21} & b_{22} & b_{23} & -C_{r2}c_{1} \\ b_{31} & b_{32} & b_{33} & -C_{r2}(b_{2} + c_{2}) \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

$$b_{11} = C_f + C_{r1} + C_{r2},$$
  
$$b_{12} = \frac{C_f a_1 - C_{r1} b_1 - C_{r2} c_1}{u} - m_1 u$$

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$$b_{13} = \frac{-C_{r2}(b_2 + c_2)}{u} - m_2 u \, b_{21} = C_f a_1 - C_{r1} b_1 - C_{r2} c_1$$

$$b_{22} = \frac{C_f a_1^2 + C_{r1} b_1^2 + C_{r2} c_1^2}{u} + m_2 u c_1 \, b_{23} = \frac{C_{r2}(b_2 + c_2) c_1}{u}$$

$$b_{31} = -C_{r2} b_2 - C_{r2} c_2 \, b_{32} = \frac{C_{r2} c_1 b_2 + C_{r2} c_1 c_2}{u} + m_2 u c_2, \quad b_{33} = \frac{C_{r2} (b_2 + c_2)^2}{u}$$

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In the above equations,  $C_f$  and  $C_{r1}$  are the front and rear tire cornering stiffness of tractor respectively;  $C_{r2}$  is the tire cornering stiffness of semitrailer;  $\alpha_f$  and  $\alpha_{r1}$  is the front and rear tire slip angle of tractor respectively;  $\alpha_{r2}$  is the tire slip angle of semitrailer;  $\theta$  is the tractor and semitrailer center line angle.

#### **OPTIMAL CONTROLLER**

Figure 2 Represents the principle of the optimal controller.



Figure 2 : Schematic diagram of the optimal controller principle

Taking the reference state variables, the reference vehicle model state response can be written as:

$$A_d \dot{X}_d = B_d X_d + C_d U_1 \tag{9}$$

where, 
$$A_d = A_{ac}$$
;  $B_d = B_{ac}$ ;  $C_d = \begin{bmatrix} -C_f & -C_f a_1 & 0 & 0 \end{bmatrix}^T$ .  
To track the reference vehicle model, let  $X = \begin{bmatrix} \Delta \beta & \Delta \gamma_1 & \Delta \gamma_2 & \Delta \theta \end{bmatrix}^T = X_{ac} - X_d$ , we can get:

$$A_{ac}\dot{X} = B_{ac}X + C_{ac2}U_2 \tag{10}$$

The quadratic cost function associated with system (17) can be defined as:

$$J = \int_{0}^{\infty} \left( X^{T} Q X + u^{T} R u \right) dt$$
<sup>(11)</sup>

By solving the riccati equation, we can get the optimal control law is:

$$u = -KX = -R^{-1}A_{ac}^{-1}B_{ac}^{T}PX$$
(12)

Where,  $Q \ge 0$ ,  $R \ge 0$ , P is definite constant matrix.

#### **RESULT AND DISSCUSS**

The major parameters are as follows:

 $m_1 = 6870 \ Kg$ ;  $m_2 = 6181 \ Kg$ ;  $I_{Z1} = 20441 \ Kg \cdot m^2$ ;  $I_{Z2} = 81912 \ Kg \cdot m^2$ ;  $a_1 = 1.96 \ m$ ;

 $c_{\alpha r1} = 143.33 \, kN/rad \; ; \\ c_{\alpha r2} = 80.312 \, kN/rad \; ; \\ c_{\alpha f} = 143.33 \, kN/rad \; ; \\ b_1 = 2.35 \, m \; ; \\ b_2 = 3.30 \, m \; ; \\ c_1 = 2.05 \, m \; ; \\ c_2 = 5.23 \, m \; . \\ c_3 = 5.23 \, m \; ; \\ c_4 = 5.23 \, m \; ; \\ c_5 = 5.23 \, m \; ; \\ c_6 = 5.23 \, m \; ; \\ c_7 = 5.23 \, m \; ; \\ c_8 = 5.23 \, m$ 

The tractor-semitrailer speed is 30 m/s. The maximum amplitudes of the sinusoidal input in Figure 3 are 0.058rad. The simulation time is 10s.



Figure 3 : The input of front steering angle





# Figure 4 : Responses of the tractor slip angle, tractor yaw rate, semitrailer yaw rate and tractor and semitrailer center line angle

The simulation results show that when the maximum amplitudes of the sinusoidal input is 0.058rad, uncontrolled vehicle loses stability about 3s, at this time the tractor slip angle, tractor yaw rate, semitrailer yaw rate and tractor and semitrailer center line angle increase sharply. The other three are able to maintain stable. For direct yaw moment control vehicle, the tractor slip angle is bigger than the other two, and the direction of the tractor slip angle is opposite to the traveling direction, at the same time, the amplitude of tractor yaw rate, semitrailer yaw rate and tractor and semitrailer center line angle changes bigger than the other two. For 4WS and 4WS direct yaw moment control vehicle, the direction of the tractor slip angle is the same as the traveling direction, and the responses of 4WS direct yaw moment control vehicle are bigger than 4WS, which can improve the handling stability effectively.

#### CONCLUSIONS

In this paper, Gim tire model was adopted to set up the nonlinear four-wheel steering dynamic model of tractorsemitrailer, then a four-wheel steering with direct yaw-moment control scheme was proposed. Based on the established dynamic model, simulations in Matlab/Simulink software environment were described. The simulation results suggest the four-wheel steering direct yaw-moment control tractor-semitrailer can improve handling and stability performance effectively, which makes the driver drive the vehicle normally.

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