# Research on the joint decision problem about facilities locating and routes designing with common routes 

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#### Abstract

Mathematical models of double routes including desirable facilities locating and undesirable routes joint decision problems with common routes on the basis of analyzing desirable facilities locating and undesirable routes problems. During the solution of these models, in order to solve the conflict between the two objects of desirable facilities locating and undesirable routes designing joint design problem, comprehensive heuristic algorithm based on adjusting arrangement was designed by setting the upper limit of affected residents and minimizing the main object of weighed distance between facilities and residential area. The calculating results and sensitivity analysis showed that these models and calculation methods were available and effective.

\section*{KEYwORDS}

Common routes; Facilities locating; Routes designing; Up limit of affected residents; Heuristic algorithm.


## INTRODUCTION

Facilities locating and routes designing problems have been existed for a long time ${ }^{[1-4]}$. But in our usual life there is a common special situation that people hope to get closer to the facilities which provides services, and to get farther from the routes between the facilities. For example the subway system, people want to live closer to the subway station to travel conveniently, but they would like to live farther away from the subway routes to avoid the disturbance of noises. This constitutes the joint decision problem of desirable facilities and undesirable routes.

In this work, the joint decision problem of desirable facilities locating, undesirable routes choice problem, and double routes desirable facilities and undesirable routes with common routes were researched and analyzed. Because this problem is a multi-objective optimization which needs minimization of weighed distance as well as the number of affected residents, and the two objectives have a certain extent of conflict, this research designed a constraint of affected residents upper limit and put it into the site selection models. In the comprehensive algorithm, this research designed a kind of comprehensive heuristic algorithm which was adjusted and arranged under dissatisfying the affected residents' limits. The calculation case showed the adjusting method was effective to solve the joint decision problem of facilities locating and routes designing in real life.

## PROBLEM DESCRIPTION

## Desirable facilities locating problem with locating constraint areas

In a continuous plane, a candidate area is given previously for convenient problem description and solution with the assumption that each candidate area is rectangle. Residential areas distribute discretely in the continuous plan. It is hypothesized that the residents in each residential area distribute uniformly and the geometric center of each residential area is chosen to replace the site of residential area. The weighed distance between all residential areas and service facilities is minimized as the goal, and reasonable location is chosen for desirable facilities of candidate area.

## Undesirable routes problem

After the location of desirable facilities is given, the undesirable routes choice and design between facilities can be carried out to minimize the influence and threaten to surrounding residents. It is assumed that residents within the distance of $r$ can be affected, and the links between facilities are regular arcs in order to make the problem soluble at the same time. Thus, undesirable link locations can be represented by the angle of osculation of the desirable facilities arcs ${ }^{[5]}$, as shown in Figure 1.


Figure 1: The example of undesirable link
The point $T$ and $P$ in Figure 1 are adjacent desirable facilities, whose coordinates are $T(-d / 2,0)$ and $P(d / 2,0)$, where $d$ is the Euclidean distance between $T$ and $P, r$ is the affected distance of routes to surrounding residents, $m_{1}, m_{2}, m_{3}$, and $m_{4}$ are representing four residential areas, and arc $T S P$ is a random arc from point $T$ to $P$. Coordinate system is built as shown in Figure 1, in which the coordinate of center $Q$ of $\operatorname{arc} T S P$ is $(0, d / 2 \tan \alpha)$, radius of $\operatorname{arc} T S P$ is $R(\alpha), R(\alpha)=d / 2 \sin \alpha$. Designing problem of undesirable link can be described as: to obtain the best angle of osculation and minimize the number of residents within the distance of $r$ in two sides of arc $T S P$. That is to make the residential weighed sum smallest within the dash line in Figure 1, and to obtain the smallest affect to residential area by adjusting the angle of osculation $\alpha$ of arc TSP.

In the situation of multi-routes, if the two facilities connected by one route are both common facilities, this route is common route. The common link of two routes is taken as an example and shown in Figure 2.


Figure 2: Diagram of a common route

## Double routes design problem with common desirable facilities and common undesirable routes

In a given plane $G$, the residential areas' number is $J$. The set is assigned as $M=\left\{m_{j}\right\}=\left\{\left(a_{j}, b_{j}\right)\right\}$, in which the coordinate of residential area $m_{j}$ is $\left(a_{j}, b_{j}\right)$, and the population weight of residential area $m_{j}$ is $\omega_{j}, j=1,2, \ldots, J$. The desirable facilities' number is $I$, providing service to residents. The given rectangle areas' number is $I$, which are used as locating for facilities and one facility will be built in one area. The facilities set $R=\left\{r_{i}\right\}=\left\{\left(x_{i}, y_{i}\right)\right\}$, the range area of each facility is $\left\{\left(x_{i}\right.\right.$ , $y_{i}$ ) $\left.c_{i} \leq x_{i} \leq d_{i}, e_{i} \leq y_{i} \leq f_{i}\right\}, c_{i}, d_{i}, e_{i}, f_{i}$ is constant, $i=1,2, \ldots, I$. Two undesirable routes will be designed now, named as $l_{1}$ and $l_{2}$. It has been known that two common facilities named as $S$ and $U$ exist in the two routes and the link $S-U$ between $S$ and $U$ is common undesirable link. The set of all undesirable links, whose number is $V$, in the two routes is $T=\left\{t_{v}\right.$ $\}$, and $\Omega_{i}$ is represented as the set of residential areas which is provided service by the facility $i$. The facilities locating and routes designing will be carried out to make the weight distance between all the facilities including common facilities and residential areas in the two routes as small as possible, and to make the affected residents number to be less than the required upper limit.

It can be inferred from the actual experience that in multi-routes designing it is impossible for each route to serve all residential areas like single routes. It is not only impractical but also inefficient. At the same time, when the residential area is served by many routes, the residents will choose one route according to a certain proportion. Therefore, we assume that:
(1) Routes $l_{1}$ and $l_{2}$ have given facilities set named as $R_{1}$ and $R_{2}$, meeting the requirements of $R_{1} R_{2}=R$, and $R_{I} \cap R_{2}=\{S, U\}$.
(2) Routes $l_{1}$ and $l_{2}$ have given residential areas set named as $M_{1}$ and $M_{2}$, meeting the requirements of $M_{1} \quad M_{2}=M$.
(3) Routes $l_{1}$ and $l_{2}$ have given links set named as $T_{1}$ and $T_{2}$, meeting the requirements of $T_{1} \quad T_{2}=T$.
(4) The proportion of two routes chosen by residents from each residential area is given as $P_{j}=\left\{p_{l j}, p_{2 j}\right\}$, meeting the requirements of $p_{n j}=\left\{\begin{array}{ll}0 & m_{j} \notin M_{n} \\ 0\left\langle p_{n j} \leq 1\right. & m_{j} \in M_{n}\end{array},(\mathrm{n}=1,2)\right.$.

## MATHEMATICAL MODEL

The routes designing problem of linking desirable facilities and undesirable facilities with locating area constraint is involving two kinds of ordered problems. Firstly, the locating problem of desirable facilities with locating area constraint need to be decided with the objective of minimizing the weighting distance between the facilities and residential area. Secondly, in the given desirable situation, choose reasonable angle of osculation and minimize the number of affected residents.

The two objectives are obviously ordered, and the secondary objective is restricted by facilities locating and it can only be decided after the decision of facilities locating. At the same time, the facilities with the smallest weighing distance obtained in the first step cannot always ensure the smallest number of affected residents in whole, the two objectives have a certain extent of contradiction. Therefore, it should be considered to set a constraint for the number of affected residents and solve the main objective of minimizing the weighing distance between facilities and residential areas.

The weighing number of residential area can be adjusted in the existed two routes. To clearly describe the feature of common facilities $S$ and $U$, three variables $\lambda_{i j}$, $\lambda_{n i j}$, and $z_{v j}$ are set in the range of 0-1.
$\lambda_{i j}=\left\{\begin{array}{l}1 \text { if served for the resident area } m_{j} \text { to } r_{i} \\ 0\end{array} \quad\right.$ or else $\quad i=1,2,3 \ldots, I$, and $\neq\{S, U\}, j=1,2,3 \ldots, J$.
$\lambda_{n i j}=\left\{\begin{array}{ll}1 \text { if common facility } i \text { is located in route } n \text { to serve resident area } j \\ 0 & \text { or else }\end{array}, n=1,2 . i=\{S, U\}, j=1,2,3 \ldots, J\right.$.
$z_{v j}=\left\{\begin{array}{l}1 \text { if } R\left(\mu_{v}\right)-r \leq d\left(j, Q\left(\mu_{v}\right)\right) \leq R\left(\mu_{v}\right)+r \\ 0 \\ \text { or else }\end{array}, v=1,2,3, \ldots, V . j=1,2,3 \ldots, J\right.$.
Where $\mu_{v}$ is the angle of osculation of the arc $v, R\left(\mu_{v}\right)$ is the radius of arc $v$ when the angle of osculation is $\mu_{v}, Q\left(\mu_{v}\right)$ is the center point of arc $v$ when the angle of osculation is $\mu_{v}$. It can be seen from Figure 1 that the coordinates of $Q\left(\mu_{v}\right)$ is ( $0,-$ $\left.d_{i, i+1} / 2 \tan \mu_{v}\right), d_{i, i+1}$ is the distance between facilities $i$ and $i+1 . d_{i, i+1}=\sqrt{\left(x_{i}-x_{i+1}\right)^{2}+\left(y_{i}-y_{i+1}\right)^{2}}, d\left(j, Q\left(\mu_{v}\right)\right)$ is the distance between residential area $m_{j}$ to the center point $Q\left(\mu_{v}\right) \cdot d\left(j, Q\left(\mu_{v}\right)\right)=\left[a_{j}^{2}+\left(b_{j}+d_{i, i+1} / 2 \tan \mu_{v}\right)^{2}\right]^{1 / 2}$.

Thus, the mathematical model of double routes designing problem with common desirable facilities and common undesirable routes can be represented by:

$$
\begin{align*}
& \min D=\sum_{n=1}^{2} \sum_{j=1}^{J} \sum_{i \in R, i \neq S, U} p_{n j} \omega_{j} \lambda\left[\left(x_{i}-a_{j}\right)^{2}+\left(y_{i}-b_{j}\right)^{2}\right]^{7 / 2}+\sum_{n=1}^{2} \sum_{j=1}^{J} p_{n j} \omega_{j} \lambda_{n S j}\left[\left(x_{s}-a_{j}\right)^{2}+\left(y_{s}-b_{j}\right)^{2}\right]^{1 / 2}+ \\
& \sum_{n=1}^{2} \sum_{j=1}^{J} p_{n j} \omega_{j} \lambda_{n U j}\left[\left(x_{U}-a_{j}\right)^{2}+\left(y_{U}-b_{j}\right)^{2}\right]^{1 / 2} \\
& \text { s.t. } \min _{(A, B),(X, Y)} \sum_{v=1}^{V} \sum_{j=1}^{J} z_{v j} \omega_{j} \leq \bar{\theta}  \tag{1}\\
& \sum_{i \in R_{n}, i \neq S, U} \lambda_{i j}+\lambda_{n S j}+\lambda_{n U j}=1, n=1,2, j \in M_{n}  \tag{2}\\
& \lambda_{i j}=0, i \in R_{n}, j \notin M_{n}, n=1,2  \tag{3}\\
& \lambda_{n i j}=0, j \notin M_{n}, n=1,2, i=S, U  \tag{4}\\
& \sum_{j=1}^{J} \lambda_{i j}>0, i \in R, i \neq S, U  \tag{5}\\
& \sum_{n=1}^{2} \sum_{j=1}^{J} \lambda_{n i j}>0, i=S, U \tag{6}
\end{align*}
$$

The difference between this model and single route designing model is that the residential area can be served by at most two facilities, but this two facilities should not belong to the same route. Besides the common facilities $S$ and $U$, all facilities cannot serve the residential areas which locate outside from being covered by the routes themselves. The variable $\lambda_{n i j}$ is set in order to simply realize the two extra constrains.

Equation (1) shows the number of affected residents cannot exceed their upper limit $\bar{\theta}$, and equation (2) is ensuring any residential area in each route can have and only can have one facility to serve for them. Equation (3) represents that if residential area $m_{j}$ is not served by route $n$, then all of the contained non-common facilities in route $n$ will not serve for them. But it is not saying the common facilities do not serve for residential area $m_{j}$, because it is possible to serve for residential area $m_{j}$ in other route. Equation (5) and (6) show that each facility should serve one residential area at least.

## SOLUTION OF MODEL

## Heuristic algorithm in given area locating

Algorithms for facilities locating have been researched by Andersen ${ }^{[6]}$, Chen et $\mathrm{al}^{[7]}$, Lei Fang et al ${ }^{[8-10]}$, and Peixi Zhao et al ${ }^{[17,12]}$. If Weiszfeld algorithm is used to solve the locating problem in continuous plane with locating area constraint,
the obtained optimized solution might not be in the optional area, leading to the solution is not available. Therefore, we should firstly figure out the solution without constraint outside constraint area, and then get the optimized solution.

The main idea of common facilities locating algorithm is to divide the common facilities locating and their serving residential areas into two different levels. Locating facilities are carried out to all serving residential areas, while inside division change of each route are carried out to residential areas served by common facilities in their routes. Then union set of the changed sets is obtained and the resulting set is new all serving residential areas. Because every facility in the model can be seen as common facility, every facility is secondary divided in the algorithm and a certain constraint is implied to distinguish common facility from non-common facility. In other words, residential areas set served by each facility is divided into several subsets, each of the subset represent the assigned serving residential area set when this facility is belong to each route. If the facility is not belong to one route, the corresponding subset is empty.

As for the locating problem in continuous with locating area constraint, Hansen et al ${ }^{[13]}$ have proved that when single facility locating is carried out in a given convex region under continuous plane, their optimized solution must be on the edge of the convex region. Based on the theorem of Hansen et al and the former analysis, the common facilities locating model algorithm can be described as below:

Step 1: The residential points whose number is $J$ are divided into $I$ sets, represented by $M=\Omega^{0}=\left\{\Omega_{1}^{0}, \Omega_{2}^{0}, \cdots \Omega_{I}^{0}\right\}$. Then the including $\Omega_{S}^{0}$ and $\Omega_{U}^{0}$ are divided into two sets, named as $\Omega_{S}^{0}=\left\{\Omega_{S, 1}^{0}, \Omega_{S, 2}^{0}\right\}$ and $\Omega_{U}^{0}=\left\{\Omega_{S, 1}^{0}, \Omega_{S, 2}^{0}\right\}$.
Where $\left(\bigcup_{i \in R_{n}, i \neq S, U} \Omega_{i}^{0}\right) \cup \Omega_{S, n}^{0} \cup \Omega_{U, n}^{0}=M_{n}, n=1,2$.
Step 2: $k=0$.
Step 3: $\eta=0$.
Step 4: As for $i=1,2, \ldots, I$. the following steps are carried out and $\left(X^{k}, Y^{k}\right)$ is obtained.
(1) Based on division of $\Omega^{k}$, the optimized locating coordinates of every areas under non-constraint conditions based on Weiszfeld algorithm are obtained as: $\left(X^{k}, Y^{k}\right)_{\text {non }}=\left\{\left(x_{1}^{k}, y_{1}^{k}\right)_{\text {non }}, \cdots,\left(x_{I}^{k}, y_{I}^{k}\right)_{\text {non }}\right\}$.
(2) If all of the $\left(x_{i}^{k}, y_{i}^{k}\right)_{\text {non }} \in\left\{\left(x_{i}, y_{i}\right) \mid c_{i} \leq x_{i} \leq d_{i}, e_{i} \leq y_{i} \leq f_{i}\right\}, i=1,2, \cdots, I$, step 5 is carried out, or else go to nest step.
(3) As for all of the $\left(x_{i}^{k}, y_{i}^{k}\right)_{\text {non }} \notin\left\{\left(x_{i}, y_{i}\right) \mid c_{i} \leq x_{i} \leq d_{i}, e_{i} \leq y_{i} \leq f_{i}\right\}$, named as set $\quad\left(X^{k}, Y^{k}\right)_{\bar{R}}$, $\left(x_{i}^{k}, y_{i}^{k}\right)_{n o n} \in\left\{\left(x_{i}, y_{i}\right) \mid c_{i} \leq x_{i} \leq d_{i}, e_{i} \leq y_{i} \leq f_{i}\right\}$, named as $\left(X^{k}, Y^{k}\right)_{R}^{1}$.
(4) As for all of the $\left(x_{i}^{k}, y_{i}^{k}\right)_{n o n} \in\left(X^{k}, Y^{k}\right)_{\bar{R}}$, search for four apex angle points $\left(v_{i}^{1}, u_{i}^{1}\right),\left(v_{i}^{2}, u_{i}^{2}\right),\left(v_{i}^{3}, u_{i}^{3}\right),\left(v_{i}^{4}, u_{i}^{4}\right)$ in region $\left\{\left(x_{i}, y_{i}\right) \mid c_{i} \leq x_{i} \leq d_{i}, e_{i} \leq y_{i} \leq f_{i}\right\}$. Under the condition of $\left(v_{i}^{*}, u_{i}^{*}\right)=\arg \min _{j=1,2,3,4} d\left(v_{i}^{j}, u_{i}^{j}\right)$, where $d\left(v_{i}^{j}, u_{i}^{j}\right)$ is Euclidean distance between $\left(x_{i}^{k}, u_{i}^{k}\right)_{n o n}$ and $\left(v_{i}^{j}, u_{i}^{j}\right), j=1,2,3,4$. Take $\left(v_{i}^{*}, u_{i}^{*}\right)$ as apex angle points and two adjacent edges of optional rectangle region are named as $e^{1}\left(v_{i}^{*}, u_{i}^{*}\right)$ and $e^{2}\left(v_{i}^{*}, u_{i}^{*}\right)$. Search inside $e^{1}\left(v_{i}^{*}, u_{i}^{*}\right)$ and $e^{2}\left(v_{i}^{*}, u_{i}^{*}\right)$, the optimized solution $\left(x_{i}^{k}, y_{i}^{k}\right)_{R_{I}}$ on the borders are obtained. All $\left(x_{i}^{k}, y_{i}^{k}\right)_{R_{I}}$ are named as set $\left(X^{k}, Y^{k}\right)_{R}^{2}$.
(5) Make $\left(X^{k}, Y^{k}\right)=\left(X^{k}, Y^{k}\right)_{R}^{1} \cup\left(X^{k}, Y^{k}\right)_{R}^{2}$.

Step 5: For all $j=1,2, \cdots, J, d_{i j}=\left[\left(x_{i}^{k}-a_{j}\right)^{2}+\left(y_{i}^{k}-b_{j}\right)^{2}\right]^{1 / 2},\left(x_{i}^{k}, y_{i}^{k}\right) \in\left(X^{k}, Y^{k}\right)$ are calculated.
Step 6: For each residential point $m_{j,}$ and a random facilitate $i^{\prime}$ in route $\mathrm{n}(\mathrm{n}=1,2), m_{j} \in \Omega_{i^{\prime}}^{k}, d_{i^{\prime \prime} j}=\min _{i \in R_{n}} d_{i j}<d_{i^{\prime} j}$. Then when $i^{\prime \prime} \neq S, U$ and $i^{\prime} \neq S, U$, make $\Omega_{i^{\prime \prime}}^{k}=\Omega_{i^{\prime \prime}}^{k} \bigcup\left\{m_{j}\right\}, \Omega_{i^{\prime}}^{k}=\Omega_{i^{\prime}}^{k} \backslash\left\{m_{j}\right\}$, when $i^{\prime \prime} \neq S, U$ and $i^{\prime}=S, U$, make $\Omega_{i^{\prime \prime}}^{k}:=\Omega_{i^{\prime \prime}}^{k} \cup\left\{m_{j}\right\}, \quad \Omega_{i^{\prime}, n}^{k}:=\Omega_{i^{\prime}, n}^{k} \backslash\left\{m_{j}\right\}$, when $i^{\prime \prime}=S, U$ and $i^{\prime} \neq S, U, \Omega_{i^{\prime \prime}, n}^{k}:=\Omega_{i^{\prime \prime}, n}^{k} \cup\left\{m_{j}\right\}, \Omega_{i^{\prime}}^{k}:=\Omega_{i^{\prime}}^{k} \backslash\left\{m_{j}\right\}$, when $i^{\prime \prime}=S, U$ and $i^{\prime}=S, U$, make $\Omega_{i^{\prime \prime}, n}^{k}:=\Omega_{i^{\prime \prime}, n}^{k} \bigcup\left\{m_{j}\right\}, \Omega_{i^{\prime}, n}^{k}:=\Omega_{i^{\prime}, n}^{k} \backslash\left\{m_{j}\right\}, \eta=\eta+1$.

Step 7: If $\eta=0$, stop calculation and output $(X, Y)=\left(X^{k}, Y^{k}\right)$. Or else $\Omega^{k+1}:=\Omega^{k}, k=k+1$, and return to step 3.

## Undesirable links algorithms

On the basis of algorithms in part 3.1, desirable facilities locating points meeting the constraints can be obtained. According to the link designing calculation method given by Pan Zheng ${ }^{[5]}$, the optimized routes can be obtained:

Step 1: Based on the given facilities location $\left(x_{i}, y_{i}\right)$ and $\left(x_{i+1}, y_{i+1}\right)$, the straight link line between $\left(x_{i}, y_{i}\right)$ and $\left(x_{i+1}, y_{i+1}\right)$ are set as standard, the possible residential area set affected by link between $\left(x_{i}, y_{i}\right)$ and $\left(x_{i+1}, y_{i+1}\right)$ are obtained and named as $\Psi_{i}, i=1,2, \cdots, I-1$.

Step 2: Based on the obtained residential set $\Psi_{i}$, the range of angle of osculation affected by every residential area can be obtained: $\left(\alpha_{\varphi}, \beta_{\varphi}\right), \varphi \in \Psi_{i}, i=1,2, \cdots, I-1$.

Step 3: Based on the acquired angle of osculation $\left(\alpha_{\varphi}, \beta_{\varphi}\right)$ of each affecting residential area, and the corresponding number of residents $\theta_{\varphi}, \varphi \in \Psi_{i}$, the optimized angle of osculation range is obtained as $A_{i}=\arg \min _{(\alpha, \beta)} \sum_{\varphi \in \Psi_{i}} \theta_{\varphi} 1_{\left\{(\alpha, \beta) \in\left(\alpha_{\varphi}, \beta_{\varphi}\right)\right\}}, i=1,2, \cdots, I-1$. If $(\alpha, \beta) \in\left(\alpha_{\varphi}, \beta_{\varphi}\right), 1_{\left\{(\alpha, \beta) \in\left(\alpha_{\varphi}, \beta_{\varphi}\right)\right\}}$ is 1 , or else it is 0 .

## Comprehensive algorithms

After obtaining heuristic algorithm of desirable multi-facilities locating with locating area constraint and obtaining the optimized angle of osculation algorithm in given facilities locations, comprehensive algorithms could be given for designing routes.

Facilities locating points with smallest weighing distance are given by algorithm 3.1. Algorithm 3.2 gives the optimized angle of osculation strategy under the facilities points of 3.1. But these algorithms do not always meet the constraints of affecting residential areas. Therefore, in the designing of comprehensive algorithms, the improved measurement without residential areas constraints and their corresponding iterative process should be considered. A kind of heuristic algorithms which can adjust and arrange without residential areas constraints are designed here, and routes designing comprehensive algorithms are provided combining with algorithms 3.1 and 3.2. The detailed steps are as follows.

Step 1: $k=0$
Step 2: The initial optimal position of multi-facilities obtained by heuristic algorithm 3.1 is $\left(X^{0}, Y^{0}\right)=\left\{\left(x_{1}^{0}, y_{1}^{0}\right), \cdots,\left(x_{I}^{0}, y_{I}^{0}\right)\right\}$. The total weighing distance of $\left(X^{0}, Y^{0}\right)$ is obtained as $D^{0}$. At the same time, the undesirable link algorithms in 3.3.2 are adopted to obtain the optimal angle of osculation range of adjacent facilities $A^{0}=\left\{A_{1}^{0}, \cdots, A_{I-1}^{0}\right\}$ and the affecting residents number $\theta^{0}$ corresponding to $A^{0}$.

Step 3: If $\theta^{0} \leq \bar{\theta}$, , the calculation is stopped. $\left(X^{0}, Y^{0}\right)$ is output. $A^{0}$ is the optimal facilities position and routes range. Or else turn to step 4.

Step 4: The following steps are carried out separately.
(1) According to the optimal position $\left(X^{k}, Y^{k}\right)=\left\{\left(x_{1}^{k}, y_{1}^{k}\right), \cdots,\left(x_{I}^{k}, y_{I}^{k}\right)\right\}$, obtain the total weighing distance $D^{k}$ corresponding to $\left(X^{k}, Y^{k}\right)$ and the weighing distance $D_{i}^{k}$ corresponding to each $\left(x_{i}^{k}, y_{i}^{k}\right)$. At the same time, the undesirable link optimal algorithm is adopted to obtain the optimal angle of osculation range of adjacent facilities $A^{k}=\left\{A_{1}^{k}, \cdots, A_{I-1}^{k}\right\}$, and the affected people numbers corresponding to $A^{k}$ and $A_{i}^{k}$ are named as $\theta^{k}$ and $\theta_{i}^{k}$, $i=1,2, \cdots, I$.
(2) As for all $j$ of $\Omega_{i-1}^{k}$, the optimal location coordinate $\left(x_{i}^{k+1}, y_{i}^{k+1}\right)$ and $\left(x_{i-1}^{k+1}, y_{i-1}^{k+1}\right)$ of new division of $\Omega_{i}^{k+1}:=\Omega_{i}^{k} \cup\{j\}$ and $\Omega_{i-1}^{k+1}:=\Omega_{i-1}^{k} \backslash j_{i}^{k}$ can be obtained by Weiszfeld algorithm ${ }^{[5]}$. The distance variation due to movement of $j$ is $\Delta \tilde{d}_{i-1, i, j}=\left(D_{i}^{k+1}-D_{i}^{k}\right)+\left(D_{i-1}^{k+1}-D_{i-1}^{k}\right)$. The optimal angle of osculation range of adjacent facilities of adjacent nodes $i-2, ~ i-1, ~ i, ~ i+1$ are $A_{i-2}^{k+1}, ~ A_{i-1}^{k+1}, ~ A_{i}^{k+1}$. And the $\theta_{i-2}^{k+1}, \theta_{i-1}^{k+1}$, and $\theta_{i}^{k+1}$ are named as $\Delta \theta_{j}^{k+1}=\left(\theta_{i-2}^{k}-\theta_{i-2}^{k+1}\right)+\left(\theta_{i-1}^{k}-\theta_{i-1}^{k+1}\right)+\left(\theta_{i}^{k}-\theta_{i}^{k+1}\right)$, representing the population affected by movement of node $j$.
(3) Based on the obtained $\Delta \tilde{d}_{i-1, i, j}$ and $\Delta \theta_{j}^{k+1}$, the following linear programming is solved.
$\min \sum_{i=1}^{I} \sum_{j=1}^{J} x_{i j} \Delta \tilde{d}_{i-1, i, j}$
s.t. $\sum_{i=1}^{I} \sum_{j=1}^{J} x_{i j} \Delta \theta_{j}^{k+1} \leq \bar{\theta}-\theta^{k}$

Where $x_{i j}$ is variable of $0,1 . x_{i j}=1$ represents the optional $j$ is turned into division $\Omega_{i}^{k}$, or else it is 0 . The corresponding optimal solution $X^{k}$ is obtained.

Step 5: The division $\Omega_{i}^{k}$ can be adjusted based on the obtained $X^{k}$. Make $\Omega_{i}^{k+1}=\Omega_{i}^{k} \cup\left\{x_{i j} j\right\}$, $\Omega_{i-1}^{k+1}=\Omega_{i-1}^{k}-\left\{x_{i j} j\right\}, i=1,2, \cdots, I$. New division groups are obtained $\Omega^{k+1}=\left\{\Omega_{1}^{k+1}, \Omega_{2}^{k+1}, \cdots, \Omega_{I}^{k+1}\right\}$.

Step 6: Based on the new division $\Omega^{k+1}$, the optimal position of multi-facilities is given by heuristic algorithm 3.1 $\left(X^{k+1}, Y^{k+1}\right)=\left\{\left(x_{1}^{k+1}, y_{1}^{k+1}\right), \cdots,\left(x_{I}^{k+1}, y_{I}^{k+1}\right)\right\}$. And the total weighing distance $D^{k+1}$ corresponding to $\left(X^{k+1}, Y^{k+1}\right)$ is obtained. At the same time, the undesirable links optimal algorithm in 3.2 is adopted and the optimal angle of osculation range $A^{k+1}$ and affected population $\theta^{k+1}$ is obtained.

Step 7: If $D^{k}-D^{k+1} \leq \varepsilon$, and $\theta^{k+1} \leq \bar{\theta}$, stop and output $\left(X^{k+1}, Y^{k+1}\right)$ and the corresponding optimal angle of osculation range $A^{k+1} . \varepsilon$ is a given small value, or else make $k=k+1$ and turn to step 4.

## NUMERICAL CALCULATE EXAMPLE

In the plane of abscissa [0-5] and ordinate [0-3], there are 180 residential areas distributed discretely and 6 desirable facilities should be chosen in the continuous plane to constitute two undesirable A-B-C-D (Route 1) and E-D-C-G (Route 2), in which C and D are common facilities and C and D are common routes.

Firstly single route facilities locating algorithm are used to locate the desirable facility with constraint area for route 1 and route 2 . When the link distance with affected by undesirable routes $r=0.15$, angle of osculation is less than $45^{\circ}$, upper limit of affected population $\bar{\theta}=30 \%$, the obtained results are shown in TABLE 1, TABLE 2, TABLE 3, and TABLE 4.

TABLE 1: Locating results of route 1

| Facilities | Locating coordinates |
| :---: | :---: |
| A | $(0.3700,2.3800)$ |
| B | $(1.3056,1.0976)$ |
| C | $(3.0844,0.8800)$ |
| D | $(4.1256,1.9400)$ |

TABLE 2: Locating results of route 2

| Facilities | Locating coordinates |
| :---: | :---: |
| E | $(2.3557,2.0970)$ |
| D | $(4.0805,1.9400)$ |
| C | $(2.8640,0.8800)$ |
| G | $(4.2500,0.5830)$ |

TABLE 3: Routes designing results of route 1

| Routes | Optimal angle of osculation range |
| :---: | :---: |
| A-B | $[42.8740-45.0000]$ |
| B-C | $[42.0133-43.3709]$ |
| C-D | $[37.6755-42.1158]$ |

TABLE 4: Routes designing results of route 2

| Routes | Optimal angle of osculation range |
| :---: | :---: |
| E-D | $[0.3419-0.6096]$ |
| D-C | $[44.4434-45.0000]$ |
| C-G | $[-45.0000-41.9437]$ |

It can be shown that the separate optimization of two routes results to two different location points for common facilities C and D , and leads to two different optimal angle of osculation ranges for route $\mathrm{C}-\mathrm{D}$. We locate facilities C and D using improved facilities locating models to obtain actual optimal locating point, and then design the lines. The final location of facility C calculated by algorithm 3.3 is $(3.0426,0.8800)$, final location of common facility $D$ is $(4.10924,1.9400)$, and optimal angle of osculation range of common route C-D is [36.9120-45.0000], which have been shown in Figure 3.


Figure 3: Final common routes

## CONCLUSIONS

In this work, a mathematical model was given to solve the joint decision problem about desirable facilities locating and undesirable routes designing with common facilities and common routes using double lines in a continuous plane. The affected residents' number upper limit was set as a constraint and a comprehensive heuristic algorithm is adjusted and arranged. The case study proved that the model and algorithm proposed in our paper could not only ensure the change of residential area set, but also ensure the iteration develop to a good direction. The joint decision problem about common routes locating and route designing was effectively solved at last. The improvement of this algorithm and their application on multi-facilities desirable facilities locating and undesirable routes designing problem are still future work.

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