# Bio Technology 

# Pricing strategy of dual-channel supply chain under joint promotion and retailer's capital constraint 

Zhou Yong-Wei ${ }^{1,2^{*}}$, Fan He-Hua ${ }^{1}$, Bao Yan-Wen ${ }^{1}$<br>${ }^{1}$ Department of Mathematics and Physics, Zhengzhou Institute of Aeronautical Industry Management, No. 2 Daxue Middle Road, Zhengzhou, (CHINA)<br>${ }^{2}$ Henan coordinative innovation center for aviation economic development, Henan aviation economy research center, No. 2 Daxue Middle Road, Zhengzhou, (CHINA)<br>E-mail: zwwfhh@126.com


#### Abstract

Under the condition of joint promotion and capital constraint on retailer, a dual-channel supply chain system with one risk neutral supplier and one risk neutral retailer selling the single product was analyzed in this paper, which set up the profit model of supplier dominated dual-channel system, studied to the optimal pricing strategies of decentralized and centralized supply chain, and obtained the sufficient conditions of optimal solution to the two kinds of supply chain system. It is observed from the optimal solution that the demand uncertainty of two distribution channels, deferred-payment rate, promotion cost and the retailer's sharing proportion of promotion cost etc, are all have certain influence relation on the pricing strategies of supply chain system. Numerical experiment has showed the effectiveness of the conclusions.


## KEYWORDS

Joint promotion; Capital constraint; Pricing strategy; Deferred payment.

## INTRODUCTION

With the rapid development of e-commerce, many brand manufacturers have begun to set up direct sales by introducing online channel. Such as Lenovo, Dell and Estee lauder so on, are all introduced online sales channel, they satisfied consumers requirements by the so-called "mouse + cement" dual-channel sales model ${ }^{[1]}$. But the introducing of online channel also gives rise to the channel conflict between supplier and retailer ${ }^{[2]}$, and consequently lowers the efficiency of supply chain ${ }^{[3] .}$ The dual-channel sales model also raised the new issue to supplier, under the condition that the coexistence of two sales channels, what way can the supplier adopt to stimulate the retailer develop market requirement, and realize the harmonious coexistence of traditional and internet direct sales channel? To stimulate the retailer developing the market, more and more suppliers proceeded joint promotion with retailer, such as shared a portion of promotion cost, sent their employees to participate in the sales promotion of retailers, and so on. At the same time, many enterprises faced with the problem of capital scarcity in reality, especially some small enterprises. Under the joint promotion and retailer's capital constraint, whether the price game between supplier and retailer has the equilibrium, which factors will affect their product pricing, the paper will attempt to resolve these problems.

Presently there have some research results on joint promotion, Zhang Juliang, Chen Jian ${ }^{[4]}$ studied how to design the contract to maximize the profit of supply chain system when the production demand is random and rely on the retailer's efforts, but it is a single channel system. Yu Aimin, Liu Liwen ${ }^{[5]}$ studied the competing and coordination strategies under the joint promotion. Aiming at the coexistence of traditional retail channel and the online sales channel within the supply chainn under e-commerce, Chen shuzhen etc ${ }^{[6]}$ analysis the optimal promotion investment and pricing strategy of centralized and decentralized supply chain under the certain demand. Qu Daogang, Guoyajun ${ }^{[7]}$ studied when the retailer's quantity demand is related to his promotion effort degree, how to realize the coordination of order quantity and sales effort by sharing contract.

There are also some researches on capital constraint; some researches ${ }^{[8-9]}$ mainly focus on its influence on inventory control and pledge decision. There are also some research findings on supply chain financing, Lee etc ${ }^{[10]}$ studied the coordination of price discount, buy-back, two-part tariff and revenue sharing four coordination mechanism to supply chain financing, subsequently, they studied supply chain with the trading credit from the supplier's view ${ }^{[11]}$, and raised the coordinate contract under the condition of on sale and compensation mechanism. Gupta etc ${ }^{[12]}$ analysis the optimal inventory decision by the relative algorithm of continuous time models, and it indicated that the supplier would automatically choose credit terms to coordinate the supply chain. Lai etc ${ }^{[13]}$ indicated that when the retailer's capital is insufficient, the supplier would prefer to the sales model of combing retailer's advanced order and consignment sales; Based on the different roles of third party logistics in a capital constrained supply chain, Chen Xiangfeng etc ${ }^{[14]}$ divided the structures of supply chain into three different types: convention model, delegation model and control model, and investigated the operational and financial decisions of firms under the supply chian with different type. Under the certain demands in traditional and online channels, Zhang Xiaojuan etc ${ }^{[15]}$ studied a dual-channel supply chain model with the retailer's capital constrained, the deferredpayment model and the loan-payment model are analyzed respectively, obtained the optimal decisions of the supplier and retailer under the two modes, and proved that no matter how much the producing-budget owns, the supplier is always willing to provide the retailer a deferred-payment policy.

Although there are many research results on joint promotion or capital constraint dual-channel supply chain, the study on the dual-channel supply chain with the joint promotion and retailer's capital constrain is seldom, the paper studied the dual-channel supply chain combing with capital constraint and joint promotion, and built the profit model of dual-channel supply chain, studied the influence of joint promotion and retailer's capital constraint on supplier and retailer's pricing decision.

## MODEL DESCRIPTION

We examine a dual-channel supply chain composed of one risk neutral supplier and one risk neutral retailer, the supplier may sell the products to the retailer, as well as to end customers directly. Customers can choose either the traditional or the online channel to buy the product. In order to express conveniently, all the relevant symbols are summarized in TABLE 1. Retailer and supplier's inventory levels to their random demand are $x$ and $y$, respectively. The supplier's production mode to retailer is make-to-order; therefore the supplier and retailer are responsible for their inventory level, respectively.

The retailer's capital is insufficient, which financing solution way is accepting the deferred-payment mode provided by supplier, the retailer pays money $B$ when order goods, the remaining payment and interest are paid after sale, the risk-free interest rate of business capital is 0 .

Then the demand functions ${ }^{[16]}$ of two channels are determined by:
$Q_{r}=\rho A-\alpha_{r} p_{r}+\beta_{r} p_{d}+e+\varepsilon_{r}$
$Q_{d}=(1-\rho) A-\alpha_{d} p_{d}+\beta_{d} p_{r}+\varepsilon_{d}$

It is assumed that $\alpha_{r}>\beta_{r}>0, \alpha_{d}>\beta_{d}>0$, which is very common in economics and operations management literatures, in order to simplified calculation, the paper assumes that $\alpha_{r}=\alpha_{d}=\alpha, \beta_{r}=$
$\beta_{d}=\beta$, and $\alpha>\beta$. e represents the retailer's promotion effort degree, retailer's sales promotion will enlarge the demand of traditional retail channel, and promotion cost $C(e)=0.5 \eta e^{2}$, where $\eta>0$. To obtain the greater profits, the supplier would willing to share some promotion cost (such as training the retailer's promotion staff, sending the relate employees to participate in the sales promotion of retailers, and so on) to encourage the retailer's production promotion. Where $0<t<1$ depends on the results of the negotiations between supplier and retailer, so it is assumed that $t$ is an exogenous variable.

So the expected demand functions in two channels are:
TABLE 1: Summarized symbols and their definitions

| Symbols | Definitions | Symbols | Definitions |
| :--- | :--- | :--- | :--- |
| $p_{r}$ | Price in the retail channel | $p_{d}$ | Price in the online channel |
| $t$ | Promotion cost proportion to retailer | $Q_{r}$ | Total demand of retail channel |
| $Q_{d}$ | total demand of online channel | $A$ | The potential demand |
| $\varepsilon_{r}$ | The random demand part of retail channel | $\mu_{r}$ | The mean of $\varepsilon_{r}$ |
| $\sigma_{r}{ }^{2}$ | The variance of $\varepsilon_{r}$ | $\left(m_{r}, n_{r}\right)$ | The value interval of $\varepsilon_{r}$ |
| $f(u)$ | The probability density function of $\varepsilon_{r}$ | $F(u)$ | The distribution function of $\varepsilon_{r}$ |
| $\varepsilon_{d}$ | Random demand part of online channel | $\mu_{d}$ | The mean of $\varepsilon_{d}$ |
| $\sigma_{d}{ }^{2}$ | The variance of $\varepsilon_{d}$ | $\left(m_{d}, n_{d}\right)$ | The value interval of $\varepsilon_{d}$ |
| $g(u)$ | The probability density function of $\varepsilon_{d}$ | $G(u)$ | The distribution function of $\varepsilon_{d}$ |
| $c$ | The supplier's unit production cost | $s_{m}$ | Supplier's unit shortage cost |
| $h_{m}$ | Supplier's unit inventory cost | $s_{r}$ | Retailer's unit shortage cost |
| $h_{r}$ | Retailer's unit inventory cost | $B$ | Retailer's initial capital quantity |
| $\rho$ | Customers' preference for retail channel | $\beta_{r}, \beta_{d}$ | Cross-price elasticity |
| $\alpha_{r}$ | Self-price elasticity of retail channel | $\eta$ | Promotion cost |
| $\alpha_{d}$ | Self-price elasticity of online channel | $w$ | Wholesale price to retailer |

$$
\begin{equation*}
E\left(Q_{r}\right)=\rho A-\alpha p_{r}+\beta p_{d}+\mu_{r}+e \tag{3}
\end{equation*}
$$

$E\left(Q_{d}\right)=(1-\rho) A-\alpha p_{d}+\beta p_{r}+\mu_{d}$
When the capital is sufficient, the expected profit functions in two channels are as follows:

$$
\begin{align*}
& E\left(\Pi_{r}\right)=\left(p_{r}-w\right) E\left(Q_{r}\right)-h_{r} \int_{m_{r}}^{x}(x-u) f(u) d u-s_{r} \int_{x}^{n_{r}}(u-x) f(u) d u-t C(e)  \tag{5}\\
& E\left(\Pi_{m}\right)=\left(p_{d}-c\right) E\left(Q_{d}\right)+(w-c) E\left(Q_{r}\right)-h_{m} \int_{m_{d}}^{y}(y-u) g(u) d u-s_{m} \int_{y}^{n_{d}}(u-y) g(u) d u-(1-t) C(e) \tag{6}
\end{align*}
$$

The total expected profit of two channels can be written as follows:
$E(\Pi)=E\left(\Pi_{r}\right)+E\left(\Pi_{m}\right)$

## MODEL ANALYSIS

## Decentralized pricing strategy with joint promotion and capital constraint

This section focused on the decentralized dual-channel system, in which the retailer and supplier maximize their own profits respectively, the paper models the decision as a sequential, Stackelberg game, with the supplier as the leader and the retailer as the follower. Retailer's insufficient capital refers to the retailer's initial capital quantity satisfy $B<w^{*} E\left(Q_{r}\right)$, to obtain more profit, retailer accepts the deferred-payment mode provided by supplier. On the ordering stage, the retailer paid part payment $B$, and $B / w$ products are obtained at wholesale price $w$, the payment for the remaining $E\left(Q_{r}\right)-B / w$ products
is $w E\left(Q_{r}\right)-B$, the supplier promises retailer pay deferred at price $w+w \delta(\delta \geq 0)$ after the sale, then the expected profit functions in two channels are as follows:
$E\left(\Pi_{r}\right)=\left(p_{r}-w\right) \frac{B}{w}+\left(p_{r}-w-\delta w\right)\left(E\left(Q_{r}\right)-\frac{B}{w}\right)-t C(e)-h_{r} \int_{m_{r}}^{x}(x-u) f(u) d u-s_{r} \int_{x}^{n_{r}}(u-x) f(u) d u$
$E\left(\Pi_{m}\right)=\left(p_{d}-c\right) E\left(Q_{d}\right)+(w-c) \frac{B}{w}+(w+\delta w-c)\left(E\left(Q_{r}\right)-\frac{B}{w}\right)-(1-t) C(e)$
$-h_{m} \int_{m_{d}}^{y}(y-u) g(u) d u-s_{m} \int_{y}^{n_{d}}(u-y) g(u) d u$
Then we have the following theorem
Theorem 1 For the decentralized dual-channel supply chain with joint promotion and retailer's capital constraint, the retailer adopt deferred-payment mode with rate $\delta$, the supply chain system exist optimal solution if (i) $\alpha \eta t>\frac{1}{2}$, (ii)
$\eta t\left(\beta^{2}-2 \alpha^{2}\right)+\alpha<0$, (iii) $\xi>0$.

Where $\xi=4 \alpha \eta t\left(\alpha^{2}-\beta^{2}\right)(2 \alpha \eta t-1)-\beta^{2}$. And
$p_{d}{ }^{*}=\frac{\alpha \beta \eta t(4 \alpha \eta t-1) \rho+2 \alpha^{2} \eta t(2 \alpha \eta t-1)(1-\rho)}{\xi} A+\frac{\alpha \beta \eta t(4 \alpha \eta t-1)}{\xi} \mu_{r}+$
$\frac{2 \alpha^{2} \eta t(2 \alpha \eta t-1)}{\xi} \mu_{d}-\left(\frac{\alpha \beta \eta t(\alpha-\beta)+\beta^{2} / 2}{\xi}-\frac{1}{2}\right) c$
$w^{*}=\frac{1}{1+\delta}\left\{\frac{[\beta(1-\rho)+\eta t[\beta(2 \alpha-\beta)+2 \alpha(\alpha-\beta)(2 \alpha \eta t-1)]] \rho+4 \alpha \beta \eta t(\alpha \beta \eta t-1)}{\xi} A+\right.$
$\left.\frac{2 \alpha^{2} \eta t(2 \alpha \eta t-1)-\beta^{2} \eta t}{\xi} \mu_{r}+\frac{\beta(2 \alpha \eta t-1)^{2}}{\xi} \mu_{d}+\frac{\left(\alpha^{2}-\beta^{2}\right) \eta t\left(4 \alpha^{2} \eta t-\alpha+\beta\right)-\alpha^{2} \eta t(\alpha-\beta)-\alpha \beta}{\xi} c\right\}$
$y^{*}=G^{-1}\left(\frac{s_{m}}{h_{m}+s_{m}}\right)$
$p_{r}^{*}=\eta t \frac{\rho A+\mu_{r}+\beta p_{d}^{*}-\alpha w^{*}(1+\delta)}{2 \alpha \eta t-1}+w^{*}(1+\delta)$
$e^{*}=\frac{\rho A+\mu_{r}+\beta p_{d}^{*}-\alpha w^{*}(1+\delta)}{2 \alpha \eta t-1}$
$x^{*}=F^{-1}\left(\frac{s_{r}}{h_{r}+s_{r}}\right)$
Proof. Please see the Appendix for proof.
Based on our results, we obtain the following propositions:
Proposition 1 The effect of traditional retail channel's demand uncertainty on supply chain's decision is determined by
$\frac{\partial p_{r}^{*}}{\partial \mu_{r}}>\frac{\partial p_{d}{ }^{*}}{\partial \mu_{r}}>0, \frac{\partial w^{*}}{\partial \mu_{r}}>0$.
From Proposition 1, we know that the effect of traditional retail channel's demand uncertainty on retailer's pricing is greater than that of supplier, and the more of $\mu_{r}$, the higher of retailer and supplier's pricing, and the higher of supplier's wholesale price of unit product to retailer.

Proposition 2 The effect of online channel's demand uncertainty on the supply chain's pricing decision is determined by
$\frac{\partial p_{d}{ }^{*}}{\partial \mu_{d}}>\frac{\partial p_{r}{ }^{*}}{\partial \mu_{d}}>\frac{\partial w^{*}}{\partial \mu_{d}}>0$.
It is observed that the effect of online channel's demand uncertainty on supplier's pricing is greater than that of supplier's unit product wholesale price to retailer and the retailer's pricing, and the greater of $\mu_{d}$, the higher of supplier's wholesale price to retailer, and also the higher of retailer's pricing.
Proposition 3 The effect of traditional and online channels' demand uncertainty on the wholesale price of retailer is determined by
$\frac{\partial w^{*}}{\partial \mu_{r}}>\frac{\partial w^{*}}{\partial \mu_{d}}>0$.
From Proposition 3, It can be seen that the traditional and online channels' demand uncertainty are all have positive effects on the wholesale price, the greater of demand uncertainty, the higher of wholesale price of retailer, and the effect of traditional channel's demand uncertainty on wholesale price is greater than that of online channel. Proposition 4 The effect of deferred-payment rate on supply chain's decision is determined by

$$
\frac{\partial p_{d}{ }^{*}}{\partial \delta}=\frac{\partial p_{r}{ }^{*}}{\partial \delta}=0, \frac{\partial w^{*}}{\partial \delta}<0
$$

It can be seen from proposition 4 that the deferred-payment rate has no effects on the direct channel's sale price and the traditional channel's retail price, but it has effect on the wholesale price to retailer, and the greater of deferred-payment rate, the lower of wholesale price to retailer.

Proposition 5 The effect of promotion cost and the retailer's sharing proportion to promotion cost on the retailer's decision is given by
$\frac{\partial e^{*}}{\partial \eta}<0, \frac{\partial e^{*}}{\partial t}<0$.
Proposition 5 showed that the promotion cost and the retailer's sharing proportion had a negative effect on the retailer's promotion effort degree, the retailer's promotion effort degree is decreased with the promotion cost and the retailer's sharing proportion to promotion cost.

## Centralized pricing strategy with joint promotion and capital constraint

In the centralized dual-channel supply chain, where the supplier and retailer are vertically integrated, the supplier determines the retail price $p_{r}$, promotion degree $e$ and direct sale price $p_{d}$ simultaneously to maximize the supply chain's total profit. In the deferred-payment mode, the supplier promises retailer pay deferred at rate $\delta$, it indicated that supplier has sufficient fund for production, so the dual-channel supply chain's total profit is as follows:

$$
\begin{align*}
& E(\Pi)=\left(p_{r}-c\right) E\left(Q_{r}\right)+\left(p_{d}-c\right) E\left(Q_{d}\right)-C(e)-h_{r} \int_{m_{r}}^{x}(x-u) f(u) d u-s_{r} \int_{x}^{n_{r}}(u-x) f(u) d u \\
& -h_{m} \int_{m_{d}}^{y}(y-u) g(u) d u-s_{m} \int_{y}^{n_{d}}(u-y) g(u) d u \tag{16}
\end{align*}
$$

So we have the following theorem:
Theorem 2 For the centralized dual-channel supply chain with joint promotion and retailer's capital constraint, the retailer adopt deferred-payment mode with rate $\delta$, the supply chain system exist optimal solution if $\zeta=2\left(\alpha^{2}-\beta^{2}\right) \eta-\alpha>0$, and

$$
\begin{equation*}
x_{c}^{*}=F^{-1}\left(\frac{s_{r}}{h_{r}+s_{r}}\right) \tag{17}
\end{equation*}
$$

$y_{c}^{*}=G^{-1}\left(\frac{s_{m}}{h_{m}+s_{m}}\right)$

$$
\begin{align*}
& p_{r c}^{*}=\frac{[\beta(1-\rho)+\alpha \rho] \eta}{\zeta} A+\frac{\alpha \eta}{\zeta} \mu_{r}+\frac{\beta \eta}{\zeta} \mu_{d}+\frac{\left(\alpha^{2}-\beta^{2}\right) \eta-\alpha}{\zeta} c  \tag{19}\\
& p_{d c}^{*}=\frac{(2 \alpha \eta-1)(1-\rho)+2 \beta \eta \rho}{2 \zeta} A+\frac{\beta \eta}{\zeta} \mu_{r}+\frac{2 \alpha \eta-1}{2 \zeta} \mu_{d}+\frac{(\alpha+\beta)[1-2 \eta(\alpha-\beta)]}{2 \zeta} c  \tag{20}\\
& e_{c}^{*}=\frac{\beta(1-\rho)+\alpha \rho}{\zeta} A+\frac{\alpha}{\zeta} \mu_{r}+\frac{\beta}{\zeta} \mu_{d}+\frac{\beta^{2}-\alpha^{2}}{\zeta} c \tag{21}
\end{align*}
$$

The proof is similar to Theorem 1, so we omit it.
According to our theorem 2, we obtain the following propositions:
Proposition 6 The effect of deferred-payment rate on the supply chain's decision is determined by
$\frac{\partial p_{d c}{ }^{*}}{\partial \delta}=\frac{\partial p_{r c}{ }^{*}}{\partial \delta}=\frac{\partial e_{c}{ }^{*}}{\partial \delta}=0$.
From Proposition 6 we can see that the deferred-payment rate has no effects on the two channels’ retail price and promotion degree, it means that in the centralized dual-channel supply chain with retailer's deferred-payment mode, retailer's capital constraint problem is the internal problem of system, which has no effects on the optimal pricing of supplier and retailer.

Proposition 7 The effect of traditional retail channel's demand uncertainty on the channel's pricing decision is greater than that of online channel, which is given by
$\frac{\partial p_{r c}{ }^{*}}{\partial \mu_{r}}>\frac{\partial p_{r c}{ }^{*}}{\partial \mu_{d}}>0, \frac{\partial e_{c}^{*}}{\partial \mu_{r}}>\frac{\partial e_{c}^{*}}{\partial \mu_{d}}>0$.
It is observed that that the two channels' demand uncertainty are all have effects on the retailer's optimal pricing and promotion degree from Proposition 7, and the influence of traditional retail channel's demand uncertainty on retailer's optimal pricing and promotion degree is greater than that of direct channel, and the greater of two channels' demand uncertainty, the higher of retailer's optimal pricing, and the higher of retailer's promotion degree.
Proposition 8 The effect of retail channel's demand uncertainty on the supplier and retailer's decision is given by
$\frac{\partial p_{r c}{ }^{*}}{\partial \mu_{r}}>\frac{\partial p_{d c}{ }^{*}}{\partial \mu_{r}}>0$.
It is observed that the effect of retail channel's demand uncertainty on retailer's pricing decision is greater than that of supplier from Proposition 8, and the greater of retail channel's demand uncertainty, the higher of retailer and supplier's optimal pricing.

Proposition 9 The effect of retailer's promotion cost on the retailer's decision is given by
$\frac{\partial e_{c}^{*}}{\partial \eta}<0$.

## NUMERICAL EXAMPLES

In this section we perform a numerical analysis to verify our results and illustrate the effect of joint promotion and the retailer's capital constraint on the optimal pricing decisions for the dual-channel supply chain. It will complement our analytical results and provide us with more managerial insights.

In our example, it is assumed that the random demand of traditional channel $\varepsilon_{r} \sim U(0,4)$, the random demand of online channel $\varepsilon_{d} \sim U(0,6)$, other parameters in the supply chain system take in the following values:
$A=20, \rho=0.5, c=3, \alpha=0.3, \beta=0.1, h_{r}=4, s_{r}=8, h_{m}=3, s_{m}=6, \delta=0.05, \eta=7, t=0.5$.

## Decentralized supply chain

According to the theoretical results of decentralized supply chain and parameter values mentioned above, we obtained $w^{*}=26.697, p_{r}{ }^{*}=51.403, p_{d}{ }^{*}=37.548, e^{*}=6.677$, and the retailer and supplier's inventory levels for uncertainty demand
are $x^{*}=2.67, y^{*}=4$, respectively. The effect of two channels' uncertainty demand, deferred-payment rate, promotion cost and the retailer's sharing proportion of promotion cost on the channel's optimal pricing is as Figure1-Figure5, they are consistent with the theoretical analysis of Proposition1 to Proposition5.


Figure 1: The influence of retail channel's uncertainty demand on pricing decision


Figure 3: The influence of retailer's deferred payment rate on pricing

## Degree



Figure 5: The influence of the retailer's sharing proportion for promotion cost on it's promotion effort


Figure 2: The influence of direct channel's uncertainty demand on pricing decision


Figure 4: The influence of promotin cost on the decision retailer's promotion effort


Figure 6: The influence of retail channel's uncertainty demand on centralized pricing decision

## Centralized supply chain

According to the theoretical results of centralized supply chain and parameter values mentioned above, we obtained that $p_{r c}^{*}=42.781, e_{c}^{*}=5.683, p_{d c}{ }^{*}=36.927$, and the retailer and supplier's inventory levels for uncertainty demand are
$x^{*}=2.67, y^{*}=4$, respectively. The effect of two channels' demand uncertainty and promotion cost on the optimal pricing is as Figure 6-Figure 8, they are consistent with the theoretical analysis of Proposition 6 to Proposition 9.

It can be seen from the above numerical analysis that the inventory levels for uncertainty demand of retailer and supplier are identical in centralized and decentralized case, but the total profit in centralized case $(E(\Pi)=437.095)$ is greater than that $\left(E\left(\Pi_{r}\right)+E\left(\Pi_{m}\right)=420.859\right)$ of decentralized one. So the supplier and retailer can consider the designing of contract for the coordination of supply chain, which will get a Pareto improvement on the profits of supplier and retailer.


Figrue 8: The retailer's proportion effort changing with the promotion cost


Figure 9: The influence of two channel's demand uncertainty on retailer's centralized pricing decision

## APPENDIX

We solve the Stackelberg equilibrium solution of supply chain by inverse order method. Firstly, to maximize the retailer's profit, given the supplier's direct sale price $p_{d}$, wholesale price $w$, and the inventory level $y$, retailer determines the retail price $p_{r}$, the promotion degree $e$ and inventory level $x$, the retailer's decision:
$\max E\left(\Pi_{r}\right)=\max \left[\left(p_{r}-w\right) E\left(Q_{r}\right)-\delta\left(w E\left(Q_{r}\right)-B\right)-t C(e)-h_{r} \int_{m_{r}}^{x}(x-u) f(u) d u-s_{r} \int_{x}^{n_{r}}(u-x) f(u) d u\right]$
As (A1) can be divided into $\left(p_{r}-w\right) E\left(Q_{r}\right)-\delta\left(w E\left(Q_{r}\right)-B\right)-t C(e)$ and $-\left[h_{r} \int_{m_{r}}^{x}(x-u) f(u) d u+\right.$ $\left.s_{r} \int_{x}^{n_{r}}(u-x) f(u) d u\right]$ two parts, and the second part is only relate to inventory level $x$, so find the optimal value of retail price $p_{r}$, the promotion degree $e$ and inventory level $x$ on $E\left(\Pi_{r}\right)$ respectively, and can obtain the retailer's optimal decisions $p_{r}{ }^{*}$, $e^{*}$ and $x^{*}$, as
$\frac{\partial^{2} E\left(\Pi_{r}\right)}{\partial p_{r}^{2}}=-2 \alpha_{r}<0, \quad \frac{\partial^{2} E\left(\Pi_{r}\right)}{\partial e^{2}}=-\eta t<0, \frac{\partial^{2} E\left(\Pi_{r}\right)}{\partial x^{2}}=-\left(h_{r}+s_{r}\right) f(x)<0$
So there exist maximum on $E\left(\Pi_{r}\right)$, according the FOC of $E\left(\Pi_{r}\right)$, We obtained (15) and
$p_{r}^{*}=\frac{1}{2 \alpha}\left(\rho A+\beta p_{d}+\mu_{r}+e+\alpha w(1+\delta)\right)$
$e^{*}=\frac{p_{r}-w(1+\delta)}{\eta t}$
Substituting (A2) and (A3) into (3) and (4), and obtained:

$$
\begin{equation*}
E\left(Q_{r}\right)=\frac{\alpha \eta t}{2 \alpha \eta t-1}\left(\rho A+\beta p_{d}+\mu_{r}\right)+\frac{\alpha w(1+\delta)}{2 \alpha \eta t-1} \tag{A4}
\end{equation*}
$$

$E\left(Q_{d}\right)=\left[\frac{\beta \eta t}{2 \alpha \eta t-1} \rho+(1-\rho)\right] A+\left(\frac{\beta^{2} \eta t}{2 \alpha \eta t-1}-\alpha\right) p_{d}+\frac{\beta \eta t}{2 \alpha \eta t-1} \mu_{r}+\mu_{d}+\beta w(1+\delta) \frac{\alpha \eta t-1}{2 \alpha \eta t-1}$
Substituting (A4) and (A5) into (9), and find the first partial derivatives on $E\left(\Pi_{m}\right)$ to direct sale price $p_{d}$, wholesale price $w$, and the inventory level $y$, respectively, and let them equal to 0 , so we obtain the game equilibrium strategy of supplier. As (9) can be divided into $\left(p_{d}-c\right) E\left(Q_{d}\right)+(w-c) B / w$
$+(w+\delta w-c)\left(E\left(Q_{r}\right)-\frac{B}{w}\right)-(1-t) C(e)$ and $-h_{m} \int_{m_{d}}^{y}(y-u) g(u) d u-s_{m} \int_{y}^{n_{d}}(u-y) g(u) d u$ two parts, which are the functions of $\left(p_{d}, w\right)$ and inventory level $y$, respectively. And

$$
\frac{\partial^{2} E\left(\Pi_{m}\right)}{\partial p_{d}{ }^{2}}=\frac{2 \beta^{2} \eta t}{2 \alpha \eta t-1}-2 \alpha<0, \frac{\partial^{2} E\left(\Pi_{m}\right)}{\partial y^{2}}=-\left(h_{m}+s_{m}\right) g(y)<0, \frac{\partial^{2} E\left(\Pi_{m}\right)}{\partial w^{2}}=-\frac{2 \alpha^{2} \eta t(1+\delta)^{2}}{2 \alpha \eta t-1}<0
$$

So there exist maximum on $E\left(\Pi_{m}\right)$, then according the FOC of $E\left(\Pi_{m}\right)$ can get (10), (11) and (12).
In the following, to prove $E\left(\Pi_{m}\right)$ reaches maximum at $\left(p_{d}^{*}, w^{*}, y^{*}\right)$, we just prove when $y=y^{*}, E\left(\Pi_{m}\right)$ reaches maximum at $\left(p_{d}^{*}, w^{*}\right)$, as $\frac{\partial^{2} E\left(\Pi_{m}\right)}{\partial p_{d}{ }^{2}}=\frac{2 \beta^{2} \eta t}{2 \alpha \eta t-1}-2 \alpha<0$, its Hessian matrix:

$$
|H|=\left|\begin{array}{ll}
\left.\frac{\partial^{2} E\left(\Pi_{m}\right)}{\partial p_{d}{ }^{2}}\right|_{\left(p_{d}^{*}, w^{*}\right)} & \left.\frac{\partial^{2} E\left(\Pi_{m}\right)}{\partial p_{d} \partial w}\right|_{\left(p_{d}^{*}, w^{*}\right)} \\
\left.\frac{\partial^{2} E\left(\Pi_{m}\right)}{\partial w \partial p_{d}}\right|_{\left(p_{d}^{*}, w^{*}\right)} & \left.\frac{\partial^{2} E\left(\Pi_{m}\right)}{\partial w^{2}}\right|_{\left(p_{d}^{*}, w^{*}\right)}
\end{array}\right|=\frac{(1+\delta)^{2} \xi}{(2 \alpha \eta t-1)^{2}}>0
$$

So $E\left(\Pi_{m}\right)$ reaches maximum at $\left(p_{d}^{*}, w^{*}, y^{*}\right)$.
Substituting (10) and (11) into (A2) and (A3), and can obtain formula (13) and (14).

## CONCLUDING REMARKS

Joint promotion and capital constraint is a very important issue in dual-channel supply chain management, this paper examines the pricing decision in a dual-channel supply chain with one supplier and one retailer under joint promotion and retailer's capital constraint. And draws the conclusion that the demand uncertainty of two distribution channels, deferredpayment rate, promotion cost and the retailer's sharing proportion of promotion cost etc, are all have certain influence relation on the pricing strategies of supplier and retailer. Furthermore, in the decentralized supply chain, the effect of traditional retail channel's demand uncertainty on retailer's pricing is greater than that of supplier; and the more of uncertainty demands in traditional channel, the higher of retailer and supplier's pricing, and the higher of supplier's wholesale price of unit product to retailer; the greater of uncertainty demands in direct channel, the higher of supplier's wholesale price to retailer, and also the higher of retailer's pricing; the traditional and online channels' demand uncertainty are all have positive effects on the wholesale price; the deferred-payment rate has no effects on the direct channel's sale price and the traditional channel's retail price, but it has effect on the wholesale price to retailer; the promotion cost and the retailer's sharing proportion had a negative effect on the retailer's promotion effort degree, the retailer's promotion effort degree is decreased with the promotion cost and the retailer's sharing proportion to promotion cost. In the centralized supply chain, the two channels' demand uncertainty are all have effects on the retailer's optimal pricing and promotion degree; the effect of retail channel's demand uncertainty on retailer's pricing decision is greater than that of supplier. Numerical experiment has verified the effectiveness of the conclusions.

There are several directions for further analysis that will achieve a better understanding of dual-channel supply chain. For instance, one may present a revenue sharing contract, which can relieve the channel conflict, realize the win-win outcome between supplier and retailer, and get a Pareto improvement on the profits of two sides, in addition, it would be interesting to examine the supply chain's unite decision of product pricing under joint promotion and retailer's capital constraint, and it is insightful to consider how the supplier determine deferred payment rate and joint promotion strategies to realized the coordination of supply chain system under the condition that the system is composed of more retailers and one supplier with different retailers have different capital constraints.

## ACKNOWLEDGEMENT

This work was supported by Natural Science Foundation of Henan Province (122300410181), Project of Youth Backbone Teachers of Colleges in Henan Province (2013GGJS-142), Fund of Aeronautics Science (2013ZD55006), Natural Science Foundation of Henan Education Department (13A110399, 14A630019, 14B110023), Students Research Fund of Zhengzhou Institute of Aeronautical Industry Management (2014-23).

## REFERENCES

[1] K.Takahashi, T.Aoi, D.Hirotani, et al; Inventory control in a two-echelon dual-channel supply chain with setup of production and delivery, International Journal of Production Economics, 133, 403-415 (2011).
[2] K.L.Webb; Managing channels of distribution in the age of electronic commerce, Industrial Marketing Management, 31(2), 95-102(2013).
[3] Wei-yu Kevin Chiang; Product availability in competitive an cooperative dual-channel distribution with stock-out based substitution, European Journal of Operational Research, 200, 111-126(2010).
[4] Zhang Juliang, Chen Jian; A coordinating contract of supply chain with sale-effort dependent demand, Chinese Journal of Management Science, 12(4), 50-56(2004).
[5] Yu Aimi, Liu Liwen; Competing and coordination strategies for the dual channel under stochastic demand and cooperative promotion, Journal of Industrial Engineering/ Engineering Mangagement, 26(1),151-155(2012).
[6] Chen Shuzhen, Xiong Zhongkai, Liang Xi, Contract Design for A Dual-Channnel Supply Chain Coordination with Incentive Compensation, China Journal of Management Science, 17(1), 64-75, (2009).
[7] Qu Daogang, Guo Yajun, Coordination of supply chain with hybrid distribution channels when retailer's demand relies on its sales effort, China Journal of Management Science, 16(3), 89-94, (2008).
[8] J.A.Buzacott, R.Zhang ; Inventory management with asset-based financing, Management Science, 50(9), 1274-1292 (2004).
[9] N.N.Yan, B.W.Sun; Optimal strategies for supply chain financing system based on warehouse receipts financing with credit line, Systems Engineering-Theory \& Practice, 31(9), 1674-1679 (2011).
[10] C.H.Lee, B.D.Rhee; Coordination contracts in the presence of positive inventory financing costs, Internal Journal of Production Economics, 124(2), 331-339 (2010).
[11] C.H.Lee, B.D.Rhee; Trade credit for supply chain coordination, European Journal of Operational Research, 214, 136146(2011).
[12] D.Gupta, L.Wang; A stochastic model with trade credit, Manufacturing and Service Operations Management, 11(1), 418 (2009).
[13] G.Lai, G.D.Laurens, Katia Sycarab; Sharing inventory risk in supply chain: The implication of financial constraint, Omega, 37(4), 811-825 (2009).
[14] X.F.Chen, D.L.Zhu; System value of 3PL to the supply chain with capital constraints, Journal of Systems Engineering, 23(6), 666-673 (2008).
[15] Zhang Xiaojuan, Wangyong; Dual-channel supply decision with retailers capital constraint, Control and Decision, 29(2), 299-306 (2014).
[16] Yun Chu Chen, Shu-Cherng Fang, Ue-Pyng Wen; Pricing policies for substitutable products in a supply chain with internet and traditional channels, European Journal of Operational Research, 24,542-551(2013).

