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Matrix eigenvalues of jacobi iterative method for solving

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ABSTRACT

Nowadays, the development in information technology is rapid. Mathematics is applied more widely in research. Eigenvalue is also successfully applied in many fields. This paper describes the definition of eigenvalues and gives concrete examples in the first place. Next, it illustrates the theoretical knowledge of Jacobi iteration method. From the perspective of representation methods, it discusses the representation method of linear equations and matrix representation method. Meanwhile, the Jacobi iteration matrix is proposed,and it tells the iterative process Jacques ratio. Again, under conditions of actual teaching situation and the use of fuzzy comprehensive evaluation method,the model evaluates the representation of linear equations and matrix representation method for practical teaching method applicability. Calculated by the software programming, the obtained results shows that from a comprehensive point of view to consider teaching carried out, presentation method of linear equations iterative method is the most appropriate. Finally, specific examples of Jacobi iteration method is given.

KEYWORDS

Matrix eigenvalue; Jacobi iteration method representation of linear equations; Matrix representation; Fuzzy comprehensive evaluation.

INTRODUCTION

Eigenvalues of matrix plays important roles in many areas such as image processing. Based on the eigenvalues of the matrix, it can be used as the processing of the image. In the mechanical system, the eigenvalues of the matrix can be used as the basis to determine the equilibrium point.

In 2013, Wenshi Liao in the "Show-shaped boundary estimate its eigenvalues", emphasized that the eigenvalues of the matrix has many important applications in quantum mechanics and image in processing. The author focuses on the question of the distribution of eigenvalues and the estimation problem. It's distribution area shows oval. The paper studys on the distribution of the eigenvalues matrix, and concentrate on the distribution estimating the range of eigenvalues. For the inequalities of eigenvalues square modulus, The model developes a judgment based on the stability and the stability of linear and nonlinear dynamical systems. The concept also shows the stability of interval matrices.

In 2006, Xiaoqian Wu in "Jacobi matrix inverse eigenvalue problems and other inverse problems", illustrates the inverse problem of classification and application. Describes the three-diagonal matrix, Jacobi matrices, orthogonal polynomials, Gaussian integral and Jacobi matrix eigenvalues are the basis on the Jacobi matrix solving. The authors proposes three eigenvalues Jacobi algorithm. They can avoid re-construct the first Jacobi matrix of order n principal submatrix drawbacks. These three algorithms are based on Newton interpolation and dichotomy,Gauss quadrature formula,orthogonal matrix decomposition style units feature vector to Jacobi matrix form the basis. Laplace equation for the inverse problem is by solving the Laplace equation to solve. Using several piecewise linear function solutions of linear combinations approximates boundary function. If the boundary is close to the boundary linear function, you can find similarities with the exact solution of the approximate solution.

In 2006,Lijun Liu in"neural network seeking symmetric matrix eigenvalues",studys computing eigenvalues of symmetric matrix problems in the theoretical basis of neural networks and gives B- norm unchanged RNNs model. According to the calculation of the maximum and minimum eigenvalues, he designed the entire program to calculate the eigenvalues and solved the results of numerical experiments. According to the theory of stochastic approximation method,he solves the maximum principal component and algorithms minimum principal elements. On the issue of calculating the generalized symmetric positive definite eigenvalues,he proposed model of generalized eigenvalue RNNs.

The paper describes the concept of matrix eigenvalues and general solving methods, and discusses the formulation in an iterative method, and gives a reasonable example.

MATRIX EIGENVALUE

If A for n order matrix, it exists a value of m, the n non-zero-dimensional column vector x, enabling the establishment of $Ax = mx$, and then it is said that m is the eigenvalues to A. Column vector x is A of the feature vector. The method for solving eigenvalues is so many, such as the use of $|mE - A| = 0$, m is the characteristic value of A. If the n order matrix A exists matrix polynomial equations $g(m) = 0$. then the eigenvalues of the matrix must comply. In addition, there is Jacobi iterative methods, QR algorithms, and G-S iterative method.

Specific examples:

Solving matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ eigenvalues.

Compages characteristic equation $|\lambda E - A| = 0$

$$|\lambda E-A|=\begin{vmatrix}\lambda-1 & -1 & 0\\ -1 & \lambda & -1\\ 0 & -1 & \lambda-1\end{vmatrix}=\begin{vmatrix}\lambda-1 & 0 & 1-\lambda\\ -1 & \lambda & -1\\ 0 & -1 & \lambda-1\end{vmatrix}=(\lambda-1)(\lambda+1)(\lambda-2)$$

Therefore, eigenvalues is $1,-1,2$ 。

JACOBI ITERATIVE METHOD

Iterative method representation includes linear equations representation and matrix equation representation. In solving practical problems, the conventional eigenvalue solution is not very practical. Iterative method for solving is most reasonable, the calculated results is relatively accurate.

Linear equations representation

Assuming the existence of the n order equations

$$\begin{cases} a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n=b_1\\ a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n=b_2\\ \cdots\cdots\\ a_{n1}x_1+a_{n2}x_2+\cdots+a_{nn}x_n=b_n\end{cases} \qquad (1)$$

If the matrix is non-singular matrix of coefficients, and $a_{ii}\neq 0(i=1,2\cdots,n)$ and then the equation can be rewritten as

$$\begin{cases} x_1=\dfrac{1}{a_{11}}(b_1-a_{12}x_2-a_{13}x_3-\cdots-a_{1n}x_n)\\ x_2=\dfrac{1}{a_{21}}(b_2-a_{22}x_2-a_{23}x_3-\cdots-a_{2n}x_n)\\ \cdots\cdots\\ x_n=\dfrac{1}{a_{nn}}(b_{1n}-a_{n1}x_1-a_{n2}x_2-\cdots-a_{n,n-1}x_{n-1})\end{cases} \qquad (2)$$

Be rewritten as iterative format

$$\begin{cases} x_1^{(k+1)}=\dfrac{1}{a_{11}}(b_1-a_{12}x_2^{(k)}-a_{13}x_3^{(k)}-\cdots-a_{1n}x_n^{(k)})\\ x_2^{(k+1)}=\dfrac{1}{a_{21}}(b_2-a_{21}x_1^{(k)}-a_{23}x_3^{(k)}-\cdots-a_{2n}x_n^{(k)})\\ \cdots\cdots\\ x_n^{(k+1)}=\dfrac{1}{a_{nn}}(b_{1n}-a_{n1}x_1^{(k)}-a_{n2}x_2^{(k)}-\cdots-a_{n,n-1}x_{n-1}^{(k)})\end{cases} \qquad (3)$$

Simplified as

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left(b_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij} x_j^{(k)} \right)(i = 1,2,\cdots,n) \quad \textbf{(4)}$$

When given initial value $X^{(0)} = \left(x_1^{(0)}, x_2^{(0)}, \cdots x_n^{(0)}\right)^T$, after repeated iterative process to derive a set of vectors $X^{(k)} = (x_1^{(k)}, \cdots x_n^{(k)})^T$, if $X^{(k)}$ converges to $X^* = (x_1^*, x_2^*, \cdots x_n^*)^T$, then x_i^* is the solution of equations, this solution method is called Jacobi iteration.

Matrix representation

Matrix equation $Ax = b$, which

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Construct a stationary iteration order:

$$\begin{cases} x^{(0)} \text{The initial vector} \\ x^{(k+1)} = Bx^{(k)} + f\,(k = 0,1,\cdots) \\ \text{Among them } B = M^{-1}N = M^{-1}(M-N) = I - M^{-1}A \\ f = M^{-1}b \end{cases} \quad (5)$$

Assuming $a_{ii} \neq 0$, $\quad i = 1,2,3,\cdots,n$

Select the diagonal $M = D$, A split to $A = D - N$, you can get the $Ax = b$ iterative method, as follows:

$$\begin{cases} x^{(0)} \text{the initial vector:} \\ x^{(k+1)} = Bx^{(k)} + f\,(k = 0,1,\cdots) \\ \text{Among them } B = I - D^{-1}A = D^{-1}(L+U) \equiv J \\ f = D^{-1}b \end{cases} \quad (6)$$

Jacobi iteration matrix

If there are three matrices L、D、U ,satisfy the following conditions,

$$L = \begin{pmatrix} 0 & & & & & \\ a_{21} & 0 & & & & \\ a_{31} & a_{32} & 0 & & & \\ \cdots & \cdots & \cdots & 0 & & \\ a_{n-1,1} & a_{n-1,2} & \cdots & & 0 & \\ a_{n,1} & a_{n,2} & \cdots & & a_{n,n-1} & 0 \end{pmatrix} \quad (7)$$

$$D = \begin{pmatrix} a_{11} & & & & & \\ & a_{22} & & & & \\ & & a_{33} & & & \\ & & & \cdots & & \\ & & & & a_{n-1,n-1} & \\ & & & & & a_{n,n} \end{pmatrix} \quad (8)$$

$$U = \begin{pmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1,n-1} & a_{1,n} \\ & 0 & a_{23} & \cdots & a_{2,n-1} & a_{2,n} \\ & & 0 & a_{34} & \cdots & a_{3,n} \\ & & & 0 & \cdots & \cdots \\ & & & & 0 & a_{n-1,n} \\ & & & & & 0 \end{pmatrix} \quad (9)$$

Well $A = L + D + U$, that is divided into a lower triangular matrix, diagonal and upper triangular matrix and thus illustrate the Jacobi iteration matrix of the form

$$J = -D^{-1}(L+U) \quad (10)$$

Jacobi iteration process

Jacobi iterative process includes a plurality of process, entering the initial conditions. And the results can be obtained. Judge error of between the calculated and the exact value. If the error is too large, they want to continue the iterative process. when the error is in the allowable range, you can determine a reasonable calculation results shown in Figure 1.

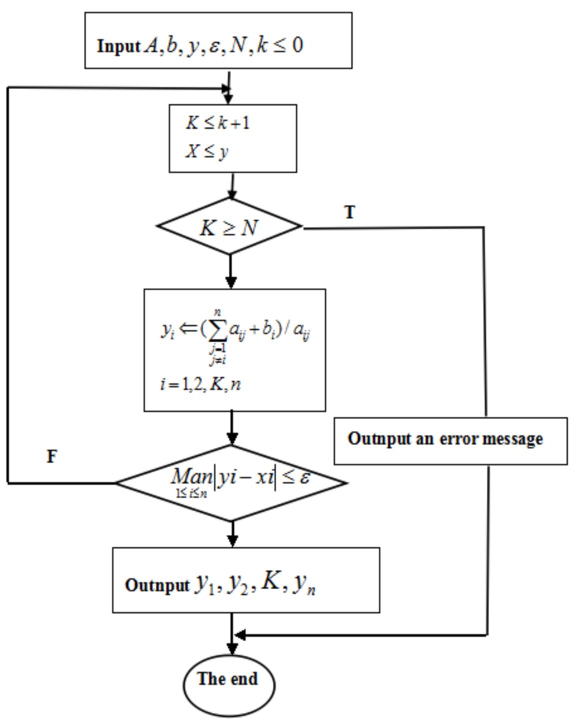

Figure 1 : Jacobi iteration process

FUZZY COMPREHENSIVE EVALUATION METHOD

In general, the amount of fuzzy comprehensive evaluation involves three. The existence of factors related to the object being evaluated to n set up, denoted by $U=\{u_1,u_2,\cdots,u_n\}$, called factors set. They set up m all the possible existence of a comment, denoted by $V=\{v_1,v_2,\cdots,v_m\}$, called judge set. Because each factor's status is not the same, and its role is not the same, so there are metrics that weight, denoted by $A=\{a_1,a_2,\cdots,a_n\}$.

Comprehensive Evaluation

Fuzzy comprehensive evaluation conducted as follows

(1) set of factors set $U=\{u_1,u_2,\cdots,u_n\}$

(2) Setting evaluation set $V=\{v_1,v_2,\cdots,v_m\}$

(3) Single factor judge was $r_i=\{v_{i1},v_{i2},\cdots,v_{im}\}$

(4) Construct comprehensive evaluation matrix:

$$R=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \tag{11}$$

(5) Evaluation: For weights $A=\{a_1,a_2,\cdots,a_n\}$, computing $B=A\circ R$, and need to be evaluated in accordance with the principle of maximum degree of membership.

Definition Operators

When comprehensive evaluation, according to the operator $\circ$ to define different models different there.

(1) Model Ⅰ: $M(\wedge,\vee)$−main factor determining the type

$$b_j=\max\{(a_i\wedge r_{ij}), i=1,2,\cdots,n\}(j=1,2,\cdots,m) \tag{12}$$

The model evaluation decide on the factors playing a major role in the overall evaluation. other factors will not affect the judgment. Relatively speaking, this model is suitable for comprehensive evaluation of the optimal single judgment optimal situation.

(2) model Ⅱ: $M(\bullet,\vee)$−the main factor prominent type

$$b_j=\max\{(a_i\bullet r_{ij}), i=1,2,\cdots,n\}(j=1,2,\cdots,m) \tag{13}$$

The model and Ⅰ model is similar, but it is more refined than Ⅰ model. Not only does it highlight the main factors, but also take into account other factors. This model is applicable to the range of model Ⅰ not applicable in a variety of factors that is not open to different circumstances, but when the need for refinement.

(3) model Ⅲ: $M(\bullet,+)$— Weighted average type

$$b_j=\sum_{i=1}^{n} a_i\bullet r_{ij}(j=1,2,\cdots,m) \tag{14}$$

This model is in accordance with the importance of the various factors influencing factors for all full consideration. Relatively speaking, it applyes situation that require comprehensive optimal.

(4) Model Ⅳ: $M(\wedge,\oplus)-$ take a small upper bound and type

$$b_j = \min\left\{\left(1, \sum_{i=1}^{n}\left(a_i \wedge r_{ij}\right)\right)\right\}(j = 1,2,\cdots,m) \tag{15}$$

The model in use, special attention is: Each can not get too large, otherwise all of the circumstances may occur; each can not get too small, there would be equal to the sum of each case, which will lead to a single evaluation factors relevant information is lost.

(5) Model Ⅴ : $M(\wedge,+)-$Balanced Average Type

$$b_j = \sum_{i=1}^{n}\left(a_i \wedge \frac{r_{ij}}{r_0}\right)(j = 1,2,\cdots,m) \tag{16}$$

$$r_0 = \sum_{k=1}^{n} r_{kj}$$

The model is suitable for the situation that comprehensive evaluation matrix element is too large or too small. Model established in this paper using the main factor determining the type of operator.

Reasonable way of expression

For representation method iterative method, one representation method of linear equations, the second is representation of matrix equations method. In the teaching process, exactly what method is more suitable for students to understand. This issue for the expansion work of teachers is essential. From the student's understanding of the application of the convenience and ease of teachers in the teaching of these three considerations. The paper does comprehensive evaluation for these two.

After do a lot of investigation for the relevant teachers and students, we can find finishing as shown in TABLE 1.

TABLE 1 : Survey results

	Equations	Matrix
Students' understanding degree	0.4	0.6
Ease of application	0.8	0.2
Difficulty of teachers taught	0.3	0.7

jAccording to the actual situation,for ease of student's understanding degree,application convenience and teachers in the teaching of these three weights were assigned $A = (0.2,0.6,0.2)$.

Through software programming, the results can be calculated $B = (0.32,0.24)$. It can be seen from a comprehensive perspective of the work carried out to consider teaching, and presentation method of linear equations iterative method is the most appropriate.

EXAMPLES

Example 1: The ratio using the Jacques iterative method is calculated $\begin{cases} x_1 + x_2 + x_3 = 3 \\ x_1 + x_2 = 2 \\ 2x_1 + 3x_3 = 5 \end{cases}$

Solution 1: Using the representation method for solving linear equations as follows,

$$\begin{cases} x_1 = (-x_2 - x_3 + 3) \\ x_2 = (-x_1 + 2) \\ x_3 = \dfrac{1}{3}(-2x_1 + 5) \end{cases} \tag{17}$$

$$\begin{cases} x_1^{(k+1)} = (-x_2^{(k)} - x_3^{(k)} + 3) \\ x_2^{(k+1)} = (-x_1^{(k)} + 2) \\ x_3^{(k+1)} = \dfrac{1}{3}(-2x_1^{(k)} + 5) \end{cases} \tag{18}$$

The initial value $(0,0,0)^T$ given to the results shown in TABLE 2.

TABLE 2 : Iteration Results Table

Iterations	x_1	x_2	x_3
1	0	0	0
2	3	2	1.67
3	-0.667	-1	-0.333
4	4.33	2.67	2.11
5	3.42	2.35	1.89
6	2.89	2.12	1.67
7	2.32	1.54	1.35
8	1.93	1.43	1.25
9	1.45	1.28	1.14
10	1.23	1.12	1.08
11	1.03	1.06	0.96

Solution 2:

$$A = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 3 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{19}$$

$$= D - L - U$$

$$J = D^{-1}(L + U) \tag{20}$$

$$= \begin{pmatrix} 1 & & \\ & 1 & \\ & & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & 0 \\ -\frac{2}{3} & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & 0 \\ -\frac{2}{3} & 0 & 0 \end{pmatrix}$$

CONCLUSION

Eigenvalues of matrix is an important science and engineering theory. Eigenvalues conventional methods are not suitable for solving practical problems, resulting in a Jacobi iteration method. QR iteration method is suitable for the calculation methods to solve practical problems. However, in the actual teaching process, iterative method is not conducive to student understanding and learning.

In this paper, fuzzy mathematics combine to linear algebra. By fuzzy comprehensive evaluation method, from the student's understanding degree, application of the convenience and ease of teachers in the teaching of these three considerations, which includes Jacobi iterative method of matrix representation method. Linear equations representation method is suitable for teaching work commenced representation method for teaching related work. It provides a theoretical basis.

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REFERENCES

[1] Hongmei Bai; Comparative Analysis of Jacobi and Gauss-Seidel Iterative Method Convergence of Iterative Methods [J], Hulunbeier College, **6**, 55-58 **(2009)**.

[2] Hengji Du, Kunliang Xu; Jacobi and Gauss-Seidel Iteration Method for Solving Linear Equations Analysis and Application[J], Qujing Teachers College, **3**, 46-49 **(2011)**.

[3] Changhe Liu; Compare Jacobi Iterative Method and Projection Method of Solving Linear Equations [J], Beijing Institute of Architectural Engineering, **4**, 64-66 **(2003)**.

[4] Xiaoqian Wu; Jacobi Matrix Inverse Eigenvalue Problems and other Inverse Problem [D], Shanghai University PhD thesis, **(2006)**.

[5] Wenshi Liao; Estimates and their Exhibition-Shaped Boundary Eigenvalues[D], Chongqing University master's degree thesis, **(2013)**.

[6] Lijun Liu; Symmetric Matrix Eigenvalues Neural Networks [D], Dalian University of Technology PhD thesis, **(2006)**.

[7] Jinwu Zhuo; *Matlab* applications in Mathematical Modeling [M], Beijing: Beijing University of Aeronautics and Astronautics Press, **(2010)**.

[8] Yongzheng Zhou; Mathematical Modeling [M], Shanghai: Tongji University Press, **(2010)**.

[9] Xinghuo Wan; Probability Theory and Mathematical Statistics [M], Beijing: Science Press, **(2007)**.

[10] Xiaoyin Wang; Mathematical Modeling and Mathematical Experiment [M], Beijing: Science Press, **(2010)**.