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Stability criteria for discrete-time linear systems with state saturation nonlinearity

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ABSTRACT

The stability analysis for a class of discrete-time linear systems with state saturation nonlinearity is studied in this paper. By introducing a free matrix whose infinity norm is less than or equal to 1 and a diagonal matrix with non-positive diagonal elements, the discrete-time state under saturation constraint is confined in a convex hull. In this way, a stability criterion for discrete-time linear systems with state saturation to be asymptotically stable is obtained in terms of bilinear matrix inequalities that can be resolved using the presented iterative linear matrix inequality algorithm. The state feedback control law synthesis problem is also resolved and the corresponding iterative linear matrix inequality synthesis algorithm is given. Two numerical examples show that the presented method is applicable and effective.

KEYWORDS

Discrete linear system; Stability analysis; State saturation nonlinearity; Iterative linear matrix inequality.

INTRODUCTION

State saturation nonlinearity is commonly encountered in control engineering, such as computer storage facilities with finite precision, mechanical systems with position and speed restrict and artificial neutral networks[1]. Such systems are defined in a hypercube since all the states are confined in the unit hypercube. The state saturation is commonly ignored in analysis and design procedure for simplicity. However, it has been verified that the closed-loop asymptotical stability cannot be guaranteed if this saturation constraint is neglected. For this reason, the stability analysis for such systems is a topic of recurring interest in recent years.

The asymptotical stability of linear systems with state saturation is first investigated in[1], and be generalized to filter design problem[2],[3]. Regarding the second-order system as a special case, the corresponding stability criterion was presented in[4], which was further extended to the *n*th order systems[5]. Based on these results, the asymptotic stability problem for linear systems with partial state saturation nonlinearity was straightforwardly given[6]. The obtained results can be classified into two categories. The first category is to study the norm characteristics of the system matrix, such as $\|A_2\| < 1$ and $\|A_\infty\| < 1$ [7],[8], and the second category is to restrict the Lyapunov matrix P to be special forms[9]. Recently, Fang *et. al.* introduced a diagonal dominant matrix with negative diagonal elements to bound the state under saturation constraint within a convex hull and used the robust control theory to obtain a sufficient stability criterion for continuous-time linear systems with state saturation nonlinearity[10]. This result is further generalized to discrete-time systems and an asymptotical stability criterion was given[11].

This paper attempts to give some stability criteria for discrete-time linear systems with state saturation nonlinearity. Borrowing the idea of [10], [11], we introduce a free matrix whose infinity form is less than or equal to 1 and a diagonal matrix with non-positive diagonal elements to bound the discrete-time state with saturation constraint within a convex hull. Then, the robust stability theory on ploytopic-type uncertain systems is applied to obtain a sufficient criterion for such systems to be asymptotically stable. Based on this criterion, the state feedback control law synthesis method can be easily given. The obtained results are formulated in terms of bilinear matrix inequalities that can be resolved numerically using the presented iterative linear matrix inequality algorithm. Two numerical simulations show that the presented method is applicable and effective.

The notations used in this paper are fairly standard. The superscript T stands for matrix transposition, while superscript “-1” presents the inverse of a matrix. $\Re^n$ denotes the n dimensional Euclidean space and $\Re^{n\times m}$ is the set of all n by m matrices. I_n is an $n\times n$ identity matrix. $\|\cdot\|_\infty$ denotes the infinity norm of a matrix, that is, $\|G\|_\infty = \max_{1\le i\le m} \sum_{j=1}^{n} |g_{ij}|$ for matrix $G=[g_{ij}]\in\Re^{m\times n}$. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \ge Y$) means that $X - Y$ is positive definite (respectively, positive-semidefinite).

PROBLEM FORMULATION

Considers the following discrete-time linear system with state saturation nonlinearity,

$$x(k+1) = h(Ax(k)) \tag{1}$$

where $x(k)\in \mathrm{D}^n := \{x(k)=[x_1(k)\ x_2(k)\cdots x_n(k)]^T \in\Re^n : -1\le x_i(k)\le 1, i\in[1,n]\}$ is the state vector $A = [a_{ij}]\in\Re^{n\times n}$ is a constant matrix, $h(\cdot)$ is the standard saturation function described by

$$h(Ax(k)) = \begin{bmatrix} h_1\left(\sum_{j=1}^{n} a_{1j}x_j(k)\right) \\ h_2\left(\sum_{j=1}^{n} a_{2j}x_j(k)\right) \\ \vdots \\ h_n\left(\sum_{j=1}^{n} a_{nj}x_j(k)\right) \end{bmatrix} \tag{2}$$

with, for each $i\in[1,n]$,

$$h_i(\zeta)=\begin{cases}1, & \text{if} \quad \zeta>1 \\ \zeta, & \text{if} \quad -1\le\zeta\le 1 \\ -1, & \text{if} \quad \zeta<-1\end{cases} \tag{3}$$

Before proceeding further, we first give some lemmas which will be used in the proof of our main results.

Lemma 1 (Hu *et. al.*[12]): Let $u,u^1,u^2,\ldots,u^{\mathrm{I}} \in \Re^{n_1}$, $v,v^1,v^2,\ldots,v^{\mathrm{J}} \in \Re^{n_2}$. $u\in \mathrm{co}\{u^i, i\in[1,\mathrm{I}\ ]\}$, $v\in \mathrm{co}\{v^j, j\in[1,\mathrm{J}\ ]\}$, then

$$\begin{bmatrix} u \\ v\end{bmatrix}\in \mathrm{co}\left\{\begin{bmatrix} u^i \\ v^j\end{bmatrix}: i\in[1,\mathrm{I}\ ],\ j\in[1,\mathrm{J}\ ]\right\} \tag{4}$$

where $\mathrm{co}\{\cdot\}$ denotes the convex hull.

Let D^n be the set of $n\times n$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^n elements in D^n and we denote its element as D_i, $i\in[1,2^n]$. Denote $D_i^-=I_n-D_i$. It is easy to see $D_i^-\in \mathrm{D}^n$, if $D_i\in\mathrm{D}^n$. For example, if $n=2$, then

$$\mathrm{D}^2=\left\{\begin{bmatrix}0&0\\0&0\end{bmatrix},\begin{bmatrix}1&0\\0&0\end{bmatrix},\begin{bmatrix}0&0\\0&1\end{bmatrix},\begin{bmatrix}1&0\\0&1\end{bmatrix}\right\}.$$

Lemma 2: Let $G=[g_{ij}]\in\Re^{n\times n}$ with $\|G\|_\infty\le 1$ and $\varepsilon=\mathrm{diag}\{\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n\}$ with $\varepsilon_i\le 0$, $i\in[1,n]$, then

$$h(Ax(k))\in \mathrm{co}\left\{D_iAx(k)+D_i^-(G+\varepsilon A)x(k), i\in[1,2^n]\right\} \tag{5}$$

Proof: Noting $\|G\|_\infty\le 1$ and $|x_i(k)|\le 1$, $i\in[1,n]$, we have $|G_ix(k)|\le\sum_{j=1}^{n}|g_{ij}x_j(k)|\le\sum_{j=1}^{n}|g_{ij}|\le\|G\|_\infty\le 1$ and $-1\le G_ix(k)\le 1$. Noting also $\varepsilon_i \le 0$, $i\in[1,n]$, we have $\varepsilon_iA_ix(k)\le 0$ when $A_ix(k)\ge 1$ and $\varepsilon_iA_ix(k)\ge 0$ when $A_ix(k)\le -1$. Then, we have $(G_i+\varepsilon_iA_i)x(k)\le 1$ when $A_ix(k)\ge 1$ and $(G_i+\varepsilon_iA_i)x(k)\ge -1$ when $A_ix(k)\le -1$. In the absence of state saturation, $h_i(A_ix(k))=A_ix(k)$, it is obvious $h_i(A_ix(k))\in\mathrm{co}\{A_ix(k),(G_i+\varepsilon_iA_i)x(k)\}$, i.e., $h_i(A_ix(k))=\alpha(A_ix(k))+(1-\alpha)(G_i+\varepsilon_iA_i)x(k)$ with $\alpha=0$ satisfying $0\le\alpha\le 1$. In the event of state saturation, $h_i(A_ix(k))=1$ when $A_ix(k)>1$, or $h_i(A_ix(k))=-1$ when $A_ix(k)<-1$. When $A_ix(k)>1$, we can obtain $h_i(A_ix(k))=1\in\mathrm{co}\{A_ix(k),\ (G_i+\varepsilon_iA_i)x(k)\}$ since $G_i+\varepsilon_iA_i\le 1$. When $A_ix(k)<-1$, we can also obtain $h_i(A_ix(k))=-1\ \in\mathrm{co}\{A_ix(k),(G_i+\varepsilon_iA_i)x(k)\}$ since $(G_i+\varepsilon_iA_i)x(k)\ge -1$. The desired result follows immediately by using Lemma 1.

Remark 1: System (1) is a nonlinear system whose analysis and synthesis problems are difficult to perform. We here introduce a matrix G satisfying $\|G\|_\infty\le 1$ and a diagonal matrix $\varepsilon=\mathrm{diag}\{\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n\}$ with $\varepsilon_i\le 0$ to bound the discrete state $x(k)$ under saturation constraint in a convex hull $\mathrm{co}\{D_iAx(k)\ +D_i^-(G+\varepsilon A)x(k)\}$. This makes the original nonlinear system (1) bounded by a discrete-time linear system with ploytopic-type uncertain parameters whose analysis and synthesis problem are fairly easy using the robust control theory on polytopic-type uncertain systems.

STABILITY ANALYSIS

The following theorem gives a sufficient condition for system (1) to be asymptotically stable.

Theorem 1: The discrete-time linear system (1) with state saturation is asymptotically stable, if there exists a symmetric positive-definite matrix P and matrices G and X, $\varepsilon=\mathrm{diag}\{\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n\}$ with $\varepsilon_i\le 0$, $i\in[1,n]$ such that

$$\|G\|_\infty\le 1 \tag{6a}$$

$$\Xi_i=\begin{bmatrix}-P & ((D_i+\varepsilon D_i^-)A+D_i^-G)^TX^T \\ * & -X-X^T+P\end{bmatrix}<0, i\in[1,2^n] \tag{6b}$$

Proof: It follows from Lemma 2 that

$$h(Ax(k)) \in \mathrm{co}\{D_i Ax(k) + D_i^- (G + \varepsilon A)x(k)\}, i \in [1, 2^n] \quad \textbf{(7)}$$

that is, system (1) can be written as

$$x(k+1) = \sum_{i=1}^{2^n} \eta_i ((D_i + \varepsilon D_i^-)A + D_i^- G)x(k) \quad \textbf{(8)}$$

where $\eta_i \geq 0$, $i \in [1, 2^n]$, $\sum_{i=1}^{2^n} \eta_i = 1$.

For polytopic-type uncertain system (8), we design a Lyapunov functional as

$$V(x(k)) = x^T(k)Px(k), P > 0 \quad \textbf{(9)}$$

and the forward difference of this functional can be given as

$$\begin{aligned} \Delta V(x(k)) &= V(x(k+1)) - V(x(k)) \\ &= \left(\sum_{i=1}^{2^n} \eta_i ((D_i + \varepsilon D_i^-)A + D_i^- G)x(k) \right)^T P \left(\sum_{i=1}^{2^n} \eta_i ((D_i + \varepsilon D_i^-)A + D_i^- G)x(k) \right) - x^T(k)Px(k) \\ &= x^T(k)\Theta x(k) \end{aligned} \quad \textbf{(10)}$$

where

$$\Theta = \left(\sum_{i=1}^{2^n} \eta_i ((D_i + \varepsilon D_i^-)A + D_i^- G) \right)^T P \left(\sum_{i=1}^{2^n} \eta_i ((D_i + \varepsilon D_i^-)A + D_i^- G) \right) - P \quad \textbf{(11)}$$

Noting the fact that

$$(X - P)P^{-1}(X - P)^T \geq 0 \quad \textbf{(12)}$$

holds for any matrix X and symmetric positive-definite matrix P , we have

$$XP^{-1}X^T \geq X + X^T - P \quad \textbf{(13)}$$

Noting this fact and inequality (6), we have

$$\sum_{i=1}^{2^n} \eta_i \begin{bmatrix} -P & ((D_i + \varepsilon D_i^-)A + D_i^- G)^T X^T \\ * & -XP^{-1}X^T \end{bmatrix} < 0$$

and then

$$\begin{bmatrix} -P & \sum_{i=1}^{2^n} \eta_i ((D_i + \varepsilon D_i^-)A + D_i^- G)^T X^T \\ * & -XP^{-1}X^T \end{bmatrix} < 0 \quad \textbf{(14)}$$

By using Schur complement lemma[13], this inequality further implies $\Theta < 0$ and then $\Delta V(x(k)) \leq -\grave{o}\|x(k)\|^2$ with $\grave{o} = -\lambda_{\max}(\Theta) > 0$, and then system (1) is asymptotically stable. This completes the proof.

In Theorem 1, the diagonal elements of matrix ε should satisfy $\varepsilon_i \leq 0$. As a special case, we can set $\varepsilon = 0$ and obtain the following corollary.

Corollary 1: The discrete-time linear system (1) with state saturation is asymptotically stable, if there exist a symmetric positive-definite matrix P and matrices G, X satisfying (6a) and

$$\begin{bmatrix} -P & (D_i A + D_i G)^T X^T \\ * & -X - X^T + P \end{bmatrix} < 0, i \in [1, 2^n] \quad \textbf{(15)}$$

Furthermore, if we choose $X = P$ as a special case, we can obtain the following corollary.

Corollary 2: The discrete-time linear system (1) with state saturation is asymptotically stable, if there exist a symmetric positive-definite matrix P and a matrix G satisfying (6a) and

$$\begin{bmatrix} -P & (D_i A + D_i G)^T P \\ * & -P \end{bmatrix} < 0, i \in [1, 2^n] \quad \textbf{(16)}$$

Remark 2: In Theorem 1, the Lyapunov functional (9) is with a constant Lyapunov matrix P. Motivated by the facts that nonlinear system (1) is transformed into polytopic-type uncertain system (8) and that the parameter-dependent Lyapunov functional is very popular for polytopic-type systems, we can design a Lyapunov functional dependent on η_i for system (8). Noting also that η_i denotes the saturation factor for system (1), we refer to this parameter-dependent Lyapunov functional as saturation-dependent Lyapunov, that is,

$$V(x(k)) = x^T(k)\left(\sum_{i=1}^{2^n} \eta_i P\right)_i x(k)$$

where P_i, $i \in [1, 2^n]$ are symmetric positive-definite matrices to be determined. Following the same philosophy as in the proof of Theorem 1, we can straightforwardly obtain the corresponding stability criterion. We can refer to references [11], [14] for detail.

Noting that stability criterion (6) is a bilinear matrix inequality condition and thus it is difficult to resolve numerically using the matured numerical algorithm. To transform it to a numerically tractable one, we follow the philosophy in[11] and transform $G = [g_{ij}] \in \Re^{n \times n}$ with $\|G\|_\infty \le 1$ as

$$h_i G y_j \le 1,\ i \in [1, n],\ j \in [1, 2^n]$$

where H is the set of n dimensional row vectors which has only one nonzero element which is 1, $h_i \in \mathrm{H}$ is an element whose *i*th element is 1, Y is the set of n dimensional column vectors whose elements are 1 or -1. There are 2^n elements in Y and we denote its *j*th element as y_j. Then, the following iterative linear matrix inequality algorithm can be given.

Algorithm 1: Asymptotical stability for the discrete-time linear system (1) with state saturation.

Step 1) Select a matrix G satisfying $\|G\|_\infty \le 1$ and $\varepsilon = 0$. Set $k = 0$, $\gamma_k > 0$ be a sufficiently big scalar.

Step 2) Solve the following linear matrix inequality optimization problem for X, P and γ,

$$\min_{X,P} \gamma$$

$$\text{s. t. } \Xi_i < \gamma, \forall i \in [1, 2^n]$$

If $\gamma < 0$ or $\gamma > \gamma_k$, go to Step 4). Otherwise, set $k = k + 1$, $\gamma_k = \gamma$, go to the next step.

Step 3) Using X and P obtained previously, solve the following linear matrix inequality optimization problem for ε, G and γ,

$$\min_{\varepsilon, G} \gamma$$

$$\text{s. t. } \begin{cases} \Xi_i < \gamma, \forall i \in [1, 2^n] \\ h_i G y_j \le 1, i \in [1, n], j \in [1, 2^n] \\ \varepsilon \le 0 \end{cases}$$

If $\gamma < 0$ or $\gamma > \gamma_k$, go to Step 4). Otherwise set $k = k+1$, $\gamma_k = \gamma$, go to Step 2).

Step 4) If $\gamma < 0$, system (1) is asymptotically stable. Otherwise, no conclusion can be drawn. Different ε and G should be chosen and algorithm can be repeated from Step 1).

We are now in the position to consider the synthesis problem for system (1). Consider system (1) with control input $u(k)$, that is

$$x(k+1) = h(Ax(k) + Bu(k)) \tag{17}$$

where $u(k) \in \Re^m$ is the control input vector, and $B = [b_{ij}] \in \Re^{n\times m}$ is a constant matrix. Our objective is to design a state feedback control law $u(k) = Fx(k)$ for the resultant closed-loop system to be asymptotically stable. The result of Theorem 1 can be straightforwardly applied to resolve this problem by replacing A by $A+BF$ in (6), which gives the following sufficient conclusion.

Theorem 2: Consider the discrete-time linear system (17) with state saturation. This system is asymptotically stabilizable, if there exists a symmetric positive-definite matrix P, matrices G, X, F, $\varepsilon = \mathrm{diag}\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}$ with $\varepsilon_i \le 0$, $i \in [1, n]$ satisfying (6a) and

$$\Gamma_i = \begin{bmatrix} -P & ((D_i + \varepsilon D_i^-)(A+BF) + D_i^- G)^T X^T \\ * & -X - X^T + P \end{bmatrix} < 0, i \in [1, 2^n] \tag{18}$$

Moreover, a suitable state feedback control law is given as

$$u(k) = Fx(k)$$

The result can be straightforwardly obtained by replacing A by $A+BF$ and using the result of Theorem 1. This concludes the proof.

This condition also involves bilinear matrix inequalities and thus it is difficult to check numerically. The following algorithm gives an iterative linear matrix inequality approach to obtain a suitable state feedback gain F, which is somewhat similar to Algorithm 1.

Algorithm 2: State-feedback control law design algorithm for system (17).

Step 1) Select a matrix $Q > 0$ and solve P from the following Lyapunov function,

$$(A+BF)^T P + P(A+BF) = -Q$$

where F is chosen such that $A+BF$ is Hurwitz stable. Set $X = P$, $k = 0$ and $\gamma_k > 0$ be a sufficiently big scalar.

Step 2) Using X and P obtained previously, solve the following linear matrix inequality optimization problem for ε, G, γ,

$$\min_{\varepsilon, G} \gamma$$

$$\text{s. t.} \begin{cases} \Gamma_i < \gamma, \forall i \in [1, 2^n] \\ h_i G y_j \le 1, i \in [1, n], j \in [1, 2^n] \\ \varepsilon \le 0 \end{cases}$$

If $\gamma < 0$ or $\gamma > \gamma_k$, go to Step 5). Otherwise, set $k = k+1$, $\gamma_k = \gamma$, go to the next step.

Step 3) Using ε, G and F obtained previously, solve the following linear matrix inequality optimization problem for X, P, γ,

$$\min_{X,P} \gamma$$

$$\text{s. t. } \Gamma_i < \gamma, \forall i \in [1, 2^n]$$

If $\gamma < 0$ or $\gamma > \gamma_k$, go to Step 5). Otherwise, set $k = k+1$, $\gamma_k = \gamma$, go to Step 2).

Step 4) Using ε, P and X obtained previously, solve the following linear matrix inequality optimization problem for G, F, γ,

$$\min_{G,F} \gamma$$

$$\text{s. t.} \begin{cases} \Gamma_i < \gamma, \forall i \in [1, 2^n] \\ h_i G y_j \le 1, i \in [1, n], j \in [1, 2^n] \end{cases}$$

If $\gamma < 0$ or $\gamma > \gamma_k$, go to Step 5). Otherwise, set $k = k+1$, $\gamma_k = \gamma$, go to Step 2).

Step 5) If $\gamma < 0$, system (17) is asymptotically stabilizable, and a suitable state feedback control is given as $u(k) = Fx(k)$. Otherwise, no conclusion can be drawn. Different F and Q can be chosen and algorithm can be repeated from Step 1).

NUMERICAL EXAMPLES

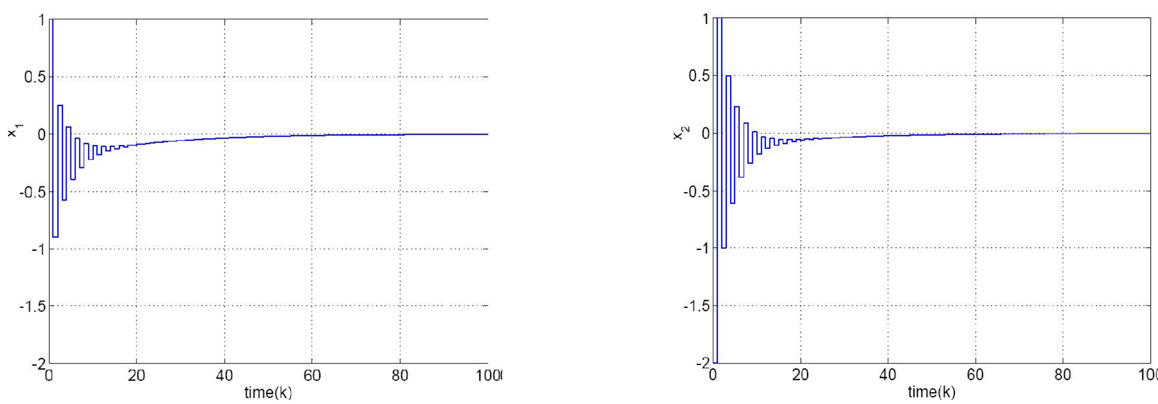
In this section, two numerical examples are given to show the applicability of the presented method.
We first consider system (1) with

$$A = \begin{bmatrix} 0.5 & 0.7 \\ 0.9 & -0.3 \end{bmatrix}$$

Applying Algorithm 1, we can show that this system is asymptotically stable. A solution to matrix inequality (6) is given as

$$P = \begin{bmatrix} 3.4302 & 0.1563 \\ 0.1563 & 2.6827 \end{bmatrix}, X = \begin{bmatrix} 3.3205 & 0.0352 \\ 0.1446 & 2.8039 \end{bmatrix}, G = \begin{bmatrix} 0.4123 & 0.5089 \\ 0.5172 & -0.3743 \end{bmatrix}, \varepsilon = \begin{bmatrix} -1.1423 & 0 \\ 0 & -0.5658 \end{bmatrix}$$

With the initial state $x(0) = [1 \ -2]^T$, the state trajectory of this system is shown in Figure 1.



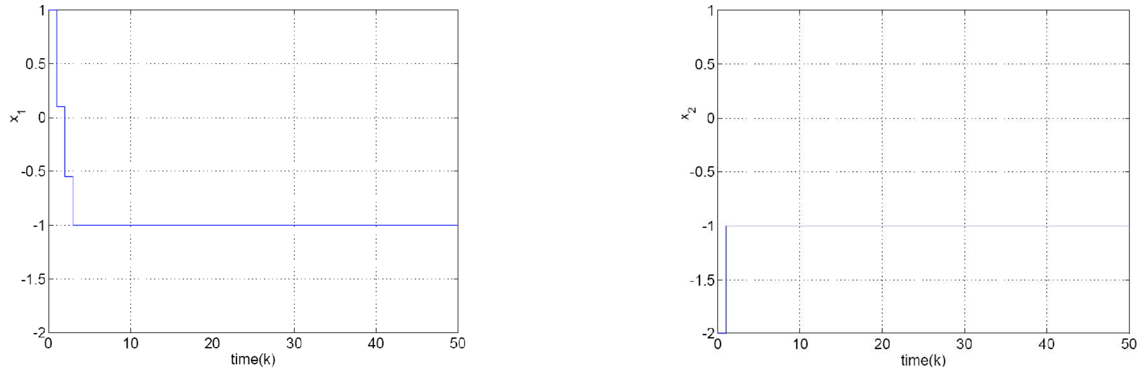
(a) State trajectory of $x_1(k)$ **(b) State trajectory of $x_2(k)$**

Figure 1 : State trajectory of discrete linear system with state saturation

The next example deals with system (17) with the following parameters,

$$A = \begin{bmatrix} 1.5 & 0.7 \\ 0.9 & 1.3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The eigenvalues of matrix A is 2.2 and 0.6, and then the open-loop system is instable even if state saturation does not occur. With the initial state $x(0) = [1 \ -2]^T$, the open-loop state trajectory of this system is shown in Figure2. It can be shown the system state converges to equivalent point $[-1 \ -1]^T$.



(a)Open-loop trajectory of state $x_1(k)$ **(b) Open-loop trajectory of state $x_2(k)$**

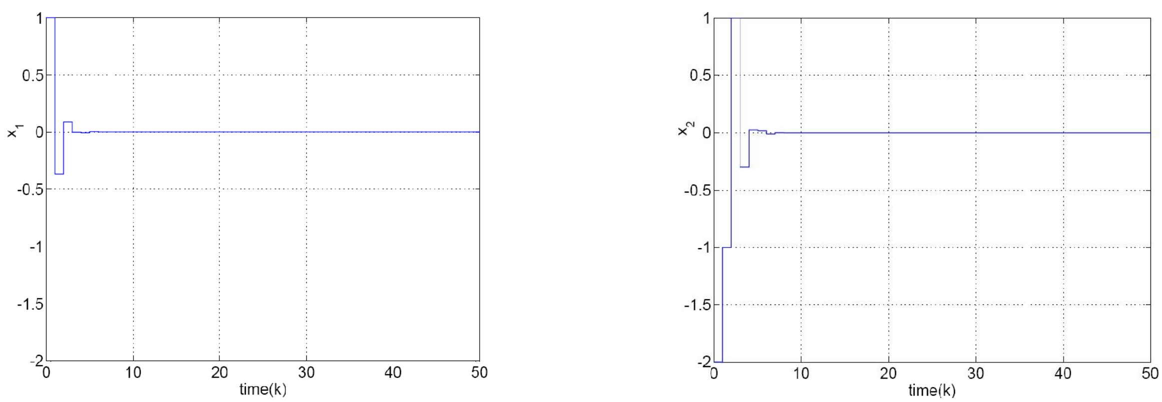
Figure 2 : State trajectory of discrete linear system with state saturation

Applying Algorithm 2 gives the following solution to inequality (18) as

$P = \begin{bmatrix} 7.2894 & -0.3426 \\ -0.3426 & 0.7204 \end{bmatrix}, X = \begin{bmatrix} 6.1896 & 0.1423 \\ -0.5517 & 0.8086 \end{bmatrix}, G = \begin{bmatrix} -0.4135 & 0.1275 \\ 0.1569 & -0.7814 \end{bmatrix}, \varepsilon = \begin{bmatrix} -0.2433 & 0 \\ 0 & -0.4145 \end{bmatrix}$ and a suitable sate feedback control law is given as

$$u(k) = \begin{bmatrix} -1.8172 & -0.6743 \end{bmatrix} x(k)$$

With this state feedback control law, the closed-loop state trajectory of this system is shown in Figure3, which implies the asymptotical stability of the closed-loop system.



(a) Closed-loop trajectory of state $x_1(k)$ **(b) Closed-loop trajectory of state $x_2(k)$**

Figure 3 : State trajectory of closed-loop discrete linear system with state saturation

CONCLUSION

The stability analysis problem for a class of discrete-time linear systems with state saturation nonlinearity is considered in this paper. With the introduction of a free matrix whose infinity norm is less than or equal to 1 and a diagonal matrix with non-positive diagonal elements, the discrete state under saturation nonlinearity is confined in a convex hull, and then a stability criterion is given for such system to be asymptotically stable. This criterion is also applied to resolve the state feedback control law synthesis problem. These results are presented in terms of bilinear matrix inequalities and the corresponding iterative linear matrix inequality algorithm is given.

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