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# Pricing decision of closed loop supply Chains in hybrid recycling channels by different market powers under fuzzy environment

Subo Xu<sup>1,2</sup>, Jiayi Sun<sup>1</sup>, Chunxian Teng<sup>1</sup>, Mujing Wei<sup>1</sup> <sup>1</sup>Institute of System Engineering, Harbin University of Science and Technology, Harbin 150080, (CHINA) <sup>2</sup>College of Accounting, Heilongjiang BaYi Agricultural University, Daging 163319, (CHINA)

## ABSTRACT

According to the hybrid recycling closed-supply chain system composed of one manufacturer and one retailer, the manufacturer and the retailer simultaneously recycle waste products in the consumer market. Under decentralized decision-making conditions, build the closed-loop supply chain game model of different market powers and centralized decision-making model, give the decision variable values respectively, and confirm the conclusion of this paper through numerical example. It showed that the centralized decisions are always superior to decentralized decisions. In decentralized decisions, structure without leader is superior to the retailer-oriented structure which is at the same time superior to the manufacturer-oriented structure. But a structure without a leader is unstable so that the manufacturer and the retailer all compete for being the leader. With the increase of recovery and utilization rate, the whole expected profits of the manufacturer, the retailer and the supply chain also increase.

# **KEYWORDS**

Fuzzy environment; Closed loop supply Chains; Recoverability rate; Hybrid recycling channels; Pricing decision.

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### **INTRODUCTION**

In a society of rapid economic development, the shortage of resources and environment pollution is increasingly affecting the development of enterprises. In the hope of achieving sustainable development of economy with minimal resources and environmental costs, the close loop supply chain with the feature of cyclical mode of production comes into being.

According to channels of recycling waste products, the way of recycling can be divided into four sorts: The manufacturer recycling, Retailer recycling, Third party recycling, and Hybrid Recycling. Hybrid Recycling includes several kinds of ways. Savaskan et al studied about the choice of the remanufacturing close loop supply chain. In a decentralized structure, they made a comparison among manufacturer recycling, Retailer recycling and third party recycling. They got the conclusion that retailer recycling was the best choice. In addition, Savaskan et al studied the influence on recycling channel and the choice of supply chain structure by recycling cost and competitive intensity between two competing retailers. Yuyan WANG have thought about Coordination mechanism of dual channel closed loop supply chain under the governmental interference. The above researches on the recycling channel of closed loop supply chain did not take the effect on its members by different market powers into consideration. Yuyin YI et al studied the influence of different market powers on the recycling channel the game result of joint recovery of closed-loop supply chain by retailers and third party under four different market powers<sup>[6]</sup>.

The above researches considered the Hybrid recycling of closed loop supply chain in certain environment. The uncertainty of environment did not show any influence on the decision of closed loop supply chain members. In an uncertain environment study, one kind was to use the probability distribution of parameters known to express the uncertainty existing in the closed-loop supply chain, which makes depict of uncertain variables affected by researchers subjectively. In order to be able to more effectively manage the closed-loop supply chain, fuzzy theory can better describe these uncertainties compared with the probability theory. In 1965 Zadeh first put forward the concept of fuzzy set and he described it preliminarily<sup>[7]</sup>. Kwakernaak further described the meaning of the fuzzy set and put forward the fuzzy set theory<sup>[8]</sup>. LIU, on the basis of predecessors' research, got the fuzzy optimal theory through research, described the basic concept of expected value of fuzzy variables, established fuzzy expected value model, and analyzed fuzzy expected value model with the hybrid intelligent algorithm<sup>[9]</sup>. On the basis of Zadeh's research, Wang studied further about fuzzy expected function model and got the method of solving the optimization problem for retailers by the theory of fuzzy set and expanded the method into wider application<sup>[10]</sup>.

On the basis of fuzzy theory research, Shengju SANG studied a two-level supply chain composed of a manufacturer and a retailer, built different contract models under fuzzy demand, and worked out optimal decision of various models by fuzzy set theory<sup>[11]</sup>, Francisco studied the two-stage, single- product, multi-period supply chain, used the method of fuzzy estimate to depict the uncertain demand, and built a system dynamic model<sup>[12]</sup>. Deng studied the influence of closed-loop supply chain on the development of circular economy, considered the closed-loop supply chain under the fuzzy random environment, and used the large system theory to establish coordination model of closed loop supply chain based on fuzzy variables and random variables<sup>[13]</sup>. David established fuzzy mathematical programming model to do research on supply and demand in closed loop supply chain and uncertain factors in the process. He used the triangular fuzzy variables to build the fuzzy mixed integer linear programming model, which offered available decision plans for the decision makers of different satisfaction degree<sup>[14]</sup>. Haiyun ZHOU studied monocycle closed-loop supply chain under the fuzzy condition. She characterized the uncertain factors in closed-loop supply chain as fuzzy variables, analyzed the supply chain decision problems under different decision situations, and established income-cost sharing contract to coordinate the supply chain <sup>[15]</sup>. Wei,in fuzzy environment, studied on pricing decision of closed-loop supply chain composed of a manufacturer and a retailer by game theory and fuzzy set theory<sup>[16]</sup>. The references from 13 to 16 all studied on the closed-loop supply chain under the fuzzy condition. In the process of research, the recycled products were of 100% supposed to be able to be converted into new products, which was unreasonable in real life for ignorance of the degree of damage of waste products.

On the basis of predecessors' research, this paper, according to two-level closed loop supply chain with hybrid recycling channels, took into consideration reutilization rate of waste products and formed the game model of different market powers under decentralized decisions and centralized decision-making model to study on pricing decision of closed loop supply chain members. This paper first expounded the related theory of the fuzzy environment, then described the models and built the basic model, studied four different modes of pricing decision, and finally, through numerical example, got the conclusion and proposed the future research direction.

### DESCRIPTIONS OF THE MODEL AND SYMBOLS

The article established a close-loop supply chain with a manufacturer (remanufacturer) and a retailer (recycler). The manufacturer sells products to the retailer and entrusts the recycling of waste products to the retailer, meanwhile, the manufacturer also recycles waste products in the consumer market, using them for remanufacturing. If the remanufactured product has the same quality with the new product, then we ignore the devaluation of the remanufactured product.

 $c_m$ : Unit cost of the new products of manufacturers

 $b_m$ : The recycling price of wasted products from retailers to manufacturers

- $c_r$ : Unit cost of remanufactured products of manufacturers
- $b_1$ : The recycling price of wasted products from consumers to retailers
- $b_2$ : The recycling price of wasted products from consumers to manufacturers
- w: The wholesale price of products from manufacturers to retailers
- p: The market price of products
- $\tau_1$ : The recovery rate of retailers
- $\tau_2$ : The recovery rate of manufacturers
- $k_1$ : The fixed cost coefficient of wasted products recycling of retailers
- $k_2$ : The fixed cost coefficient of wasted products recycling of manufacturers
- *s* : Unit residual value of recycled products
- $\theta$ : Proportionality coefficient of remanufacture of recycled products (recycling rate)
- D(p): The market demand as the unit price of products is p

In this article, param  $b_r$  eters  $b_m$ ,  $c_m$ , are all non-negative independent fuzzy variables. The market demand of n  $c_r$  ew products is non-negative in real life, thus  $Pos(\alpha - \beta p < 0) = 0$  is possible.

This article assumed that  $D(p) = \alpha - \beta p$ ,  $\alpha$  was the market basis of new products,  $\beta$  was the price elasticity of sale price p, namely, the extent of reaction of market demand on the sale price.  $\alpha$ ,  $\beta$  are all non-negative fuzzy variables. The number of recycle wasted product is  $\tau D(p)$ , and  $0 \le \tau \le 1$ . Similar to the description of cost-recovering by Savaskan, the cost-recovering of wasted products has two parts: one is the  $b\tau D(p)$ , the cost-recovering changing with the recycling price, another is the  $k\tau^2$ , the fixed cost generated in the recycling process. This article assumed that the residual value of the wasted products which could not be remanufactured was less than the revenue of remanufacture, namely,  $s < c_m - c_r$ .

#### THE MODEL BUILDING

In the situation of hybrid recycling, manufacturers sell products to retailers, then retailers sell them to consumers, at the same time, retailers and manufacturers recycle wasted products from the market. The wholesale price w, recovery rate of retailers  $\tau_1$ , recovery rate of manufacturers  $\tau_2$  and sale price p are decision variables. Parameters  $\alpha$ ,  $\beta$ ,  $b_m$ ,  $b_1$ ,  $b_2$ ,  $c_m$ ,  $c_r$  are all non-negative independent fuzzy variables. The market demand of new products is non-negative in real life, thus  $Pos(\alpha - \beta p < 0) = 0$  is possible.



#### Figure 1 : Structure of closed-loop supply Chain in hybrid recycling channels

So the manufacturer's profit  $\pi_m$  is as follows :

$$\pi_{m} = (\alpha - \beta p)(w - c_{m}) + (\theta(c_{m} - c_{r}) + s(1 - \theta))(\tau_{1} + \tau_{2})(\alpha - \beta p) - (b_{m}\tau_{1} + b_{2}\tau_{2})(\alpha - \beta p) - k_{2}\tau_{2}^{2}$$
(1)

The retailer's profit  $\pi_r$  is :

$$\pi_r = (\alpha - \beta p)(p - w) + (b_m - b_1)\tau_1(\alpha - \beta p) - k_1\tau_1^2$$
(2)

The manufacturer's decision variables are wholesale price W and recovery rate  $\tau_2$ , its maximized expected profit  $E[\pi_m]$  is :

 $\max_{m} E[\pi_m] = \max_{m} E[(\alpha - \beta p)(w - c_m) + \theta(c_m - c_r)(\tau_1 + \tau_2)(\alpha - \beta p)]$ 

$$+s(1-\theta)(\tau_{1}+\tau_{2})(\alpha-\beta p) - (b_{m}\tau_{1}+b_{2}\tau_{2})(\alpha-\beta p) - k_{2}\tau_{2}^{2}]$$
(3)

The retailer's decision variables are product price p and recovery rate  $\tau_1$ , its maximized expected profit  $E[\pi_m]$  is :

$$\max_{p,\tau_2} E[\pi_r] = \max E[(\alpha - \beta p)(p - w) + (b_m - b_1)\tau_1(\alpha - \beta p) - k_1\tau_1^2]$$
(4)

#### The manufacturer-oriented Stackelberg model-M model

The manufacturer-oriented market structure is composed of large manufacturers and relatively small retailers. In this case, the bargaining power of manufacturer is stronger than that of retailer. The manufacturer sets wholesale price w and recovery rate  $\tau_2$ , and the retailer sets its selling price p recovery rate  $\tau_1$  according to the wholesale price set by the manufacturer. Its equilibrium is the sub-game perfect Nash equilibrium, so we can use reverse recursive method to solve this game.

According to formula (4), we can get the retailer's expected profit as follows :

$$E[\pi_{r}] = E[\alpha](p-w) - E[\beta]p^{2} + E[\beta]pw + \tau_{1}pG_{1} + \tau_{1}G_{2} - k_{1}\tau^{2}$$
(5)

Formula(5)'s Hessian matrix is :

$$\begin{vmatrix} \frac{\partial^2 E[\pi_r^M]}{\partial p^2} & \frac{\partial^2 E[\pi_r^M]}{\partial p \partial \tau_1} \\ \frac{\partial^2 E[\pi_r^M]}{\partial \tau_1 \partial p} & \frac{\partial^2 E[\pi_r^M]}{\partial \tau_1^2} \end{vmatrix} = \begin{vmatrix} -2E[\beta] & G_1 \\ G_1 & -2k_1 \end{vmatrix}$$

In which,

$$F_{1} = \frac{1}{2} \int_{0}^{1} (\beta_{\lambda}^{U} b_{m\lambda}^{L} + \beta_{\lambda}^{L} b_{m\lambda}^{U}) d\lambda, F_{2} = \frac{1}{2} \int_{0}^{1} (\alpha_{\lambda}^{U} b_{1\lambda}^{L} + \alpha_{\lambda}^{L} b_{1\lambda}^{U}) d\lambda, G_{1} = E[b_{1}\beta] - F_{1}, G_{2} = E[b_{m}\alpha] - F_{2}$$

From this we can conclude that, when  $k_1 > \frac{G_1^2}{4E[\beta]}$ , Hessian matrix is negative definite, the retailer's expected profit function is joint concave function between *p* and  $\tau_1$ . According to the first order conditions we get :

$$p^{M} = \frac{2k_{1}(E[\alpha] + E[\beta]w) - G_{1}G_{2}}{4k_{1}E[\beta] - G_{1}^{2}}$$
(6)

$$\tau_1^M = \frac{G_1(E[\alpha] + E[\beta]w) + 2E[\beta]G_2}{4k_1 E[\beta] - G_1^2}$$
(7)

Put formula (6) and (7) into formula (3), we can get the manufacturer's expected profit as follows :

$$E[\pi_m] = wG_8 + w^2G_9 + w\tau_2G_{10} - k_2\tau_2^2 + \tau_2G_{11} + G_{12}$$
(8)

In which, 
$$F_3 = \frac{1}{2} \int_0^1 (\alpha_\lambda^U b_{2\lambda}^L + \alpha_\lambda^L b_{2\lambda}^U) d\lambda$$
,  $F_4 = \frac{1}{2} \int_0^1 (\alpha_\lambda^U b_{m\lambda}^L + \alpha_\lambda^L b_{m\lambda}^U) d\lambda$ ,  
 $F_5 = \frac{1}{2} \int_0^1 (\alpha_\lambda^U c_{m\lambda}^L + \alpha_\lambda^L c_{m\lambda}^U) d\lambda$ ,  $F_6 = \frac{1}{2} \int_0^1 (\alpha_\lambda^U c_{r\lambda}^L + \alpha_\lambda^L c_{r\lambda}^U) d\lambda$ ,  
 $G_3 = \theta(F_5 - F_6) + s(1 - \theta) E[\alpha]$ ,  $G_4 = \theta(E[c_r\beta] - E[c_m\beta]) + s(\theta - 1)E[\beta]$ ,  
 $G_5 = 2k_1 E[\alpha] + G_1 G_2$ ,  $G_6 = G_1 E[\alpha] + 2G_2 E[\beta]$ ,  $G_7 = 4k_1 E[\beta] - G_1^2$ ,

$$G_{8} = \frac{E[\beta](G_{4} + E[b_{m}\beta])(G_{1}G_{5} + 2k_{1}G_{6})}{G_{7}^{2}} + \frac{E[\beta](-G_{5} + G_{1}G_{3} + 2k_{1}E[c_{m}\beta] - F_{4}G_{1})}{G_{7}} + E[\alpha]$$

$$G_{9} = \frac{2k_{1}E[\beta]^{2}(G_{1}(G_{4} + E[b_{m}\beta]) - G_{7})}{G_{7}^{2}}, G_{10} = \frac{2k_{1}E[\beta](G_{4} + E[b_{2}\beta])}{G_{7}}, G_{10} = \frac{2k_{1}E[\beta](G_{7} + E[b_{2}\beta])}{G_{7}}, G_{10} = \frac{2k_{1}E[\beta]($$

$$G_{11} = -F_3 + G_3 + \frac{G_5}{G_7}(G_4 + E[b_2\beta]),$$

$$G_{12} = -F_5 + \frac{G_5 G_6 (G_4 + E[b_m \beta])}{G_7^2} + \frac{G_5 E[c_m \beta] + G_3 G_6 - F_4 G_6}{G_7} \,.$$

In Formula(8), Hessian matrix is as follows :

$$\begin{vmatrix} \frac{\partial^2 E[\pi_m^M]}{\partial w^2} & \frac{\partial^2 E[\pi_m^M]}{\partial w \partial \tau_2} \\ \frac{\partial^2 E[\pi_m^M]}{\partial \tau_2 \partial w} & \frac{\partial^2 E[\pi_m^M]}{\partial \tau_2^2} \end{vmatrix} = \begin{vmatrix} 2G_9 & G_{10} \\ G_{10} & -2k_2 \end{vmatrix}$$

When  $k_2 > \frac{-G_{10}^2}{4G_9}$ ,  $E[\pi_m^M]$ 's Hessian matrix is negative definite, the manufacturer's expected profit function is joint

concave function between w and  $\tau_2$ . According to the first order conditions we get :

$$w^{M*} = \frac{-2k_2G_8 - G_{10}G_{11}}{4k_2G_9 + G_{10}^2}$$
(9)

$$\tau_2^{M^*} = \frac{2G_{11}G_9 - G_8G_{10}}{4k_2G_9 + G_{10}^2} \tag{10}$$

Put formula(9) and (10) into formula (6) and (7), we get :

$$p^{M^*} = \frac{2k_1 E[\beta] w^* + G_5}{G_7}$$
(11)

$$\tau_1^{M^*} = \frac{G_1 E[\beta] w^* + G_6}{G_7}$$
(12)

#### **Retailer-oriented stackelberg model-R model**

The retailer-oriented market structure is made up of large retailers and relatively smaller manufacturers. In this case, the bargaining power of retailers is stronger than that of the manufacturer. The retailer sets selling price p and recovery rate  $\tau_1$ , and the manufacturer sets its wholesale price W and recovery rate  $\tau_2$ . accordingly. Its equilibrium is the sub-game perfect

Nash equilibrium, so we can use reverse recursive method to solve this game.

According to formula (3), we can get the expected profit of retailer as follows:

$$E[\pi_{m}] = (\tau_{1} + \tau_{2})(G_{3} + G_{4}p) - k_{2}\tau_{2}^{2} + wE[\alpha] + p\tau_{1}E[b_{m}\beta] + p\tau_{2}[b_{2}\beta] - pwE[\beta] + pE[c_{m}\beta] - \tau_{2}F_{3} - \tau_{1}F_{4} - F_{5}$$
(13)

Suppose p = w + m, *m* represents the profit brought about by the selling price being greater than the wholesale price, put p = w + m into formula (13) we can get :

$$E[\pi_{m}] = (\tau_{1} + \tau_{2})(G_{3} + G_{4}p) - k_{2}\tau_{2}^{2} + wE[\alpha] + (w + m)\tau_{1}E[b_{m}\beta] - \tau_{1}F_{4} - F_{5} + (w + m)\tau_{2}E[b_{2}\beta] - (w + m)wE[\beta] + (w + m)E[c_{m}\beta] - \tau_{2}F_{3}$$
(14)

Formula(14)'s Hessian's matrix is :

$$\begin{vmatrix} \frac{\partial^2 E[\pi_m^R]}{\partial w^2} & \frac{\partial^2 E[\pi_m^R]}{\partial w \partial \tau_2} \\ \frac{\partial^2 E[\pi_m^R]}{\partial \tau_2 \partial w} & \frac{\partial^2 E[\pi_m^R]}{\partial \tau_2^2} \end{vmatrix} = \begin{vmatrix} -2E[\beta] & G_{14} \\ 0 & -2k_2 \end{vmatrix}$$

So in the retailer-oriented Stacklberg game, the retailer expects Hessian's matrix of profit function to be negative definite, and the manufacturer's expected profit function  $E[\pi_m]$  is the joint concave function about *w* and  $\tau_2$  According to the first order conditions we get :

$$w^{R} = \frac{G_{18} + 2k_{2}\tau_{1}G_{13} - pG_{17}}{2k_{2}E[\beta]}$$
(15)

$$\tau_2^R = \frac{pG_{14} + G_{16}}{2k_2} \tag{16}$$

In which,  $G_{13} = G_4 + E[b_m\beta]$ ,  $G_{14} = G_4 + E[b_2\beta]$ ,  $G_{15} = E[\alpha] + E[c_m\beta]$ ,  $G_{16} = G_3 - F_3$ ,  $G_{17} = 2k_2E[\beta] - G_{14}^2$ ,  $G_{18} = 2k_2G_{15} + G_{16}G_{14}$ 

Put the formula (15) and (16) into formula (5), we can get Hessian matrix of retailer's expected profit function as follows:

$$\begin{vmatrix} \frac{\partial^2 E[\pi_r^R]}{\partial p^2} & \frac{\partial^2 E[\pi_r^R]}{\partial p \partial \tau_1} \\ \frac{\partial^2 E[\pi_r^R]}{\partial \tau_1 \partial p} & \frac{\partial^2 E[\pi_r^R]}{\partial \tau_1^2} \end{vmatrix} = \begin{vmatrix} 2G_{20} & G_{13} + G_1 \\ G_{13} + G_1 & -2k_1 \end{vmatrix}$$

When  $k_1 > \frac{-(G_{13} + G_1)^2}{4G_{20}}$ ,  $E[\pi_r^R]$ 's Hessian matrix is negative definite, the retailer's expected profit function is the

joint concave function between p and  $\tau_1$ , according to the first order conditions we get :

$$p^{R^*} = \frac{2k_1G_{19} + (G_1 + G_{13})G_{21}}{(G_{13} + G_1)^2 + 4k_1G_{20}}$$
(17)

$$\tau_1^{R^*} = \frac{2G_{21}G_{20} - G_{19}(G_1 + G_{13})}{(G_{13} + G_1)^2 + 4k_1G_{20}}$$
(18)

In which, 
$$G_{19} = E[\alpha] + \frac{G_{18}}{2k_2} + \frac{E[\alpha]G_{17}}{2k_2E[\beta]}$$
,  $G_{20} = -E[\beta] - \frac{G_{17}}{2k_2}$ ,  $G_{21} = G_2 - \frac{E[\alpha]G_{13}}{E[\beta]}$ .

Put formula (17) and (18) into formula (15) and (16), we can get :

$$w^{R*} = \frac{G_{18} + 2k_2\tau_1^*G_{13} - p^*G_{17}}{2k_2 E[\beta]}$$
(19)

$$\tau_2^{R*} = \frac{p^* G_{14} + G_{16}}{2k_2} \tag{20}$$

#### Nash model-N model

The manufacturer competes with the retailer Nash in the market consisting of relatively small and medium manufacturers and retailers. But both could not dominate the market. Under the condition of Nash game, they have the same bargaining power and at the same time they make the optimal decision of each.

Combine formula (5)and(13),we can get :

$$w^{N*} = \frac{1}{2E[\beta](G_{13}G_{1}k_{2} + G_{14}^{2}k_{1} + G_{1}^{2}k_{2} - 6k_{1}k_{2}E[\beta])} (2k_{2}E[\beta]G_{2}(G_{1} - 2G_{13}) +4k_{1}k_{2}E[\beta](E[\alpha] - 2G_{15}) - 2k_{2}G_{1}(G_{15}G_{1} - E[\alpha]G_{13}) -2k_{1}G_{14}(E[\alpha]G_{14} + 2E[\beta]G_{16}) + G_{14}G_{1}(G_{16}G_{1} - G_{14}G_{2}))$$
(21)

$$p^{N*} = \frac{k_2 G_2 G_1 + 2k_2 k_1 G_{15} + k_2 G_{13} G_2 + k_1 G_{14} G_6 + 2k_1 k_2 E[\alpha]}{6k_1 k_2 E[\beta] - G_{13} G_1 k_2 - G_{14}^2 k_1 - G_1^2 k_2}$$
(22)

$$\tau_1^{N^*} = \frac{2E[\alpha]k_2G_1 + 2k_2G_{15}G_1 + G_1G_{14}G_{16} - G_{14}^2G_2 + 6k_2E[\beta]G_2}{2(6k_1k_2E[\beta] - G_{13}G_1k_2 - G_{14}^2k_1 - G_1^2k_2)}$$
(23)

$$\tau_2^{N^*} = \frac{(G_{14}G_2 - G_{16}G_1)(G_1 + G_{13}) + 2G_4k_1(G_{15} + E[\alpha]) + 6G_{16}k_1E[\beta]}{2(6k_1k_2E[\beta] - G_{13}G_1k_2 - G_{14}^2k_1 - G_1^2k_2)}$$
(24)

#### **Centralized model-C model**

Add formula (1) and (2) together, we can get the profit function of supply chain:

$$\pi_{s} = \pi_{m} + \pi_{r} = (\alpha - \beta p)(p - c_{m}) + (\theta(c_{m} - c_{r}) + s(1 - \theta))(\tau_{1} + \tau_{2})(\alpha - \beta p) - (b_{1}\tau_{1} + b_{2}\tau_{2})(\alpha - \beta p) - k_{1}\tau_{1}^{2} - k_{2}\tau_{2}^{2}$$
(25)

The decision variables of supply chain is the product price  $p_1$  recycling rate  $\tau_1$  and  $\tau_2$ , so the expected profit of supply chain  $E[\pi_s]$  is :

$$\max_{p,\tau_1,\tau_2} E[\pi_m] = \max E[(\alpha - \beta p)(p - c_m) + \theta(c_m - c_r)(\tau_1 + \tau_2)(\alpha - \beta p) + s(1 - \theta)(\tau_1 + \tau_2)(\alpha - \beta p) - (b_1\tau_1 + b_2\tau_2)(\alpha - \beta p) - k_1\tau_1^2 - k_2\tau_2^2]$$
(26)

Simplified as :

$$E[\pi_{s}] = (\tau_{1} + \tau_{2})(G_{3} + G_{4}p) - k_{1}\tau_{1}^{2} - k_{2}\tau_{2}^{2} + pE[\alpha] + p\tau_{1}E[b_{1}\beta] + p\tau_{2}[b_{2}\beta] - p^{2}E[\beta] + pE[c_{m}\beta] - \tau_{2}F_{3} - \tau_{1}F_{2} - F_{5}$$
(27)

Formula(27)'s Hessian matrix as follows :

$$\begin{vmatrix} \frac{\partial^2 E[\pi_s^C]}{\partial p^2} & \frac{\partial^2 E[\pi_s^C]}{\partial p \partial \tau_1} & \frac{\partial^2 E[\pi_s^C]}{\partial p \partial \tau_2} \\ \frac{\partial^2 E[\pi_s^C]}{\partial \tau_1 \partial p} & \frac{\partial^2 E[\pi_s^C]}{\partial \tau_1^2} & \frac{\partial^2 E[\pi_s^C]}{\partial \tau_1 \partial \tau_2} \\ \frac{\partial^2 E[\pi_s^C]}{\partial \tau_2 \partial p} & \frac{\partial^2 E[\pi_s^C]}{\partial \tau_2 \partial \tau_1} & \frac{\partial^2 E[\pi_s^C]}{\partial \tau_2^2} \end{vmatrix} = \begin{vmatrix} -2E[\beta] & G_4 + E[b_1\beta] & G_4 + E[b_2\beta] \\ G_4 + E[b_1\beta] & -2k_1 & 0 \\ G_4 + E[b_2\beta] & 0 & -2k_2 \end{vmatrix}$$

 $k_1 > \frac{[G_4 + E[b_1\beta]]^2}{4E[\beta]}$ ,  $E[\pi_s^C]$  Hessian's matrix is negative definite, the expected profit function of supply chain is the

joint concave function about  $p \tau_1$  and  $\tau_2$ , according to the first order conditions we can get the following :

$$p^{C^*} = \frac{1}{k_2 G_4^2 + 2k_2 G_4 E[b_1 \beta] + k_2 E[b_1 \beta^2] + k_1 G_4^2 + 2k_1 G_4 E[b_2 \beta] + k_1 E[b_2 \beta^2] - 4k_1 k_2 E[\beta]} \cdot (-k_2 G_3 G_4 + k_2 G_4 F_2 - k_2 G_3 E[b_1 \beta] + k_2 F_2 E[b_1 \beta] - k_1 G_3 G_4 + k_1 G_4 F_3 - 2k_1 k_2 E[\alpha] - k_1 G_3 E[b_2 \beta] + k_1 F_3 E[b_2 \beta] - 2k_1 k_2 E[c_m \beta])$$
(28)

$$\tau_{1}^{C^{*}} = \frac{1}{2(k_{2}G_{4}^{2} + 2k_{2}G_{4}E[b_{1}\beta] + k_{2}E[b_{1}\beta^{2}] + k_{1}G_{4}^{2} + 2k_{1}G_{4}E[b_{2}\beta] + k_{1}E[b_{2}\beta^{2}] - 4k_{1}k_{2}E[\beta])} \cdot (-2k_{2}G_{4}E[c_{m}\beta] - G_{3}G_{4}E[b_{1}\beta] + G_{3}E[b_{2}\beta^{2}] + G_{4}^{2}F_{3} - G_{4}^{2}F_{2} - F_{2}E[b_{2}\beta^{2}] + G_{3}G_{4}E[b_{2}\beta] - 4k_{2}G_{3}E[\beta] - 2k_{2}G_{4}E[\alpha] + G_{4}F_{3}E[b_{2}\beta] + G_{4}F_{3}E[b_{1}\beta] - 2k_{2}E[\alpha]E[b_{1}\beta] - G_{3}E[b_{1}\beta]E[b_{2}\beta] + F_{3}E[b_{1}\beta]E[b_{2}\beta] - 2k_{2}E[b_{1}\beta]E[c_{m}\beta] - 2G_{4}F_{3}E[b_{2}\beta] + 4k_{2}F_{2}E[\beta]$$
(29)

$$\tau_{2}^{C^{*}} = \frac{1}{2(k_{2}G_{4}^{2} + 2k_{2}G_{4}E[b_{1}\beta] + k_{2}E[b_{1}\beta^{2}] + k_{1}G_{4}^{2} + 2k_{1}G_{4}E[b_{2}\beta] + k_{1}E[b_{2}\beta^{2}] - 4k_{1}k_{2}E[\beta])} \cdot (-2k_{1}G_{4}E[c_{m}\beta] + G_{3}G_{4}E[b_{1}\beta] + G_{3}E[b_{1}\beta^{2}] - G_{4}^{2}F_{3} + G_{4}^{2}F_{2} - F_{3}E[b_{1}\beta^{2}] - G_{3}G_{4}E[b_{2}\beta] - 4k_{1}G_{3}E[\beta] - 2k_{1}G_{4}E[\alpha] + G_{4}F_{2}E[b_{2}\beta] + G_{4}F_{2}E[b_{1}\beta] - 2k_{1}E[\alpha]E[b_{1}\beta] - G_{3}E[b_{1}\beta]E[b_{2}\beta] + F_{2}E[b_{1}\beta]E[b_{2}\beta] - 2k_{1}E[b_{1}\beta]E[c_{m}\beta] - 2G_{4}F_{3}E[b_{1}\beta] + 4k_{1}F_{3}E[\beta]$$
(30)

#### NUMERICAL EXAMPLE

All the fuzzy variables involved in this paper are supposed to be the triangular fuzzy variables. Supposing  $\alpha = (100, 150, 200)$ ,  $\beta = (0.2, 0.5, 0.8)$ ,  $c_m = (30, 35, 45)$ ,  $c_r = (9, 14, 18)$ ,  $b_m = (8, 10, 12)$ ,  $b_r = (3, 5, 6)$ ,  $b_1 = (3, 5, 6)$ ,  $b_2 = (5, 7, 9)$ As for the fixed recovery cost k, The cost of building its own recovery channel by manufacturer is higher than that of retailer, so the recovery cost of manufacturer  $k_2 = 600$ , the recovery cost of retailer  $k_1 = 500$ , the value of recycling waste products s = 4.

Chapter 2 has explained the fuzzy theory, according to it  $E[\alpha] = \frac{100 + 2 \times 150 + 200}{4} = 150$ ,  $E[\beta] = \frac{0.2 + 2 \times 0.5 + 0.8}{4} = 0.5$ . The pessimistic value and positive value of fuzzy variables :  $\alpha_{\lambda}^{L} = 100 + 50\lambda$ ,  $\alpha_{\lambda}^{U} = 200 - 50\lambda$ ,  $\beta_{\lambda}^{L} = 0.2 + 0.3\lambda$ ,  $\beta_{\lambda}^{U} = 0.8 - 0.3\lambda$ ,  $c_{m\lambda}^{L} = 30 + 5\lambda$ ,  $c_{m\lambda}^{U} = 45 - 10\lambda$ ,  $c_{r\lambda}^{L} = 9 + 5\lambda$ ,  $c_{r\lambda}^{U} = 18 - 4\lambda$ ,  $b_{m\lambda}^{L} = 8 + 2\lambda$ ,  $b_{m\lambda}^{U} = 12 - 2\lambda$ ,  $b_{r\lambda}^{L} = 3 + 2\lambda$ ,  $b_{r\lambda}^{U} = 6 - \lambda$ ,  $b_{1\lambda}^{L} = 5 + 2\lambda$ ,  $b_{2\lambda}^{U} = 9 - 2\lambda$ .







Figure 3 : Profits of Retailer of Different Market Powers Based on Recoverability Rate



Figure 4 : Profits of supply chain under different market power based on recoverability rate

From picture one, picture two, and picture three, we can conclude:

Firstly, with the increase of recycling rate- $\odot$ , through these four ways, the expected profit of manufacturers, the expected profit of retailers and the whole expected profit of supply chain are increasing. That is to say, the enhancement of recycling is beneficial for manufacturers and retailers.

Secondly, in the three ways, the comparison of the expected profit of manufacturers is as follows:  $E[\pi_m^M] > E[\pi_m^R] > E[\pi_m^R]$ , namely, the expected profit is highest among structures led by retailers, and the lowest among the structures led by manufacturers.

Thirdly, in the three ways, the comparison of the expected profit of retailers is as follows:  $E[\pi_r^R] > E[\pi_r^M] > E[\pi_r^M]$ 

namely, the expected profit of retailers is highest among structures led by retailers and the lowest among structures led by manufacturers.

Fourthly, in the four ways, the comparison of the expected profit of supply chain is as follows: when there is no leader in the market, the profit will become the highest. When  $\odot$  is beyond 0.055 or 0.467, the expected profit of supply among structures led by retailers is higher than that among structures led by manufacturers.

In conclusion, the market structure without leaders is the most superior under the distributed strategy. However, the market structure without leaders is not a stable structure, and manufacturers and retailers all have motivation to achieve the occupation of leader. Therefore, if the leader was retailer, the market structure is more superior. And if we analyze from the

perspective of supply chain, the whole expected profit of centralized is always higher than that of distributed strategy. Therefore, the centralized strategy is the best choice.

#### CONCLUSION

This paper analyzed the closed loop supply chain consisting of mixed recycling ways of one manufacturer and one retailer in an obscured environment. Through considering the recycling rate of wasted and old products and analyzing closed loop supply chain in different market forces, I concluded the best pricing strategy for manufacturers and retailers. I have tested the researcher in numerical examples. It suggested that the enlargement of recycling rate would bring much profit for manufacturers and retailers. It also concluded that centralized strategy was more superior than distributed one. Although the no-leader structure is more beneficial for supply chain, the structure is unstable for manufacturers and retailers having opportunity to be the leader, and the market structure led by the latter, in this circumstance, is the best. On the basis of the researcher of my paper, we can consider the decision problem of pricing differences of new and remanufactured products.

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