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Springboard diving technical development features kinematical simulation research based on differential equation model

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ABSTRACT

Springboard diving is favored by broad masses due to its elegant movements, is one of most popular Olympic Games events. In recent years, with diving development, athletes' competitions has grown fiercely, scientific training way is the basis that ensure athlete get excellent results. The paper targeted springboard diving technical movements, combines with differential equation knowledge and kinematics, mechanics of materials as well as other knowledge, it makes force analysis of athletes and pedal, establishes differential equation model, and combines with computer analogue simulation technique, it tests on model and knows that model has practicability. Finally it inputs athletes' kinematic parameters into differential equation model; solve differential equation model by computer technology, it gets calculation data results, so that analyzes data and makes improvement suggestions on springboard diving training with an aim to promote Chinese diving development.

KEYWORDS

Differential equation;
Mechanics of materials;
Computer simulation;
Springboard diving.

PREFACE

During the late 19th century and early 20th century, modern diving competition event was originated from Europe. In the 3rd Olympic Games organized in 1904, committee listed diving as formal competition event. In the 4th Olympic Games organized in 1908, it defined formal competition rules by committee discussing. In the 5th Olympic Games in 1912, women's diving events was listed as formal competition events. From now on, diving has been developed in global. Presently, besides Olympic Games, worldwide major matches include world championship and world cup.

Chinese diving has been rapidly developed and already in the world advanced level. But with era progress, every country diving has been rapidly developed, every country's athletes levels are much closed, therefore competition has grown more fierce. Especially in nowadays world, advances to technology come quickly, each kind of new diving training ways emerge in endlessly, any country athlete cannot ensure him to be the dominant one. Therefore, Chinese reform on diving training should also constantly strengthen, integrate new techniques in training, reform in training, make progress in training, which are the basis that guarantee Chinese diving never lag behind and always in the world advanced level. In recent years, computer analogue simulation technique has been rapidly developed, computer technology has already integrated into all walks of life, and there is no exception for sports; lots of national coaches and athletes combine computer analogue simulation technique with their own country's each sports event and further achieve new training plans. There is no exception for China; it integrates computer analogue simulation technique in football, badminton, table tennis and other lots of sports events training. Just on the basis of Chinese such status, the paper works on combining springboard diving with computer analogue simulation technique, it establishes differential equation model in the hope of speeding up Chinese diving development.

PEOPLE AND BOARD MODEL ESTABLISHMENT AND SOLUTION

At first, system partition people and board into two that are also divers' body and springboard. The paper regards diver's body as a particle that is formed by two pieces of mass less rigid slender rods hinges connecting, and let the two slender rods and vertical direction included angles to be the same, both are $\theta(t)$, as Figure 1 show, they change with time. To springboard, due to diving process springboard mainly takes vertical movements, horizontal displacement is nearly zero by observing. In this way by mechanics of materials, it is clear that it can regard springboard as elastic beam in plane, it can refer to Figure 2.

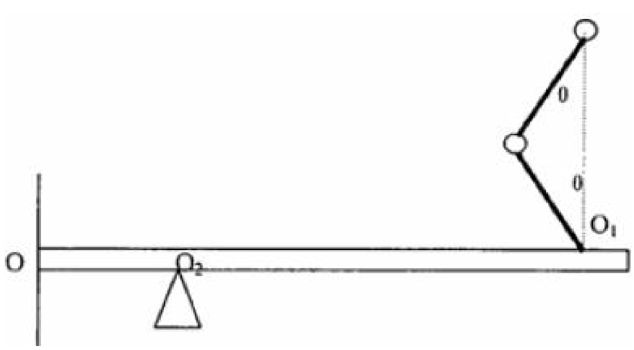

Figure 1: Human body model simplified analysis graph

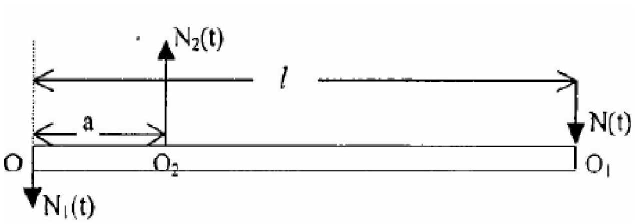

Figure 2 : Springboard model simplified analysis graph

By analyzing Figure 2, it gets that there is a support point in the middle of springboard, by adjusting support point, it can efficiently change springboard length and then achieve the target of changing its vibration period. Assume $\overline{m}$ is springboard mass, l is springboard length, a is the length from support point to fixed end, $N(t)$ is diver's body to springboard action force, $N_2(t)$ is external to support point action force, $N_1(t)$ is external to springboard supported end action force.

Springboard dynamics analysis

By Figure 2 combining with kinematics and mechanics relative theoretical analyses, create dynamical equation (springboard should meet). At first, by force and moment balance, it is clear that it has following formula:

$$\begin{cases} N_2(t) = N(t) + N_1(t) \\ N_2(t)a = N(t)l \end{cases} \quad (1)$$

Thereupon, it can get:

$$N_1(t) = \frac{l-a}{a} N(t) \quad (2)$$

$$N_2(t) = \frac{l}{a} N(t) \quad (3)$$

Due to diving springboard is a kind of elastic board, when it suffers force effects, it will occur to deform and then generate bending moment. And because springboard will not generate greatly deformation, the paper thinks it is a kind of little motion problems. Therefore the paper assumes when springboard generates deformation, the arm of force at one point will not change. Take O point as reference point, let t time springboard far from O point x area bending moment is $M(x,t)$, then it has:

$$M(x,t) = \begin{cases} N(t)(l-x) - N_2(t)(a-x), 0 \le x \le a \\ N(t)(l-x), a \le x \le l \end{cases} \quad (4)$$

By formula(3)(4)it can get:

$$M(x,t) = \begin{cases} N(t) = \frac{l-a}{a} x, 0 \le x \le a \\ N(t)(l-x), a \le x \le l \end{cases} \quad (5)$$

Combine with mechanics of materials beam's one end fixed supporting and another one free, it can list bending differential equation:

$$EJ\frac{\partial^2}{\partial x^2}y_2(x,t)=M(x,t) \tag{6}$$

According to formula(5)(6)it can get:

$$EJ\frac{\partial^2}{\partial x^2}y_2(x,t)=\begin{cases}N(t)=\frac{l-a}{a}x, 0\le x\le a\\ N(t)(l-x), a\le x\le l\end{cases} \tag{7}$$

Among them, $y_2(x,t)$ is springboard far from fixed end x position's deflection at t moment,Inertia moment is using J to express and $J=\frac{bu^3}{12}$, springboard width is using b to express, springboard thickness is using u to express. And then the paper makes use of integral computational formula (7) and gets:

$$EJ\frac{\partial}{\partial x}y_2(x,t)=\begin{cases}N(t)=\frac{l-a}{2a}x^2+c_1, 0\le x\le a\\ N(t)(lx-\frac{1}{2}x^2)+c_2, a\le x\le l\end{cases} \tag{8}$$

According to $EJ\frac{\partial}{\partial x}y_2(x,t)$ $x=a$ area continuity, it has:

$$N(t)=\frac{l-a}{2a}x^2+c_1=N(t)(lx-\frac{1}{2}x^2)+c_2 \tag{9}$$

Simplify and can get:

$$c_1=\frac{1}{2}N(t)la+c_2 \tag{10}$$

Make integral of formula (8) and can get:

$$EJy_2(x,t)=\begin{cases}N(t)=\frac{l-a}{6a}x^3+c_1x+d_1, 0\le x\le a\\ N(t)(\frac{1}{2}lx^2-\frac{1}{6}x^3)+c_2x+d_2, a\le x\le l\end{cases} \tag{11}$$

Because O point is springboard fixed point,and once O point is defined, it also remains stationary, therefore at any time t, its deflection is O, that displacement is always, O, which is also $y_2(0\ ,\ t)=0$ and $y_2(a,t)=0$.

According to formula(11)by $y_2(0\ ,\ t)=0$, it gets $d_1=0$,by $y_2(a,t)=0$, it can get:

$$N(t)=\frac{l-a}{6a}a^3+c_1a=0 \tag{12}$$

So $c_1=\frac{a(a-l)}{6}N(t)$

Combine with formula(10), it can get c_2:

$$c_2=\left(\frac{a^2}{6}-\frac{2la}{3}\right)N(t) \tag{13}$$

Now we take upward direction as positive direction,record it as $y_{01}(t)=y_2(1,t)$,that $y_{01}(t)$is t moment springboard free end position. Then it has:

$$\overline{m}y_{01}^{n}=-N(l)-ky_{01}(l) \tag{14}$$

Body dynamics analysis

It is well known that diver body movement is regarded as linear movement in plane. The paper regards diver's body as a particle, its structure is formed by two pieces of massless rigid slender rods hinges connecting. Set human body mass is m, $y_c(t)$is t moment human body mass center position. By Newton first law of motion, it has:

$$my_c''(t)=-mg+N(l) \tag{15}$$

Among them, g is gravitational acceleration.

People-Board differential equation model establishment

The paper describes springboard diving process as following: After first time hurdling, athlete body falls from height h area far from mass center, it starts to record time from athlete is next to springboard such moment, after T time passing, athlete is in the rising stage, and raised maximum height is H. In the series of process, the paper assumes that athlete body speed is V_{1O}, springboard speed is V_{2O}.After T time passing, which is also the moment athlete departs the springboard, athlete speed is V_{1out}, springboard speed is V_{2out}, then it is clear by above:

$$\begin{cases}y_c'(0)=V_{1O}\ ,\ y_{01}'=V_{2o}\\ y(t)=y_c(t)-y_{01}(t)=2L\cos\theta(t)\end{cases} \tag{16}$$

In the instant that athlete is just next to springboard, assume that athlete and springboard common mechanical energy is completely converted by his potential energy; in the instant that athlete just contacts with springboard, hid body muscles don't work. Let $y_2(x,t)$ to be springboard at t moment far from support point x area's deflection, and then beam's kinetic energy is:

$$T = \frac{1}{2}\int_0^l \bar{m} y_2(x,t)^2 dx \tag{17}$$

Therefore:

$$T_S = \frac{1}{2}\bar{m}\left(\frac{dy_2(l,t)}{dt}\right)^2 \tag{18}$$

And when $\Delta t \to 0$, it can think that beam's elastic potential energy is 0, therefore according to energy conservation law, it gets:

$$mgh = \frac{1}{2}\bar{m}\left(\frac{dy_2(l,t)}{dt}\right)^2 + \frac{1}{2}mv_{1o}{}^2 \tag{19}$$

Therefore, it has:

$$mgh = \frac{1}{2}\bar{m}v_{2o}{}^2 + \frac{1}{2}mv_{1o}{}^2 \tag{20}$$

People jump in the flat ground, one time takeoff effects merits measurement is to check takeoff height or people liftoff instant speed size. To people-board system, takeoff height is principal element, however in springboard diving process; diver's use rate of his own energy is also one of important elements. In the moment people leave springboard, body mass center takes upward movement at initial speed V_{cont}, body mass center position is y_{cont}, V_{o1ont} is springboard free end speed, y_{o1ont} is springboard free end position, now springboard potential energy is:

$$\frac{1}{2}\int_0^l EJ[\frac{\partial^2 y_2(x,t)}{\partial x^2}]^2 dx = \frac{1}{2}Ky_2^2(l,t) = \frac{1}{2}Ky_{o1out}^2 \tag{21}$$

Board kinetic energy is:

$$\frac{1}{2}\int_0^l \bar{m} y_2(x,t)^2 dx = \frac{1}{2}\bar{m}V_{o1out}^2 \tag{22}$$

People takes upcast movements at initial speed V_{cont}, set maximum lifting height is H,then:

$$mg(H - y_{cont}) = \frac{1}{2}mV_{cont}^2 \tag{23}$$

$$H = \frac{V_{cont}^2}{2g} + y_{cont} \tag{24}$$

Therefore it can make function:

$$E = \begin{cases} \dfrac{mg(H-h)}{\frac{1}{2}\bar{m}V_{o1out}^2 + \frac{1}{2}Ky_{o1out}^2 + mg(H-h)} & (H > h) \\ 0 & (H < h) \end{cases} \tag{25}$$

In above formula(25), numerator indicates people increased potential energy in this springboard pedaling period, denominator is people muscles actions in this period, E_f reflects energy use rate. When V_{cont} is certain, E_f gets bigger, then it shows pedaling and extending effects are better. When H is certain, E_f better evaluates one time pedaling and extending movement effects merits.

Control function construction

Regarding people and board system control function $y(t)$ selection, there are some kinds of common methods in the following:

(1) Analyze athlete movements' movies and videos materials, calculate body mass center changing graphs and springboard free end movement curve graphs, and further calculate control function;

(2) Take a kind of concrete function as control function, as polynomial function, trigonometric function and so on;

(3) Achieved by mouse fixed point revising on alignment curve.

By above methods obtained discrete data, the paper smoothes them, or uses curve fitting and other methods. Analytic function derivative's definition is as following:

$$y'(t) = \lim_{\Delta t \to 0}\frac{y(t+\Delta t) - y(t)}{\Delta t} \tag{26}$$

And when function expression can only use discrete points to express, we can use following methods to approximately calculating derivatives:

$$y'(t) \approx \frac{y(t+\Delta t) - y(t)}{\Delta t} \tag{27}$$

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Only when Δt is very small, it can ensure accuracy requirement. But in actual calculation, when Δt is very small, $y(t+\Delta t)$ and $y(t)$ get closer, it will cause effective data lose after subtracting, on the contrary it causes accuracy reduce. Therefore the method has no practical significance. In order to solve control function derivative, we can also use curve fitting method that uses interpolation polynomial $p_m(t)$ approximate function $y(t)$, and then uses $p_m(t)$ one point each order derivative as $y(t)$ the point each order derivative approximation.

Model simulation analysis and model test solution

According to above differential equation model and numerical handling process, the paper compiles people-board simulation system software to investigate different control functions and different parameters status influences on system. System parameters selection is as following TABLE 1 show.

TABLE 1 : Simulation system parameters values

m	$\bar{m}$	k	h	T	y_{max}	y_{min}
72.00	45.00	6543.3117	0.67	0.5129	1.20	0.49325

By running simulation system software, it gets results as following Figure 3 and Figure 4 show:

By Figure(3), (4), it gets that the paper obtained simulation results have stability, and control function $y(t)$ can

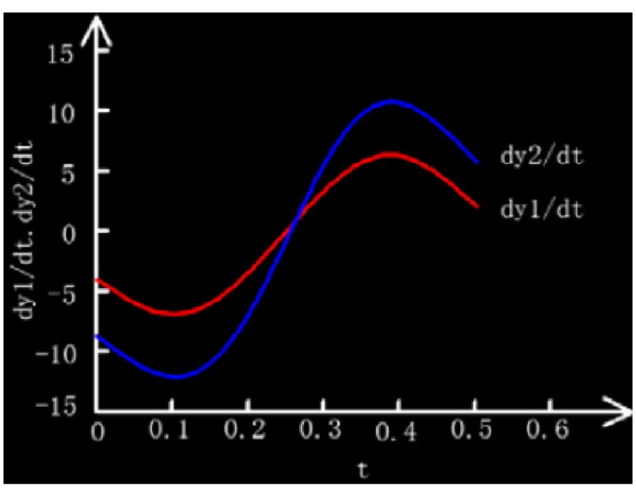

Figure 3 : When control function is sinusoidal periodic function, dy_c/dt and dy_{o1}/dt changing curves

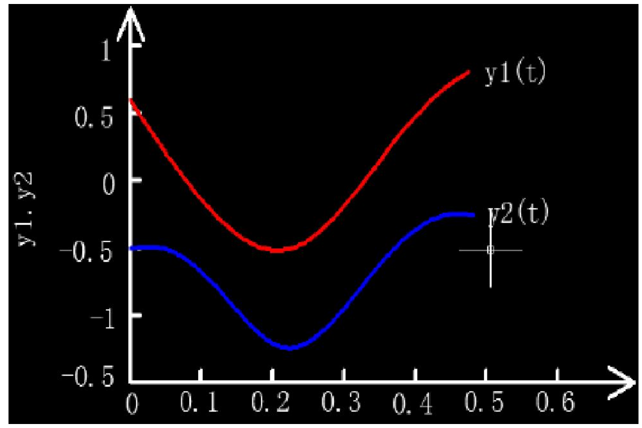

Figure 4 : When control function is sinusoidal periodic function, y_c and y_{o1} changing curves

be arbitrary selected, but control function that doesn't meet physiological limits has no practical significances. Therefore, it is clear the simulation model has reliability, input data has output that conforms to reality. Therefore, input data V_0 (V_0 value range is $0 \geq V_0 \geq -\sqrt{2gh}$) it can get calculation results as Table 2 show.

From TABLE 2 result, it is clear V_0 gets smaller, the obtained takeoff height will get higher, and $-V_0 = V_{20} - V_{10}$, and in human body contacting with board instant, board speed is very small that can approximate to 0, that V_0 gets smaller, H would get bigger. And V_0 can be thought to be decided by previous takeoff height h, therefore in practical diving, reasonable allocate initial speed V_0 is the key to influence on H size, athlete in daily training and competition should

TABLE 2 : People and board relative initial speed V_0 influences on system

V_0	y_{cout}	y_{o1out}	V_{cout}	V_{o1out}	H	E_f
-3.62	0.666	-0.559	4.199	0.583	1.566	0.381
-3.50	0.668	-0.556	4.102	0.583	1.527	0.373
-3.00	0.679	-0.541	3.697	0.700	1.377	0.341
-2.50	0.690	-0.527	3.292	0.795	1.224	0.306
-2.00	0.700	-0.513	2.888	0.889	1.026	0.271
-1.50	0.711	-0.499	2.482	0.984	0.942	0.234
-1.00	0.722	-0.484	2.078	1.079	0.875	0.198
-0.50	0.732	-0.470	1.673	1.173	0.859	0.165
-0.25	0.738	-0.463	1.470	1.221	0.848	0.151

special note that he cannot quickly squat in human body falling and contacting board instant, while he should first use foot to contact with pedal, in dropping process, body gradually curls, thigh exerts to slowly increase and achieves people and board follow-through combined common movements.

Take $V_0 = -3.5$, other parameters values selection is unchangeable,take different h values calculation results as TABLE 3 show.

TABLE 3 : First time takeoff height h influences on height

h	y_{cout}	y_{o1out}	V_{cout}	V_{o1out}	H	E_f
0.00	-0.090	-1.314	3.325	-0.171	0.474	0.055
0.25	0.242	-0.983	2.904	-0.591	0.672	0.086
0.50	0.500	-0.724	3.553	0.057	1.144	0.209
0.75	0.746	-0.487	4.378	0.882	1.725	0.477
1.00	0.986	-0.238	5.294	1.798	2.417	0.832
1.25	1.222	-0.002	6.269	-0.002	3.227	0.973
1.50	1.455	0.230	7.284	3.787	4.163	0.884
1.75	1.686	0.461	8.331	4.834	5.338	0.751
2.00	1.919	0.690	9.403	5.906	6.426	0.643

Therefore it is clear that first time takeoff height has very big relations with second time takeoff height, the first time takeoff height gets higher, and then it is beneficial to the second time takeoff. Therefore in practical springboard diving, after walking board process, diver first leaps and then treadle-jumps, it should try to make leap height arrive at maximum. It requires that athlete strengthen explosive power training in daily training, explosive power gets stronger, obtained takeoff height gets bigger.

CONCLUSIONS

The paper researches on springboard diving training, it establishes differential equation model, and carries out analogue simulation on diving movements and solves differential equation model by computer technology, it gets some movement rules and makes improvement suggestions. The paper combines differential equation modeling with recent years' rapidly developed computer technology and common uses them into sports events training as a great innovation, which provides a new guiding way for sports events physical training. By computer calculation results researching on springboard diving, it gets athlete explosive power is the important factor that affects sports performance, in future training, it should strengthen explosive power training so that let first takeoff height to be high as much as possible. For researching on human body falling and contacting with springboard phase, it is known that in future training, when athlete falls and contacts with springboard, he cannot directly exert to pedal while should let body and springboard cooperate and then fall down follow-through, so that reduce V_{20} and increase V_{10} absolute values, and let springboard subsequent takeoff height to get bigger.

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