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Relationships Between Two Gravitationally-Bound Points in Single or Multiple Systems In The Universe

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Abstract

Supported by real data, this article derives and proves relationships for any two gravitationally-bound objects in single or multiple systems in the universe. These findings have implications for simpler and more accurate calculations in related practical applications[1][2]. Normally, relative error is less than 3.35%.

***Keywords:* Gravity; Centrifugul force; N body; Gravitational field; Solar system; Exoplanet; Universe**

Introduction

Gravitationally-bound objects that orbit a central object (single system) or different central objects (multiple systems) are general phenomena in the universe. It is therefore very meaningful to find the relationships between two gravitationally-bound points in single or multiple systems in the universe.

In this article, the case of the common centre is first discussed (single system examples: planets orbiting the Sun, moons orbiting a planet) and then, the second case of multiple centres with different central masses is discussed (multiple system examples: different moons orbiting different planets).

In these two systems, results show that the relationship between gravity and centrifugal force is the same, however, the relationships between any two gravitationally-bound points are different but convertible, and normally, the maximum relative error is less than 3.35%.

Derivation and Verification

Relationships between two points around the same centre (single system):

Centrifugal force: $F_c = \frac{mV^2}{R}$ (1)

then $F_{c0} = \frac{m_0 V_0{}^2}{R_0}$ (1a), $F_{c1} = \frac{m_1 V_1^2}{R_1}$ (1b)

Where, m is the mass of an orbiting body.

Gravitation: $F_g = G\frac{Mm}{R^2}$ (2)

Then $F_{g0} = G\frac{Mm_0}{R_0{}^2}$ (2a), $F_{g1} = G\frac{Mm_1}{R_1{}^2}$ (2b)

Where, M is the mass of the centre.

$$m_0 = \frac{F_{g0}R_0{}^2}{GM} \text{ then } F_{c0} = \frac{F_{g0}R_0{}^2 V_0{}^2}{GMR_0} = \frac{F_{g0}R_0 V_0{}^2}{GM} \quad (3)$$

$$m_1 = \frac{F_{g1}R_1{}^2}{GM} \text{ then } F_{c1} = \frac{F_{g1}R_1{}^2 V_1{}^2}{GMR_1} = \frac{F_{g1}R_1 V_1{}^2}{GM} \quad (4)$$

$$\frac{Fc_0}{Fc_1} = \frac{F_{g0}R_0 V_0{}^2}{F_{g1}R_1 V_1^2} \quad (5)$$

When $\frac{R_0 V_0{}^2}{R_1 V_1^2} = 1$ (6)

Or $\frac{V_0{}^2}{V_1^2} = \frac{R_1}{R_0}$ (7)

Then $\frac{Fc_0}{Fc_1} = \frac{F_{g0}}{F_{g1}}$ (8)

Following are verifications of equation (7) : $\frac{V_0{}^2}{V_1^2} = \frac{R_1}{R_0}$ using real data.

Verifications between the 8 planets in the Solar System as orbiting satellites and the Sun as the common centre.

Where absolute error $E = \frac{V_0{}^2}{V_1^2} - \frac{R_1}{R_0}$ (9)

Relative error $E_1 = \frac{2E}{\frac{V_0{}^2}{V_1^2} + \frac{R_1}{R_0}}$ (10)

TABLE. 1. Where V1 and R1 are Earth's data, V1= 29.78(km/s), R1=149598023(km). Note: V1 and R1 can be data from any of the 8 planets(verified and confirmed)[3]

Planet	V_0 (km/s)	R_0 (km)	$\frac{V_0^2}{V_1^2}$	$\frac{R_1}{R_0}$	$\lvert E \rvert$	$\lvert E_1 \rvert$
Neptune	5.43	4495.06	0.033246832	0.03328098	3.41473E-05	0.001026557
Uranus	6.8	2872.46	0.052139689	0.052080795	5.88944E-05	0.001130189
Saturn	9.68	1433.53	0.105657743	0.104357774	0.001299968	0.012379737
Jupiter	13.06	778.57	0.192325543	0.192147142	0.000178401	0.000928031
Mars	24.07	227.92	0.653285161	0.656370656	0.003085495	0.004711918
Venus	35.02	108.21	1.382874908	1.382496997	0.000377791	0.000273317
Mercury	47.36	57.91	2.529146582	2.583318943	0.054172361	0.021192264

Verifications between Jupiter’s 4 moons.

Table. 2. Where V1 and R1 are moon Io’s data, V1= 17.334(km/s)[4], R1=421700(km)[4]. Note: V1 and R1 can be data from any of the 4 moons(verified and confirmed).

Moon of Jupiter	V_0 (km/s)	R_0 (km)	$\frac{V_0^2}{V_1^2}$	$\frac{R_1}{R_0}$	$\lvert E \rvert$	$\lvert E_1 \rvert$
Europa[5]	13.74	670900	0.628312762	0.628558652	0.000245891	0.000391274
Ganymede[6]	10.88	1070400	0.393967327	0.393964872	2.45406E-06	6.2291E-06
Callisto[7]	8.204	1882700	0.22400294	0.223986827	1.61126E-05	7.19328E-05

Verifications between Saturn’s 4 moons.

Table. 3.Where V1 and R1 are moon Mimas’ data, V1= 14.28(km/s)[8], R1=185539(km)[8]. Note: V1 and R1 can be data from any of the 4 moons(verified and confirmed).

Moon of Saturn	V_0 (km/s)	R_0 (km)	$\frac{V_0^2}{V_1^2}$	$\frac{R_1}{R_0}$	$\lvert E \rvert$	$\lvert E_1 \rvert$
Tethys[9]	11.35	294619	0.631735537	0.629759112	0.001976273	0.003133465
Rhea[10]	8.48	527108	0.352643018	0.351994278	0.00064874	0.001841344
Titan[11]	5.57	1221870	0.1521437	0.151848396	0.000295304	0.001942842

Table 1, Table 2 and Table 3 show that $E1_{max}$ is less than 2.11%, due to the source data being averages. The results proved equation (7): $\frac{V_0^2}{V_1^2}=\frac{R_1}{R_0}$, therefore $\frac{Fc_0}{Fc_1}=\frac{F_{g0}}{F_{g1}}$ (8).

Relationship Between Two Points Around the Different Centres (Multiple Systems):

There are two kinds of relationships in a multiple system, one is the relationship between any two points around the same centre (single system) and another is the relationship between any two points around different centres (multiple systems). The following discusses relationships found in multiple systems, however the formerly discussed single system relationships are still valid and applied [4-10].

According to equation (3), $F_{c0} = \frac{F_{g0} R_0 V_0^{\,2}}{GM_0}$ (11)

Where M_0 is the central mass of single system A, R_0 is the distance from the orbiting point 0 to the centre.

According to equation (4), $F_{c1} = \frac{F_{g1} R_1 V_1^2}{GM_1}$ (12)

Where M_1 is the central mass of single system B, R_1 is the distance from the orbiting point 1 to the centre.

Then $\frac{Fc_0}{Fc_1} = \frac{F_{g0} R_0 V_0^{\,2} M_1}{F_{g1} R_1 V_1^2 M_0}$ (13)

When $\frac{R_0 V_0^{\,2} M_1}{R_1 V_1^2 M_0} = 1$ (14)

Then $\frac{M_1}{M_0} = \frac{R_1 V_1^2}{R_0 V_0^{\,2}}$ (15)

When $\frac{M_1}{M_0} = \frac{R_0}{R_1} = \frac{V_1}{V_0}$ (16)

Then $\frac{Fc_0}{Fc_1} = \frac{F_{g0}}{F_{g1}}$ (17)

Following are Verifications Of Equation (16) : $\frac{M_1}{M_0} = \frac{R_0}{R_1} = \frac{V_1}{V_0}$ Using Real Data(V_0 And R_0 are Theoretical Data).

Example of detailed calculations between Jupiter's moon Io and Saturn's moon Mimas:

Where M_1, V_1 and R_1 are Jupiter's data and M_0, V_0 and V_x and R_x are Saturn's data, V_0 and R_0 are Saturn's theoretical data obtained from real data.

$\frac{M_1}{M_0} = \frac{317.8}{95.152}$ =3.33998949, R_1=421700(km)(moon Io)[9], according to equation(16): $\frac{M_1}{M_0} = \frac{R_0}{R_1}$

$R_0 = \frac{R_1 M_1}{M_0}$ =1408473.568(km),

V_x=14.28(km/s)[13], R_x=85539(km)(moon Mimas)[13], according to equation(7): $\frac{V_0^{\,2}}{V_1^2} = \frac{R_1}{R_0}$

$\frac{V_0^{\,2}}{V_x^{\,2}} = \frac{R_x}{R_0}$, then $V_0 = V_x \sqrt{\frac{R_x}{R_0}}$ $=14.28\sqrt{\frac{85539}{408473.568}}$ =5.182883687(km/s)

$\frac{V_1}{V_0} = \frac{17.334}{5.182883687}$ =3.344470192

E= $\frac{M_1}{M_0} - \frac{V_1}{V_0} = 4.48\times10^{-3}$, $E_1 = \frac{2E}{\frac{M_1}{M_0}+\frac{V_1}{V_0}} = 1.34\times10^{-3}$.

Table. 4.Verifications between Jupiter's moon Io, Saturn's moon Mimas, Uranus' moon Miranda, Mars' moon Phobos and Earth's moon. These moons orbit different centres with different central masses (this multiple system consists of 5 single systems). Note: Although M_1, V_1 and R_1 are Jupiter's data (M_1=317.8[3], V_1=17.334(km/s)[4] and R_1=421700(km)[4]), these data can be from any of the 5 single systems(verified and confirmed).

Moons	M_0	V_x(km/s)	R_x(km)	R_0(k[m)	V_0(km/s)	$\frac{M_1}{M_0}$	$\frac{V_1}{V_0}$	$\lvert E_1 \rvert$
Mimas[8]	95.152[3]	14.28	185539	1408473.568	5.18288	3.34	3.344	0.00134
Miranda[14]	14.536[3]	6.66	129390	9219610.622	0.78898	21.86	21.97	0.00488
Phobos[12]	0.107[3]	2.138	9376	1252488411	0.00585	2970	2963	0.00231
Moon[13]	1[3]	1.022	384399	134016260	0.05473	317.8	316.7	0.0035

Table 4 shows that E_{1max} is 0.00488. The results proved equation (16): $\frac{M_1}{M_0} = \frac{R_0}{R_1} = \frac{V_1}{V_0}$

and therefore, equation (8): $\frac{Fc_0}{Fc_1} = \frac{F_{g0}}{F_{g1}}$.

Conclusion

$$\frac{Fc_0}{Fc_1} = \frac{F_{g0}}{F_{g1}} \qquad (8)$$

Where, $F_{c0}$and F_{c1} are centrifugal forces of any two points around the same or different centres, F_{g0} and F_{g1} are gravitational forces with distances of R_0 and R_1 to the corresponding centres.

Relationship between two points around the same centre (single system):

$$\frac{V_0^{\,2}}{V_1^2} = \frac{R_1}{R_0} \qquad (7)$$

Where, V_0 and V_1 are cross-radial velocities with distances of R_0 and R_1 to the centre respectively.

Relationship between two points around the different centres (multiple systems):

$$\frac{M_1}{M_0} = \frac{R_0}{R_1} = \frac{V_1}{V_0} \qquad (16)$$

Where, M_0 and M_1 are corresponding central masses, V_0 and V_1 are cross-radial velocities with distances of R_0 and R_1 to the centres respectively.

Gravitationally-bound point in single or multiple systems in the universe:

According to (15): $\frac{M_1}{M_0} = \frac{R_1 V_1^2}{R_0 V_0^{\,2}}$

$$M_1 = kR_1V_1^2 \qquad (17)$$

where $k = \frac{M_0}{R_0V_0^2} = 2.50863 \times 10^{-6}$, unit of M_1 is Earth mass

Where, V_0 and V_1 are cross-radial velocities with distances of R_0 and R_1 to the centre respectively.

Table. 5. calculations of constant k based on the data of the 5 planets and their moons

Moons	M_0[3]	V_0(km/s)	R_0(km)	k	Average k
Mimas[8]	95.152	14.28	185539	2.51493E-06	2.50863E-06
Miranda[14]	14.536	6.66	129390	2.53277E-06	
Phobos[12]	0.107	2.138	9376	2.49661E-06	
Moon[13]	1	1.022	384399	2.49067E-06	
Io[4]	317.8	17.334	421700	2.50815E-06	

Table. 6. verification results of equation (17): $M_1 = kR_1V_1^2$, to calculate the mass of the Sum and error E1 based on the data of the 8 planets and the Sun(M=333000[3]) to confirm k=2.50863E-6.

Planet[3]	**V_0(km/s)**	**R_0(k[m)**	**k**	**M_{SUN}**	**E1**
Neptune	5.43	4.50E+09	2.51E-06	3.32E+05	-0.001547221
Uranus	6.81	2.87E+09	2.51E-06	3.34E+05	0.003553643
Saturn	9.69	1.43E+09	2.51E-06	3.38E+05	0.014020461
Jupiter	13.07	7.79E+08	2.51E-06	3.34E+05	0.001938855
Mars	24.13	2.28E+08	2.51E-06	3.33E+05	-0.0002545
Venus	35.02	1.08E+08	2.51E-06	3.33E+05	-0.000248513
Mercury	47.87	5.79E+07	2.51E-06	3.33E+05	-0.000292851
Earth	29.78	149598023	2.51E-06	3.33E+05	-0.000534934

Table. 7. verification results of equation (17) using exoplanet data(R, T and M), M is real central star mass, M_1 is calculation central star mass, E_1 is Error(max: 3.9%, average: 1.28%).

Exoplanet	**R(AU)**	**T(day)**	**M**	**M_1**	**E_1**
Centauri[15][16]	0.0485	11.186	0.12121	0.121612946	0.003318847
PSR B1257	0.19	25.262	1.4	1.433605191	0.023719035
[17][18][19][20]	0.36	66.5419	1.4	1.405469195	0.003898952
	0.46	98.2114	1.4	1.346027444	-0.039309553
Gliese436[21][22]	0.0291	2.6339	0.46	0.473788621	0.029532638

Explanation

This article finds and establishes the theoretical/ideal relationships between two gravitationally-bound points in single or multiple systems in the universe, and uses real data to verify the calculation errors (the real statistic impact of the N body issue[1][2], N = 9, 8 planets + Sun) in these relationships. Especially, Equations (7),(16) and (17) are independence in gravity and centrifugal force, (17) is verified and confirmed by real data from both local and exoplanets.

Normally, when the orbit of a gravitationally-bound point approximates a circle, the theoretical/ideal relationships between two gravitationally-bound points can be expressed by equations (7), (8),(16) and (17) with about 3.35% error (average: 0.6%). The N body issue exists, however, its impact is limited / even negligible.

Equation (8) represents the relationship between gravity and centrifugal force. According to equation (8), gravity and centrifugal force change in the same direction at the same ratio [11-22].

Equation (17) represents the relationship between gravitationally-bound point and central mass. According to equation (17), once we know any two of the factors: cross-radial velocity V1, the central mass M1 or the distance R1, we can calculate the corresponding distance R1 or central mass M1 or cross-radial velocity V1 of a moon, a planet or a star.

Given that k is an universal constant. Now we can use equations (17) to explain why there are many exoplanets orbit their central stars at very high speed V in a very short distance R.

In summary, these findings are properties of gravitational field, prove physical laws are universal and can be used to accurately identify exoplanets.

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