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Influence of the Magnetic Field in the Solar Interior on the Differential Rotation

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Abstract

The influence of the magnetic force in the solar interior on the differential rotation is studied. The formula for the influence of the magnetic field in the solar interior on the differential rotation at depth H below surface and co-latitude is derived. As a calculating example, the influence of the magnetic field on the differential rotation at given depth H=0.3 R below solar surface and latitude φ = 0°, 15°, 30°, 45°, 60°, 75° and 90° is calculated. The theoretical and calculated results are discussed.

Keywords: ***Solar interior magnetic field; Differential rotation; Influence***

Introduction

The study for the sun's differential rotation had been passed through over hundred years, but some studies only restrict to the surface differential rotation, a few studies for the interior differential rotation. The sun's interior differential rotation is connected with the radius velocity of the meridional circulation. At first, Kippenhahn calculated the sun's differential rotation and circulational flow by using anisotropic viscosity [1,2]. Durney developed Kippenhahn's theory in study the relation between the differential rotation and meridional circulation [3,4]. He regards that the sun's differential rotation is maintained by a large scale meridional circulation flow in the convective zone, and he point out that the observational differential rotation is resulted from the base of convective zone in where the influence of mutual action between rotation and convective zone derives the meridional circulation on the whole convective zone, and the differential rotation is arisen from the meridional circulation. Under this assumption, he deduces the sun's differential rotational velocity as the function of the depth and co-latitude. But the various disturbing force in the outside or inside of the sun, such as the tidal, magnetic and Coriolis forces affect the radius velocity of the meridional circulation, and therefore, it affects indirectly the sun's differential rotation.

Recently, Tassoul studied the meridional circulation, magnetic field's affect and rotation (spin-down) of the sun [5,6]. He obtains the significant scientific and theoretical results. The present paper studies the influence of the magnetic force on the differential rotation of the solar interior.

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The formula of the solar interior differential rotation as expressed the meridional velocity in solar interior. Durney had ever deduced the formula for the differential rotational velocity in solar interior as the function of depth and co-latitude starting from hydrodynamics of viscous fluids [3]:

$$\Omega(r,.\theta) = \Omega_o\left[1 + \omega_0(r) + \omega_2(r)P_2(\cos\theta)\right] \tag{1}$$

Where

$$\left.\begin{aligned} \omega_o(r) &= \frac{2}{3\nu}\int_r^{R_\Theta}\left(\frac{\psi}{r^2\rho}\right)dr \\ \omega_2(r) &= -0.189 + \frac{4}{3\nu}\int_r^{R_\Theta}\left(\frac{\psi}{r^2\rho}\right)dr \end{aligned}\right\} \tag{2}$$

$P_2(\cos\theta)$ is the Legendre polynomials

The components of the velocity of the meridional circulation on the meridional circle in solar interior are

$$\left.\begin{aligned} V_r &= \frac{2\psi}{r^2\rho}P_2(\cos\theta), \\ V_\theta &= -\frac{\psi'}{r\rho}\sin\theta\cos\theta. \end{aligned}\right\} \tag{3}$$

Where ψ denotes steam-function, θ is co-latitude in spherical coordinates and ν is viscosity, $\psi' = \frac{d\psi}{dr}$.

Substituting $V_r / 2P_2(\cos\theta) = \psi / r^2\rho$ of (3) into Eqs. (2), then

$$\omega_1(r) = \frac{1}{3\nu P_2(\cos\theta)}\int_0^{2\pi} V_r dr$$

$$\omega_2(r) = -0.189 + \frac{2}{3\nu P_2(\cos\theta)}\int_r^{R_r} V_r dr \tag{4}$$

Substituting (4) into (1), we obtain

$$\Omega(r.\theta) = \Omega_0\left\{1 - 0.189P_2(\cos\theta) + \frac{2}{3\nu}\left[\frac{1}{2P_2(\cos\theta)} + 1\right]\int_r^{R_0} V_r dr\right\} \tag{5}$$

We study the influence of the velocity of meridional circulation on the solar interior differential rotation by using the formula (5).

The Influence of The Magnetic Force of Solar Interior on The Differential Rotation

The influence of the magnetic force of solar interior on the differential rotation is realized through the interior meridional velocity. Kippenhahn had ever connected the magnetic force of this action with meridional velocity V_r, and he give the following formula [1]:

$$V_r = \frac{L}{4\pi GM\rho R}\frac{\nabla ad}{\nabla - \nabla ad}\chi_{mag}, \tag{6}$$

$$\text{Where } \nabla_{ad} = (\frac{d\ln T}{d\ln P}),\ \nabla = 1 - \frac{d\ln\rho}{\ln P}.$$

L. M. ρ and R denote the luminosity, mass, density, and radius of the sun respectively, χ_{mag} denotes the order of the ratio of the magnetic force to the gravitational force, and it relates with magnetic field B as follows [1,2]:

$$\chi_{mag} = \frac{B^2 r}{4\pi GM(r)\rho(r)}. \qquad (7)$$

M (r) and ρ (r) denote the mass and mean density inside the sphere of radius r.

In the expression (6), Kippenhahn puts $\frac{\nabla ad}{3(\nabla - \nabla ad)} \approx 1,$ and we let $\rho = M / \frac{4}{3}\pi R^3$, then, (6) can be written as [1]

$$\therefore V_r = \frac{LR^2}{GM^2}\chi_{mag} \qquad (8)$$

Substituting (8) into (5) and consider (7), we obtain

$$\Omega = \Omega_0 \left\{ 1 - 0.189 P_2(\cos\theta) + \frac{L_\Theta R^2}{12\pi\nu G^2 M^2} \left[\frac{1}{P_2(\cos\theta)} + 2 \right] \int_r^{R_\Theta} \frac{\vec{B}^2 r}{M(r)\rho(r)} dr, \right\}$$

$$\text{using } \rho(r) = M(r) / \frac{4}{3}\pi r^3, \text{ then}$$

$$\Omega = \Omega_0 \left\{ 1 - 0.189 P_2(\cos\theta) + \frac{L_\Theta R^2}{9\nu G^2 M^2} \left[\frac{1}{P_2(\cos\theta)} + 2 \right] \int_r^{R_\Theta} \frac{\vec{B}^2 r^4}{M(r)^2} dr. \right\} \qquad (9)$$

The results of integration of formula (9)

M(r) and B(r) in formula (9) are the function of r, the components of the magnetic field $\vec{B}$ (r) at any point in solar interior are given by Chen-Biao [7]**:**

$$B_r = R_r(r)\cos\theta,\ B_\theta = R_\theta(r)\sin\theta,\ B_\varphi = R_\varphi(r)\sin\theta. \quad (10)$$

The solution is a circulational solution in the meridional plane, and according to the calculation by chen-Biao:

$$R_r(r) = \left[a + br^2 + cr^{-3} \right] B_0$$

$$R_\theta(r) = \left[a' + b' r^2 + c' r^{-3} \right] B_0 \qquad (11)$$

$$R_\varphi(r) = 0$$

Where B_0 is the magnetic field of the sun's surface, and

a=1.4496, b=-1.2698, c=-0.1799,

a'=-1.4496, b'=2.5396, c'=-0.08995. (12)

Hence

$$B^2 = B_r^2 + B_\theta^2 + B_\varphi^2,$$

$$= (a + br^2 + cr^{-3})^2 B_0^2 \cos^2\theta + (a' + b' r^2 + c' r^{-3})^2 B_0^2 \sin^2\theta,$$

$$= (a^2 + a'^2)B_0^2 + (ab\cos^2\theta + a'b'\sin^2\theta)r^2 B_0^2 +$$

$$+ (b^2\cos^2\theta + b'^2\sin^2\theta)r^4 B_0^2 + 2(cb\cos^2\theta + b'c'\sin^2\theta)\frac{1}{r}B_0^2$$

$$+ 2(ac\cos^2\theta + a'c'\sin^2\theta)\frac{1}{r^3}B_0^2 + (c^2\cos^2\theta + c'\sin^2\theta)\frac{B_0^2}{r^6} \quad (13)$$

According to the theory of stellar interior constitution (Chandrasekhar [8]:

$$M(r) = 4\pi\int_O^r r^2\rho(r)dr,$$

$$= -4\pi\left[\frac{(n+1)K}{4\pi G}\right]^{3/2}\rho_c^{\frac{3-n}{2-n}}(\xi^2\frac{d\psi}{d\xi}). \quad (14)$$

where $K = P_c/\rho_c$, P_c and ρ_c are the central pressure and density respectively

For the sun, we put $n = \frac{3}{2} = 1.5$, then, (14) reduce to

$$\therefore M(r) = M(\xi) = -4\pi\Delta^{3/2}\rho_c^{\frac{1}{2}}(\xi^2\frac{d\psi}{d\xi})$$

$$M(r)^2 = 16\pi^2\Delta^3\rho_c\xi^4(\frac{d\psi}{d\xi})^2. \quad (15)$$

Where $\Delta = 5K/8\pi G$,

and $r = \alpha\xi = \Delta^{\frac{1}{2}}\rho_c^{-\frac{1}{6}}\xi$, $r^4 = \Delta^2\rho_c^{-\frac{2}{3}}\xi^4$,

$$r^{-2} = \Delta^{-1}\rho_c^{\frac{1}{3}}\xi^{-2},\ r^6 = \Delta^3\rho_c^{-1}\xi^6, \quad (16)$$

$$r^3 = \Delta^{3/2}\rho_c^{-\frac{1}{2}}\xi^3,\ r^8 = \Delta^4\rho_c^{-\frac{4}{3}}\xi^8,$$

Substituting $B^2(r)$ and M(r) of (13) and (15) into (9), and use $dr = \alpha d\xi$, $\alpha = \Delta^{\frac{1}{2}}/\rho_c^{\frac{1}{6}}$, and then, integrating:

$$\int_r^{R_\Theta}\frac{B^2r^4}{M(r)^2}dr = (a^2 + a'^2)\int_r^{R_\Theta}\frac{B_0^2r^4}{M(r)^2}dr + 2(ab\cos^2\theta + a'b'\sin^2\theta)B_0^2\int_r^{R_\Theta}\frac{r^6}{M(r)^2}dr$$

$$+ (b^2\cos^2\theta + b'^2\sin^2\theta)B_0^2\int_r^{R_\Theta}\frac{r^8}{M(r)^2}dr + 2(b\cos^2\theta + b'c'\sin^2\theta)B_0^2\int_r^{R_\Theta}\frac{r^3}{M(r)^2}dr$$

$$+ 2(ac\cos^2\theta + a'c'\sin^2\theta)B_0^2\int_r^{R_\Theta}\frac{r}{M(r)^2}dr + (c^2\cos^2\theta + c'^2\sin^2\theta)B_0^2\int_r^{R_\Theta}\frac{1}{M(r)^2r^2}dr$$

$$\therefore \int_r^{R_\Theta}\frac{B^2r^4}{M(r)^2}dr = \frac{B_0(a^2 + a'^2)}{16\pi^2\Delta\rho_c^{5/3}}\left[\frac{1}{\frac{d\psi}{d\xi}}\right]_{\xi=\frac{r'}{\alpha}}\int_{\xi=\frac{r'}{\alpha}}^{R\Theta/\alpha}\alpha d\xi$$

$$+\frac{B_0^2}{8\pi^2\rho_c^2}(ab\cos^2\theta+a'b'\sin^2\theta)\left[\ (\xi^2/(\frac{d\psi}{d\xi})^2)\ \right]_{\xi=\frac{r'}{\alpha}}\int_{\xi=\frac{r'}{\alpha}}^{R_\Theta/\alpha}\alpha d\xi$$

$$+\frac{B_0^2\Delta}{16\pi^2\rho_c^{7/3}}(b^2\cos^2\theta+b'^2\sin^2\theta)\left[\ (\xi^2/(\frac{d\psi}{d\xi})^2)\ \right]_{\xi=\frac{r'}{\alpha}}\int_{\xi=\frac{r'}{\alpha}}^{R_\Theta/x}\alpha d\xi$$

$$+\frac{B_0^2}{8\pi^2\Delta^{3/2}\rho_c^{3/2}}(bc\cos^2\theta+b'c'\sin^2\theta)\left[\ (1/\xi(\frac{d\psi}{d\xi})^2)\ \right]_{\xi=\frac{r'}{\alpha}}\int_{\xi=\frac{r'}{\alpha}}^{R_\Theta/x}\alpha d\xi$$

$$+\frac{B_0^2}{8\pi^2\Delta^{5/2}\rho_c^{7/6}}(ac\cos^2\theta+a'c'\sin^2\theta)\left[\ (1/\xi^3(\frac{d\psi}{d\xi})^2)\ \right]_{\xi=\frac{r'}{\alpha}}\int_{\xi=\frac{r'}{\alpha}}^{R_\Theta/x}\alpha d\xi$$

$$+\frac{B_0^2}{16\pi^2\Delta^4\rho_c^{2/3}}(c^2\cos^2\theta+c'^2\sin^2\theta)\left[\ (1/\xi^6(\frac{d\psi}{d\xi})^2)\ \right]_{\xi=\frac{r'}{\alpha}}\int_{\xi=\frac{r'}{\alpha}}^{R_\Theta/\alpha}\alpha d\xi \tag{17}$$

$$\because \int_{\xi=r'/\alpha}^{R_\Theta/\alpha}\alpha d\xi=\alpha(\frac{R_\Theta}{\alpha}-\frac{r'}{\alpha})=R_\Theta-r'=H \tag{18}$$

Where H is the depth bellow the solar surface.

The above expression (17) reduces to

$$\int_r^{R_\Theta}\frac{\vec{B}^2r^4}{M(r)^2}=\{(a'^2+a^2)Ak\ +(abBL+b^2CM+bcDN+acEP+C^2FQ)\cos^2\theta$$

$$+(a'b'BL+b'^2CM+b'c'DN+a'c'EP+c'^2FQ)\sin^2\theta\}B_0^2H$$

Substituting a, b, c and a’, b’, c’ of (12) into the above expression, it can be written as

$$\int_r^{R_\theta}\frac{\vec{B}^2r^4}{M(r)^2}dr=[4.2026A\kappa$$

-(1.8144 BL -1.5876 CM +0.2255 DN +0.2610EP-0.0324FQ)cos$^2\Theta$

-(3.6830BL-6.4516CM+0.2282DN-0.1303EP-0.0081FQ) $\sin^2\theta]\times B_0^2H$ (19)

Where

$$A=1/16\pi^2\Delta\rho_c^{5/3},\ B=1/8\pi^2\rho_c^2,\ C=\Delta/16\pi^2\rho_c^{7/3},\ D=1/8\pi^2\Delta^{3/2}\rho_c^{3/2},$$

$$E=1/8\pi^2\Delta^{2/5}\rho^{7/6}\ F=1/16\pi^2\Delta^4\rho_c^{2/3},\ K=1/(\frac{d\psi}{d\xi})^2,\ L=\xi^2/(\frac{d\psi}{d\xi})^2,\ M=\xi^4/(\frac{d\psi}{d\xi})^2,\ N=1/\xi(\frac{d\psi}{d\xi})^2,$$

$$P=1/\xi^3(\frac{d\psi}{d\xi})^2,\ Q=1/\xi^6(\frac{d\psi}{d\xi})^2,\ H=R_\Theta-r'. \tag{20}$$

Where R_Θ denotes solar radius, r’ denotes the radius from solar center.

Substituting (19) into (9), we obtain the formula for the influence of the magnetic force in solar interior on the differential rotation at the depth H below the sun’s surface and co-latitude $\theta=\frac{1}{2}\pi-\varphi$,

$$\Omega = \Omega_0 \left\{ 1 - 0.189 P_2(\cos\theta) + \frac{L_\theta R^2}{27\nu G^2 M^2} \left[\left(\frac{1}{P_2(\cos\theta)} \right) + 2 \right] (\kappa + \lambda \cos^2\theta + \mu \sin^2\theta) B_0^2 H \right\} \quad (21)$$

Where $\kappa = 4.2026AK$,

$$\lambda = -(1.8144BL - 1.5876CM + 0.225DN + 0.2610P - 0.0324FQ),$$

$$\mu = -(3.6830BL - 6.4516CM + 0.2282DN - 0.1303EP - 0.008FQ). \quad (22)$$

The Numerical Results

In the formula (21) the third term is the perturbation effect of the magnetic force in solar interior on the differential rotation. We denote as $\Delta\Omega$, that is

$$\Delta\Omega = \Omega_0 Q F(\theta) F(\theta, \lambda, \mu) B_0 H. \quad (23)$$

Here

$$Q = \frac{L_\Theta R^2}{27\nu G^2 M^2},\ F(\theta) = \frac{1}{P_2(\cos\theta)} + 2,\ F(\theta, \lambda, \mu) = (\kappa + \lambda\cos^2\theta + \mu\sin^2\theta) \quad (24)$$

As example, we use the obtained result (23) to calculate the differential rotation of solar interior at latitude φ = 0, 15°, 30°, 45°, 60°, 75° and 90° and depth H below solar surface.

For the sun, we use n=1.5, γ =5/3, the values of its M , R, L, ρ_c, P_c and ν are given by Allen [9]. So, we obtain:

$$r' = R - H = 5.1 \times 10^{10} cm,$$

$$K_{1.5} = 2.12 \times 10^{15}, \quad \Delta = 63 \times 10^{16}, \quad \alpha = 3.4 \times 10^{10}, \quad \xi = 1.5, \quad \psi = 0.68132, \quad \frac{d\psi}{d\xi} = -0.35752,$$

$$\xi^2 \frac{d\psi}{d\xi} = -0.8040$$

Substituting $\Delta = 63 \times 10^{20}$, $\rho_c = 160 g/cm^3$ and $\xi = 1.5$, $\frac{d\psi}{d\xi} = -0.35752$ into (20) we obtain:

$$\mu = 6.4516CM - 3.6830BL.$$

A=0, E=0, M=39.71, B=4.8×10^{-7}, F=0, N=5.22, C=2.9 × 10^8, K=7.84, P=2.32, D=0, L=17.65, Q=0.69.

Substituting the above values into (22), we obtain:

K=4.2026AK=0,

λ=4.2026AK=0,

μ=1.5876CM-1.8144BL, (25)

The value of $B=4.8\times10^{-7}$ may be ignored as compare with the value of the order of $C=2.9\times10^{8}$ so, $B\approx0$, the above expressions can be written as

$$\kappa=0,\ \lambda=1.5876CM=1828\times10^{7},\ \mu=6.4516CM=7431\times10^{3}. \qquad (26)$$

For the solar model $M=1.989\times10^{33}g$, $R=6.959\times10^{10}cm$, $L=3.826\times10^{33}erg/s$, $\Omega_0=2.865\times10^{-6}rad/s$ [9] $\nu=2\times10^{12}cm/s$ [10] and the general magnetic field of solar surface $B_0=25\sim50\ G$. [11]. We take the average value 37 G. and calculate depth H=0.3R below solar surface. Substitution of the above data into the formulae (23)-(24), we obtain the numerical results for $\Delta\Omega$ as shown in TABLE 1.

TABLE 1. **Numerical results for the differential rotation of the depth 0.3 H below solar surface.**

φ (deg)	θ (deg)	$F(\theta)$	Q$\times10^{-12}$	$F(\theta,\lambda,\mu)\times10^{7}$	$\Delta\Omega$ $(10^{-6}rad/s)$
90	0	+3	19.5. ×	1.83	+1.07
75	15	+3.11	19.5	2.20	+1.34
60	30	+3.60	19.5	3.23	+2.26
45	45	+6	19.5	9.26	+10.83
30	60	-6	19.5	-6.03	-7.05
15	75	-5.03	19.5	-7.05	-0.69
0	90	0	19.5	0	0
φ is the latitude, θ is the co-latitude. $\Omega_0=2.865\times10^{-6}rad/s$, $B_0=37G$					

Discussion and Conclusion

We obtain the following conclusion according to theoretical and calculated results and numerical results in TABLE.

In the TABLE $\Delta\Omega=0$ as H = 0. There is not perturbation effect. This is the case of the differential rotation of solar surface, that is, the differential rotational velocity decreases with the latitude φ. The formula (21) reduces to

$$\Omega=\Omega_0\left[(1-0.189P_2(\cos\theta)\right]$$

The effect of the depth on the differential rotation

In the sun's interior, the deeper the depth, the larger the effect on the differential rotation as H≠0, In the sun's central $r = 0$, the differential rotational velocity arrives at maximum. In addition, the stronger the magnetic field B_0, the larger the effect on the differential rotation also.

The effect of various latitudes on the defferential rotation

As $\varphi = 0$ (the case of the equator), $\Delta\Omega = 0$, this show that there is not perturbation effect or no effect on the differential rotation. The effect exhibits a retardation (negative value) at $0 \prec \varphi \leq 30°$. The effect increases successively to maximum at $\varphi = 45°$ Their effect decreases successively to minimum at $45° \leq \varphi \leq 90°$.

Comparison of effects of the internal angular velocity with surface angular velocity

The solar surface angular velocity: $\Omega_0 = 2.865 \times 10^{-6} rad/s$, as in TABLE the internal angular velocity is small than that of surface values at 15° and rotation of contrary direction. The internal rotation is large than that of surface rotation at 30° and the rotation of the contrary direction. The value of internal rotation is large than that of the surface rotation at 45°. Their values of the internal rotation are small than that of the surface rotation at 60°~ 90°.

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