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Research on the conditions of reaction-diffusion equation occurring Hopf bifurcation under prescribed boundary condition

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ABSTRACT

Reaction diffusion equation, which is well practically applied at present, is a good model to describe the natural phenomenon. Although Hopf bifurcation is easy, it is vitally important as a dynamic bifurcation. Hopf bifurcation theory has been the essential method to analyze the appearance and disappearance of the small amplitude periodic solution of the differential equation, mainly because its research theory and numerical research method are significant for the dynamic bifurcation and limit cycle. Moreover, there is inseparable relationship between Hopf bifurcation and the theory of self-excited vibration, so Hopf bifurcation is widely used in the modern engineering. First of all, this research elaborates the research status and significance of reaction diffusion equation. Then, the paper explains reaction diffusion equation and discusses its equilibrium, distribution and linear stability. In the end, the research discusses the conditions of one dimension reaction equation occurring Hopf bifurcation and analyzes the stability and conditions. Through the study of the whole process, it can be seen that reaction diffusion equation can be widely applied. Therefore, it is important for describing the natural movements and involves a number of disciplines. Furthermore, many mathematical models can be switched into reaction diffusion equation so that it is more beneficial for research. The research on one dimension reaction diffusion equation will be useful for the analysis and awareness of several natural phenomena.

KEYWORDS

Hopf bifurcation; Dynamic bifurcation; Reaction diffusion equation; Prescribed boundary.

INTRODUCTION

Reaction diffusion system is an essential branch in the research field of modern analysis, mainly because its related questions are widely occurred in the field of physics, ecosystem, ecology and chemistry with giant application background, for instance, Lotka-Volterra model, plant growth model, population distribution model, gas discharge model, and etc. One key teaching question in the study of reaction diffusion equation is the dynamic mechanical property, which has had mature and complete research results. Reaction diffusion model has become the classic mathematical model, and it can give a basic description of natural phenomenon model. Thus, its actual application range has been huge. The research on reaction diffusion equation can help people understand and perceive a lot of unknown natural phenomena.

THE BASIC THEORY AND RESEARCH STATUS

The research status of reaction diffusion equation

In recent years, with further study of reaction diffusion equation by more researchers, it has developed rapidly. Moreover, the research on reaction diffusion equation has been attracting more and more mathematicians, physicists and senior engineers. During the process, more challenging scientific questions have been raised for mathematics. Before 1960, the foreign researches were made great achievements in nonlinear kinetics in chemical reactions mainly by scientists Nayfeh, Mook, Chua and Mess, which analyzed and studied the mechanism of nonlinear chemical reactions by the knowledge of nonlinear kinetics. It also involved the design methods and model controlling, and so on. Since 1980, the research has been represented by chemical Turning waving pattern achieved by the French group De Kepper.

Although Chinese studies started late, China has made extraordinary achievements and improvements in recent years. Represented by scientific research institutions such as Jilin University, Fudan University, Chinese Academy of Science, and etc, researchers have published many monographs and journals about reaction diffusion equation. It can be predicted that China will invest more attention into the study of nonlinear kinetics in chemical reactions and promote it to a strategic position, because it will be the central pillar for Chinese study of nonlinear kinetics to step in the international arena.

The research of reaction diffusion equation is realized by building mathematic models and solving equation. Analyzing the parametric variations in the equation can describe the results of reaction diffusion more specifically. Currently, there are several kinds of reaction diffusion equation. According to their characteristics, some of equations can be controlled, while some of them are out of control; some of equations are easy, but some of them are unique.

The most important for the study of reaction diffusion equation is to analyze its long-term behaviors, especially judging if the solution keeps balance independently relying on some time. Of course, the long-term behaviors may be periodic and irregular. Once these long-term behaviors surround the solution and control all behaviors of the solution, people will take them as attractors. Attractors can not only keep stable balance, but also find possible classifications of equilibrium based on their basic stable feature for given questions.

Hopf bifurcation

Normally, Hopf bifurcation appears after the dynamic step in the system with oscillatory instability. As a normal phenomenon, Hopf bifurcation has vital significance for mastering the properties of the whole system. At present, the researches on the contents of bifurcation are extensive and abundant. The following four points are the main directions.

Firstly, confirm the bifurcation set, which means to build the necessary and sufficient conditions for forming bifurcation in the system;

Analyze the qualitative properties of bifurcation, mainly analyzing the occurrence of the parameter variation of topological structure in system as bifurcation;

Compute the solutions of bifurcation. The main computing contents are the equilibrium point and limit cycle of the system. But the difficulty is usually to solve directly the bifurcation problem in the nonlinear system. The possibility to get the results is small, so the approximation analysis is often adopted;

Finally, analyze the direct effects of different bifurcations, involving the relation characteristics between different kinetics phenomena such as bifurcation and chaos.

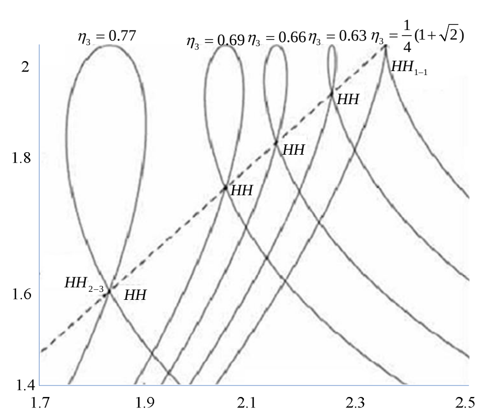

Figure 1 : The diagram of stationary bifurcation

Bifurcation includes stationary bifurcation and dynamic bifurcation, as shown in Figure 1 and Figure 2. Hopf bifurcation belongs to one of three topological structures of dynamic bifurcation. The general definition of Hopf bifurcation is that the equation of n dimension single parameter nonlinear dynamic system is x=f(x, μ). In this equation, the analytic function *f* is connected with *x* and μ, andμ is real bifurcation parameter. If the equilibrium point of the system is set as $x_{0,}$ whatever value μ is, the equation is f(x, μ)=0. If Hopf bifurcation occurred in the system, it would change around parameter λ, which would change the stability of the equilibrium point in the system. The phenomenon of Hopf bifurcation in real models is very common, for example, periodic heartbeat and periodic economic crisis.

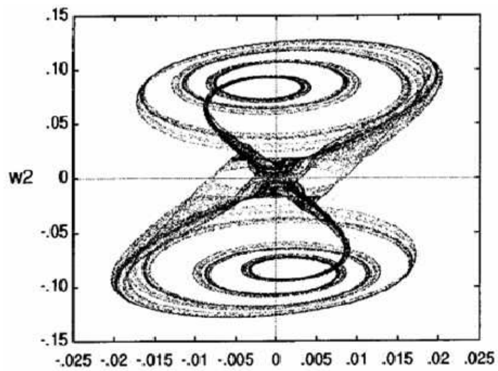

Figure 2 : The diagram of dynamic bifurcation

Hopf bifurcation occurs while changing parameter and the dynamic system will appear periodic solution at an equilibrium point. Moreover, Hopf bifurcation belongs to partial dynamic bifurcation and the researches all over the world have been achieved many improvements, including Hopf bifurcation, closed orbit bifurcation, degraded Hopf bifurcation, and etc. However, there is still lack of deeper theories in the field of double Hopf bifurcation, mainly because its study is very complicated. Thus, the study of double Hopf bifurcation is far behind with the study of Hopf bifurcation, which is demanded further improvement and development of the research methods.

THE CONDITIONS OF HOPF BIFURCATION UNDER PRESCRIBED BOUNDARY

The analytical method of linear problems

The first problem is the spectrum of differential operator. Take X as a Banach space, so $A:D\left(A\right)\to X$ is the linear operator on the field $D\left(A\right)\subseteq X$. As a result, no matter what value complex number λ is, the operational form can be conducted as following:

$$A_\lambda = A-\lambda I \ ,$$

I is the unit operator above *X*. While A_λ is reversible, it can be represented as:

$$R_\lambda(A) = A_\lambda^{-1}$$

It is the resolvent of A, while the regular point of A is λ, so:

(1) $R_\lambda\left(A\right)$ exists;

(2) $R_\lambda\left(A\right)$ has boundary;

(3) $R_\lambda\left(A\right)$ is defined in the dense subset of X.

When $R_\lambda\left(A\right)$ does not exist, the value of λ is eigenvalue of point spectrum synthesis; when $R_\lambda\left(A\right)$ exists without boundary, continuous spectrum is the set of the value of λ; when $R_\lambda\left(A\right)$ exists with boundary, the set of the value of λ becomes leftover spectrum. If the spectrum of A is presented by σ (A), any point (not the isolated eigenvalue of limited multiplicity) in $\sigma\left(A\right)$ is the eigenspectra of A.

Suppose $X = L_2\left(R\right)$, the quadratic continuous differentiable function $\mu\left(X\right)$, μ_{xx} in X would be carried, and the definition is presented as following:

$$Au = -u_{xx} + 2au_x + bu$$

Both *a* and *b* are constant. Next, extend the definition of A, turning it into the closed operator of field D (A) in X. When the spectral theory of closed operator develops well, the spectrum of A call be considered directly.

First of all, suppose that A_λ is irreversible to some λ. Therefore, there is some eigenfunction W∈X and $A_{\lambda w} = 0$. Last, suppose that the equation is the linear equation with constant coefficients. Thus, arbitrary solutions will not decay within ±∞ domain and exist in X. Moreover, there is no eigenvalue for A, but there must be an opposite R_λ for A. Then, Green function G is used to present it, so suppose $h \in D\ (R_\lambda) \subseteq X$, resulting $u = R_\lambda h$. When

$$u = \int_R G(x-s)h(s)ds$$

While the form of G is:

$$G(y) = \begin{cases} a\exp(\mu_1 y) & y \le 0 \\ a\exp(\mu_2 y) & y \ge 0 \end{cases}$$

The analysis of the distribution of eigenvalue will suppose that:

$$= e^{\lambda t} e^{\mu x}$$

$$u_{xx} = \mu^2 e^{\lambda t} e^{\mu x}$$

$$u_x = \mu e^{\lambda t} e^{\mu x}$$

$$\lambda e^{\lambda t} e^{\mu x} = \mu^2 e^{\lambda t} e^{\mu x} + 2a\mu e^{\lambda t} e^{\mu x} + b e^{\lambda t} e^{\mu x}$$

$$\lambda = -\mu^2 + 2a\mu + b$$

After some conversion, it can be seen that it is very clear of the boundary between R_λ and X. Besides, the value of P continuously depends on λ. When λ<b, there are two values of P, one of which is positive and the other is negative. It can be seen that the distribution of value in λ and k plane from Figure 3:

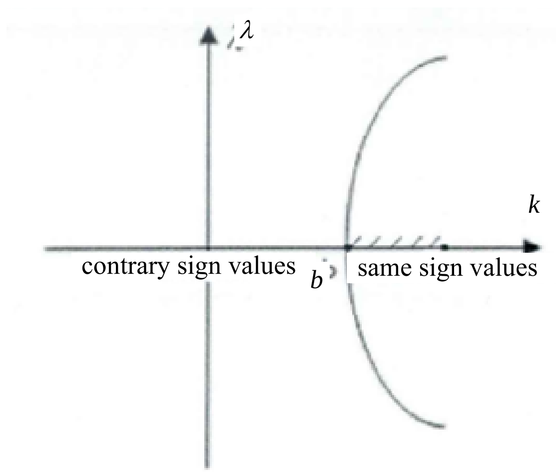

Figure 3 : The distribution of value in λ and k plane

The processing method of nonlinear stability

Generally, Hopf bifurcation at unit interval of Schnackenburg equation shall be confirmed at first, including the following forth steps:

(1) A→ X
(2) X→ product
(3) 2X+Y→ 3X
(4) B→ Y

Next, nondimensionalize reaction diffusion equation:

$$\begin{cases} \frac{\partial x}{\partial t} = \gamma(a - x + x^2 y) + \nabla^2 x \\ \frac{\partial y}{\partial t} = \gamma(b - x^2 y) + d\nabla^2 y \end{cases}$$

Then, set the boundary condition as:

$$\begin{cases} \frac{\partial x}{\partial r}(0,t) = \frac{\partial x}{\partial r}(1,t) = 0 \\ \frac{\partial y}{\partial r}(0,t) = \frac{\partial y}{\partial r}(1,t) = 0 \end{cases}$$

In this condition, $r \in [0,\ 1]$, γ is the reduction reaction rate, and the diffusion coefficient ratio of Y and X is d. Moreover, x and y are real valued functions, depending upon r and t. $\triangledown^2$ is Laplace operator and the controlled parameter of the system a,b>0. Then the even steady-state solution of the system can be resolved as $\left(a+b, \frac{b}{(a+b)^2}\right)$. The stationary point is transferred into the origin, so it can be concluded that:

$$\begin{cases} x = u + (a+b) \\ y = v + \frac{b}{(a+b)^2} \end{cases}$$

After conversion, the result can be got:

$$\begin{cases} \frac{\partial u}{\partial t} = \gamma\left[\left(\frac{2b}{a+b} - 1\right)u + (a+b)^2 v\right] + \nabla^2 u + h(u,v) \\ \frac{\partial v}{\partial t} = \gamma\left[-\frac{2b}{a+b}u - (a+b)^2 v\right] + d\nabla^2 v - h(u,v) \end{cases}$$

While $h(u,v) = \gamma(u \uparrow 2v) + b/1(a+b) \uparrow 2u \uparrow 2 + 2(a+b)uv$, so set L and K as:

$$L = \begin{bmatrix} \gamma\left(\frac{2b}{a+b} - 1\right) + \nabla^2 & \gamma(a+b)^2 \\ -\gamma\frac{2b}{a+b} & -\gamma(\mathrm{a+b})^2 + \mathrm{d}\nabla^2 \end{bmatrix} = K + D\Delta$$

$$K = \gamma\begin{bmatrix} \frac{2b}{a+b} - 1 & (a+b)^2 \\ -\frac{2b}{a+b} & -(a+b)^2 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & d \end{bmatrix}$$

When operator meets the condition u→Δu, the according eigenfunction cosnπx can be obtained. The subsequent work is conducting the stability test for bifurcation periodic solution.. Figure 4 shows Hopf bifurcation corresponding to the equilibrium point.

According to Hopf bifurcation theory, when v is satisfied with the condition $v = u \mid b < 0$, there must be a pair of pure values and the system will appear Hopf bifurcation. It can be seen from (a) and (b) of Figure 4 that the time of stable orbit limit cycle and state variable are corresponding when $V = -0.1$, $u = 0.1$, $b = 0.2$ and $a = 1$. Besides, as shown in TABLE 1, the variation of system FMs with the change of parameter v, it can be concluded that only one FM factor score is 1 when 0.5<v<2/3 and other two factor scores are less than 0; when v>2/3 appears in unstable system, one pair of conjugate FM factors is more than 0; when v=2/3, which means v is within the range of 0.52404<v<0.915959.

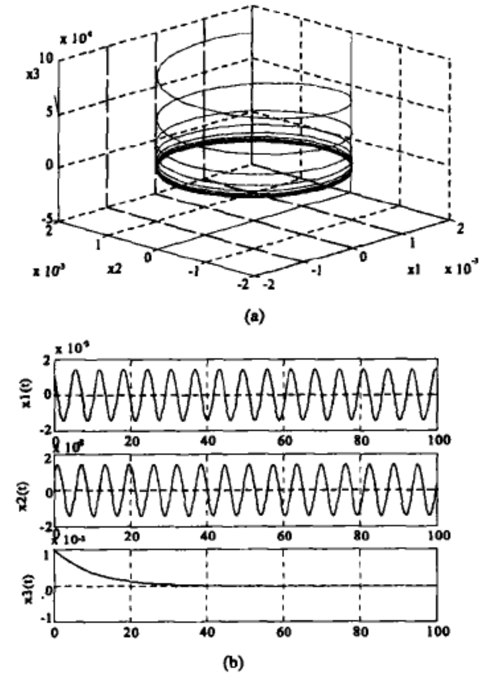

Figure 4 : Hopf bifurcation corresponding to the equilibrium point

TABLE 1 : System FMs changes with the variation of parameter v

Serial number		**1**	**2**	**3**	**4**	**5**	**6**	**7**
Parameter v		0.5	0.52182146	0.58233669	0.66664228	0.79521422	0.94939605	1
FMs	Mod(1)	0.0432139	0.16046	0.451675	0.999591	1	1	1
	Mod(2)	1	0.406343	0.451675	0.999591	3.35863	2.20639	1
	Mod(3)	1	1	1	1	3.35863	93.5005	535.492
	Arg(1)	0	0	-2.19062	-2.96173	0	0	0
	Arg(2)	-3.67053×10^{-11}	0	2.19062	2.96173	-2.84235	0	0
	Arg(3)	3.67053×10^{-11}	0	0	0	2.84235	0	0
property		Hopf Bifurcation point	stable periodic solution	stable periodic solution	torus bifurcation point	instability	instability	instability

CONCLUSION

With the rapid development of computer technology, it has provided brand new views and methods for the study of mathematics. Bifurcation is a complicated phenomenon occurred a lot in nonlinear dynamic system. The research on reaction diffusion equation has tended to be complete and perfect, so this research mainly analyzes and discusses the mathematical description of reaction diffusion equation. Furthermore, taking one dimension reaction equation as an example explains Hopf bifurcation and the distribution of eigenvalues. Besides, analyze the conditions of occurring Hopf bifurcation utilizing schnackenberg equation and the stability of its periodic solution. Hopf bifurcation has become a vitally important dynamic bifurcation question with essential theory research significance. In the real engineering system, reaction diffusion equation can describe perfectly many dynamic models. Based all above, it can be concluded that the research on Hopf bifurcation has extremely high application value.

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