# Study on the maximum deviation of basketball's shooting angle and speed based on numerical simulation 

Liyan Zhang ${ }^{\mathbf{1}}$, Shaowei Zhang ${ }^{2 *}$<br>${ }^{1}$ College of Physical Education, Northeast Normal University, Changchun 130024, Jilin, (CHINA)<br>${ }^{2}$ College of Humanities and Sciences, Northeast Normal University, Changchun 130024, Jilin, (CHINA)<br>E-mail: tiyuxi@qq.com


#### Abstract

As basketball becomes popular, the exciting atmosphere and other factors such as the aggressive defend reduce the shooting field. The basic and simple way for scoring in basketball match is shooting, the accuracy of which has decided the game to a great extent. The key points for the accuracy of shooting are the power and direct, and the speed and angle of shooting can decide the goal. By establishing the mathematic model of the ball and basket, the essay is focus on the influence of shooting field under the four different variables: the size of the ball and basket, the air resistance, shooting angle and speed and the relative maximum deviation. On a standard basketball ground, the fast the speed increases, the less deviation the angle allows; the bigger deviation the speed allows, the stricter demand we have on the angle than the speed. If the speed is fixed, the higher you shoot, the less deviation of the angle you'll have, at this time, the allowable deviation of shooting speed can be up to the limit, and the demand for shooting angle and speed is low accordingly.


© 2014 Trade Science Inc. - INDIA

## Keywords

Basketball shooting; Shooting speed; Maximum deviation; Shooting angle.

## FOREWORD

So far, the study on basketball in home and abroad is focus on the influence of various factors towards shooting from all aspects and different levels, mainly including technology, combination, skill and the nutrition of athletes to create mathematic and mechanical model for study. The teacher in basketball teaching and training, coach and the researchers have made analysis on the factors which can decide the shooting field from different perspective.

The factors that influence shooting field are about the
same, including the shooting time, technology, holding pattern, holding location, shooting ways, falling style, aiming methods, shooting angle, parabola, falling angle and so on. The principle and methods have been specified: in "the factors that influence shooting field", Liming stated the influence of shooting skilled movement on shooting field; in "the influence caused by increasing the shooting rang in jump shot" Stuart, Miller Roger and Bartlett (1993) took some photos of jump shot in different distances from the basket in the Men's basketball match of University sports in the city of Sheffield. Through data analysis, it was found that the shooting speed increased as the shoot-

## Full Paper

ing distance increased and a certain special relationship existed between shooting angle and distance ${ }^{[2]}$; it was stated in Groningen's "shooting skill" that shooting angle means the intersection angle between the level of shooting location and the moving direction the time the ball left the hands, which decides the height the ball flies in the air and the falling angle. If the shooting speed is fixed, shooting speed and moving radian would be low, and vice versa. Only had the fixed shooting angle and speed cooperated can the ball flied along a suitable track in the space and fell into the expected target, thus providing the scientific basis for the study of shooting process.

Under this kind of background, the essay discuss about the basketball shooting with sports mechanics to establish an effective shooting model, analyzing the influence of various factors towards shooting filed from the shooting angle, speed, shooting height, the level distance of the ball and the basket, and the relationship of the shooting angles. Finally, the shooting angle, speed range and the allowable maximum deviation for falling has come out.

## BASKETBALL SHOOTING MODEL

Look at the Figure 1, first set a coordinate system, and suppose V is the shooting speed when $\mathrm{t}=0$, angle $\alpha$ reflects the basketball, $\theta$ expresses shooting angle, and we can consider it a projectile motion of the particle.

Analyze the ball when it is flying after leaving the


Figure 1 : Basketball shooting model
hands, set the speed along the horizontal direction, ver-
tical speed as $v_{\mathrm{x}} 0 v_{y}$, so the formula of them is as follows:
$\left\{\begin{array}{l}v_{x}=v \cos \alpha \\ v_{y}=v \sin \alpha-g t\end{array}\right.$
Then we can calculate the location of the Centre of the basketball $(x, y)$ :
$\left\{\begin{array}{l}\mathrm{x}=\mathrm{vt} \cos \alpha \\ \mathrm{y}=\mathrm{vt} \sin \alpha-\frac{1}{2} \mathrm{gt}^{2}\end{array}\right.$
The $t$ can be eliminated from the above formula, and then we got $y=x \tan \theta-\frac{g}{2 v^{2} \cos ^{2} \theta} x^{2}$, if the centre of the basketball directly passes through that of the basket, we can put $\mathrm{p}_{0}\left(\mathrm{x}_{0}, y_{0}\right)$ into (2) and get the following value:
$\mathrm{v}^{2}=\frac{\mathrm{gx}_{0}{ }^{2}}{2 \cos ^{2} \theta\left(\mathrm{x}_{0} \tan \theta-\mathrm{y}_{0}\right)}$
We can see from the parabola in Figure 2, the angle of incidence $\alpha$ will increase as the shooting angle $\theta$ increases, and vice versa. So the shooting angle is proportional to the angle of incidence when is reduced to be $30^{\circ}$.


Figure 2 : Relationship between the speed and shooting angle

## Basketball and basket as the particles, the ball falls into the centre of the basket

Calculating the condition of the centre of the balls falling into that of the basket according to formula (3)

$$
\begin{equation*}
\tan \alpha=\frac{v^{2}}{g x}\left[1 \pm \sqrt{1-\frac{2 g}{v^{2}}\left(y-H+\frac{g x^{2}}{2 v^{2}}\right)}\right] \tag{4}
\end{equation*}
$$

Tenable condition for formula (4) is:
$1-\frac{2 g}{v^{2}}\left(y-H+\frac{g x^{2}}{2 v^{2}}\right) \geq 0$
So we can get:
$\mathbf{v}^{2} \geq \mathrm{g}\left[\mathrm{y}-\mathbf{H}+\sqrt{\mathbf{x}^{2}+(\mathbf{y}-\mathbf{H})^{2}}\right]$
We need to figure out the minimum initial speed, if you want the ball to fall into the basket, enough speed will be needed to afford it. The minimum shooting speed to support formula (5): $v_{\text {min }}$
$\mathbf{v}_{\text {min }}=\sqrt{\mathbf{g}\left[(\mathbf{y}-\mathbf{H}) \sqrt{\mathbf{x}^{2}+(\mathbf{y}-\mathbf{H}) \mathbf{2}}\right]}$
The function of $v_{\text {min }}$ towards H is as follows:


Figure 3 : the blanks of Minimum speed and the height
When considering the air resistance, horizontal resistance only:
$\mathrm{w}_{x}=-a v_{x}$
According to Newton's laws, the differential formula that the basketball meets in the horizontal direction is:

$$
\left\{\begin{array}{l}
\frac{d x}{d t}=v_{x}  \tag{6}\\
m \frac{d v_{x}}{d t}=-a v_{x}
\end{array}\right.
$$

When $\mathrm{t}=0$, we can calculate the formula (6) by using the initial condition $\mathrm{v}_{x}=v \cos \alpha, \mathrm{x}(0)=0$

$$
\mathbf{v}_{\mathrm{x}}=\mathrm{e}^{\frac{-a t}{\mathrm{~m}}} \mathrm{v} \cos \alpha
$$

And then use the initial condition to calcu-
late $\frac{d x}{d t}=v_{\mathrm{x}}$, we can get:
$\mathrm{x}=\left(1-\frac{\mathrm{m}}{\mathrm{a}} \mathrm{e}^{-\mathrm{at}} / \mathrm{m}\right) \mathrm{v} \cos \theta$
Put (7) into (4), Calculate the relationship of shooting speed and the angle.

## Ball and basket not as the particle, the ball falls into the centre of the basket

Considering the size of the ball and the basket, so the angle of incidence can not be low, otherwise the ball would shoot onto the top of the basket. The angle of incidence $\alpha=\left.\frac{d y}{d x}\right|_{x=x_{0}} \operatorname{so} \tan \alpha=\tan \theta-\frac{2 y_{0}}{x_{0}}$, different $\alpha$ for different $\theta$.

## The allowable maximum deviation for shooting angle and speed

To make sure the ball falls into the basket, the centre of the ball does not necessarily hit the centre of the basket, it can be partial front or behind (put aside partial left or right). Discussing about the allowable maximum deviation for shooting angle and speed on the premise of making sure the ball falls into the basket. When the ball falls in, the centre of the ball might deviate from the basket core. The maximum distance before deviation is $l$, which can be calculated from the angle of incidence $\theta$. And then we can get the allowable maximum deviation $l$ based on $\alpha$ and and $\theta$ in the ball core track. The maximum deviation $\Delta \mathrm{v}$ allowed by shooting speed $v$ can be calculated in the same way.

Supposed the diameter of the basketball is D, and the diameter of the basket is d , the possible maximum distance in the process of falling into the basket is:
$\mathrm{I}=\frac{\mathrm{D}}{2}-\frac{\mathrm{d}}{2 \sin \alpha}$
To get the allowable maximum deviation of shooting angle, we can use $\mathrm{x}+\Delta x$ to replace $x$ to recalculate in formula (4). It can not be calculated by analysis, because $\theta$ and $\alpha$ are included in $\Delta x$. But if we start from formula (3), and replace the height between people

## Full Paper

and the ball by $\mathrm{y}-\mathrm{H}$, we can get:
$x^{2} \frac{g}{2 v^{2} \cos ^{2} \alpha}-x \tan \alpha+y-H=0$
Make differentiation of $\alpha$, we can get:
$\frac{d x}{d \alpha}=\frac{\mathrm{x}\left(\mathrm{v}^{2}-\mathrm{gx} \tan \alpha\right)}{\mathrm{gl}-\mathrm{v}^{2} \sin \alpha \cos \alpha}$
Replace the left derivative by $\frac{\Delta x}{\Delta \alpha}$, we can get the following relationship between $\Delta \alpha$ (derivation of shooting angle) and $\Delta x$ :
$\Delta \alpha=\frac{g x-v^{2} \sin \alpha \cos \alpha}{\mathrm{x}\left(\mathrm{v}^{2}-\mathrm{gx} \tan \alpha\right)} \Delta \mathrm{x}$
We can easily figure out the relative derivation by $\Delta \alpha$ and $\alpha$.

Similarly, we can see that if we make the differentiation of $v$ in formula (9), we can get the allowable maximum speed derivation $\frac{\Delta x}{\Delta \alpha}$ the time the ball left the hands.
$\Delta \mathrm{v}=\frac{\mathrm{gx}-\mathrm{v}^{2} \sin \alpha \cos \alpha}{\mathrm{gx}^{2}} \mathrm{v} \Delta \mathrm{x}$
So the relative derivation calculated from formula (11) and (12) is:

$$
\begin{equation*}
\left|\frac{\Delta v}{v}\right|=\left|\Delta \alpha\left(\frac{v^{2}}{g x}-\tan \alpha\right)\right| \tag{13}
\end{equation*}
$$

## RESULTANALYSIS

## Ball and basket as the particles

(1) Ignoring the air resistance, $\mathrm{v}_{\text {min }}=\frac{v^{2}}{g x}$. Select the height $\mathrm{h}=1.8-2.1(\mathrm{~m})$, and then the result can be seen from TABLE 1.

TABLE 1 : Basic index statistics list

| $\mathrm{H}(m)$ | $v_{\min }(m / s)$ | $\alpha(o)$ |
| :---: | :---: | :---: |
| 1.80 | 7.66 | 52.5 |
| 1.95 | 7.55 | 51.7 |
| 2.1 | 7.42 | 50.7 |

We can see from tablet 1 , the shooting speed and angle will reduce as the shooting height increases, and the shooting speed should not be under $8 \mathrm{~m} / \mathrm{s}$ in this kind of situation.
(2) Considering the air resistance:
$\mathrm{y}=-\mathrm{m} \ln \left[\left(1-\frac{x}{v \cos \alpha}\right) \frac{a}{m}\right]-\frac{1}{2} \mathrm{~g} \frac{\mathrm{~m}}{\mathrm{a}} \ln ^{2}\left[\left(1-\frac{\mathrm{x}}{v \cos \alpha}\right) \frac{a}{m}\right]$
Substitute $p_{0}$, we can get:
$y_{0}-H=-m \ln \left[\left(1-\frac{x_{0}}{v \cos \alpha}\right) \frac{a}{m}\right]-\frac{1}{2} g \frac{m}{a} \ln ^{2}\left[\left(1-\frac{x_{0}}{v \cos \alpha}\right) \frac{a}{m}\right]$
Suppose $a / m=0.005$, the resistance speed can be guessed so we can substitute different value as the resistance speed is larger than $8 \mathrm{~m} / \mathrm{s}$, and the results can be seen in TABLE 2

TABLE 2 : Index statistics list without resistance speed

| $v(m / s)$ | $\mathrm{H}(m)$ | $\alpha(o)$ |
| :---: | :---: | :---: |
| 8.0 | 1.8 | 60.7 |
|  | 1.95 | 61.8 |
|  | 2.1 | 62.4 |
| 8.5 | 1.8 | 66.6 |
|  | 1.95 | 67.0 |
|  | 2.1 | 67.4 |
| 9.0 | 1.8 | 70.3 |
|  | 1.95 | 70.5 |
|  | 2.1 | 70.7 |

We can see from the above tablet, compared with non-resistance, the shooting angle will be influenced more as the speed increases when with resistance.

## Ball and basket not as the particles

As the radian is very low when shooting, so the ball is considered to be flying into the basket in straight line. And then we can get the limit of the angle incidence $\alpha>33.1^{\circ}$ according to the relationship of trigonometric. But actually the ball does not fall into the basket in straight line, and the parabola is convex in a very short track, so the result will not be affected. We can see TABLE 3 when $\alpha<33.1^{0}$

After calculating, we can find that all $\alpha$ is unqualified; the parabola is too low and level.
Analysis on the maximum deviation of shooting

## angle and speed

Calculate the maximum deviation of shooting angle $\Delta \alpha, \frac{\Delta \alpha}{\alpha}$ by using formula (11) and the above $\alpha$, and calculate that of shooting speed $\Delta \mathrm{v} 0 \frac{\Delta v}{v}$ by using formula (12) and (13). Then list the result of $h=1.8(m)$, $h=2.0(m)$ into the following TABLE4.

Generally speaking, the allowable deviation is al-
TABLE 3: Index statistics list while $\alpha<\sqrt{33.1^{0}}$

| $v(m / s)$ | $\mathrm{H}(m)$ | $\alpha(o)$ |
| :---: | :---: | :---: |
| 8.0 | 1.8 | 62.41 |
|  | 1.95 | 63.43 |
|  | 2.1 | 64.27 |
| 8.5 | 1.8 | 67.70 |
|  | 1.95 | 68.19 |
|  | 2.1 | 68.62 |
| 9.0 | 1.8 | 71.07 |
|  | 1.95 | 71.37 |
|  | 2.1 | 71.66 |

TABLE 4: Complex list for deviation

| $\mathbf{H ( m )}$ | $\alpha\left(^{\circ}\right)$ | $v(m / s)$ | $\Delta \alpha$ | $\Delta v$ | $\frac{\Delta \alpha}{\alpha}$ | $\frac{\Delta v}{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.8 | 62.4097 | 8.0 | -0.7563 | 0.0526 | 1.2260 | 0.6596 |
|  | 67.6973 | 8.5 | -0.5607 | 0.0691 | 0.8274 | 0.8163 |
|  | 71.0693 | 9.0 | -0.4576 | 0.0803 | 0.6432 | 0.8916 |
| 2.0 | 63.7286 | 8.0 | -0.7102. | 0.0605 | 0.0043 | 0.7513 |
|  | 68.3365 | 8.5 | -0.5407 | 0.0732 | 0.7918 | 0.8623 |
|  | 71.4706 | 9.0 | -0.4461 | 0.0836 | 0.6241 | 0.9250 |

most very low. On analysis, we can see that as the speed increases, the allowable deviation of angle reduces while that of speed increases. And it's stricter with the angle than speed. When the shooting speed is fixed, the faster the speed is, the lower the shooting angle allows, and the higher the deviation of shooting speed goes. Little requirement would be made for the shooting angle and speed at this very moment.

## CONCLUSION

The essay first establishes the mathematics model from shooting the basketball to its falling into the bas-
ket, making a relational expression of the speed and angle. Focus on the study of the ball and basket by using mathematics language which means the figures, tablets and expression symbols to express the issue shooting, and then scientifically and concretely analyze the influence of the two conditions that affect shooting field during the process of shooting towards shooting field. The essay also analyzes two aspects that influence the shooting field: whether air resistance exists and whether the ball and basket are considered as the particles. The conclusion has provided scientific basis for the basketball players' reasonable training, which also supports other researchers' further study on shooting.

## REFERENCES

[1] Zhu Hao, Bao Yun; Relation Between the Spinning Ball and Scoring[J]. Joournal of Nanjing Institute of Physical Education: Natural Science, 5(1), 3335, 32 (2006).
[2] Li Yu-Zhong; Dept. of Physical Education, Chongqing Technology and Business University, Chongqing, China.Influence of Mechanics Elements on Percentage of Basketball Hits[J]. Journal of Southwest China Normal University(Natural Science), 29(2), 300-304 (2004).
[3] Wang Wei-Ping, Yang Li-Hua; Biomechanics Factors of Influence Shooting Baskets Accuracy Rate[J]. Journal of Inner Mongolia University for the Nationalities(Natural Sciences), 18(2), 180-182 (2003)
[4] Wang Jiantao, Xu Yuming; About Spinning Balls in Goal Throwing[J]. Journal of Shanxi Teachers University, 8(1), 71-75 (1994).
[5] Zhang Shuang, Zhou Cui, Qin Yu-Ping; Analysis of the Problems in Basketball Match[J]. Journal of Neijiang Teachers College, 22(4), 36-39 (2007).
[6] Yang Tie-li, Zhang Jian-hua, Wang Bi-Qi; Successful Experiences and Revelation of NBA Professional Basketball Market[J]. Journal of Beijing Sport University, 23(3), 300-302 (2000).
[7] Wang Wu-nian, Qian Shao-wen, Yang Peng-fei; Attacks of European strong teams at 16th Men's Basketball World Championships[J]. Journal of Wuhan Institute of Physical Education, 45(8), 5458 (2011).
[8] Cai Lei, Wang Wu-Nian; Research on Scores and the Ways of Obtaining Scores in 2005-2006 NBA Playoffs[J]. Journal of Beijing Sport University, 30(5), 718-719 (2007).
[9] Huang Song-Feng; Research on Chinese Men's Basketball Team's Preparation Against Europe and American Teams in the London OlymFigure Games[J]. China Sport Science and Technology, 48(2), 29-34 (2012).

