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Improvement of the grey target decision of incomplete category preference based on incomplete information

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ABSTRACT

The MADM method based on incomplete decision alternatives information and incomplete decision alternative categories for preference is researched. By means of case-based reasoning, fuzzy processing is done for incomplete decision-making information; an attribute weight determination model based on incomplete information and incomplete program categories for preference is established through the relative overall off-target distance between programs of the same category and the average overall off-target distance between programs of different categories represented by the grey target decision-making theory. The example shows the method is effective.

KEYWORDS

Incomplete information; Incomplete categories; Grey target decision-making; Decision-making method.

INTRODUCTION

The Multiple Attribute Decision Making (MADM) problem is a process sorting limited decision alternatives and selecting the most satisfying one after comparing their attributes comprehensively[1]. It is widely applied in engineering design, military, economy, finance, etc. In actual decision-making, decision makers always give incomplete information about decision-making parameters because of the following factors: Firstly, sometimes they are not willing to give accurate information about their preferences for decision alternatives based on specific attributes (for example, technical experts evaluating product performance cannot evaluate the technical performance with which they are not familiar with); secondly, people are under a great pressure in emergencies and are therefore difficult to evaluate relevant indexes definitely in limited time and in special circumstances (for example, in earthquake disaster forecasting); thirdly, evaluation data cannot be sorted and summed up in the specified time (for example, in statistics for local annual reports, some towns and villages cannot report data in the specified time because of lagged data statistics). Fishburn[2], an expert researching decision-making based on incomplete information earliest, has established planning models based on information such as attribute value incompletion[3,6], attribute weight incompletion[7,9], and attribute value and attribute weight incompletion to rank programs in most of his researches.

In MADM, particularly when there are many decision alternatives, experts always tend to show categories for preference for some decision alternatives. This is the so-called classified decision-making. With fast social development and popularization of information of social behaviors, it is more flexible and easier for experts to classify decision alternatives in their familiar fields or classify their familiar decision alternatives into different categories. In the researches about decision-making, there have been some achievements basically including the following three types: (1) case-focusing classification methods (documents[14] to[16] introduce a case-based multi-index sorting method, a multi-attribute classification method and a language information grey target decision-making classification method, respectively); (2) classification methods focusing on decision-making alternative preference (documents[13,17,18] introduce a multi-attribute classification method based on value levels of decision alternatives, excellence ranking of decision alternatives, and ranking of decision alternatives, respectively); (3) case-focusing statistical data classification methods (documents[19,20] realize decision alternative search and data grading and classification based on statistical data). It is clear that decision alternative classification in multi-attribute decision-making is attracting increasingly more attention in the academic community.

Common circumstance in actual decision-making: Experts cannot give complete information for multiple decision alternatives in limited time in emergencies and are sensitive to some of the decision alternatives, thinking some of them are of the same category. This is actually case study and analysis based on incomplete information and incomplete decision alternative categories for preference. According to publicly reported documents, there are very few researches about this. Thus, this paper presets some new decision-making problems: Firstly, decision makers have incomplete categories for preference for some decision alternatives; secondly, the attribute values of decision alternatives have incomplete information. In this paper, the positive target center and the negative target center are determined according to application and embodiment of the “non-uniqueness” principle of grey target decision-making in the grey system theory and to the method of grey target decision-making, a categories for preference structure model based on minimum relative overall off-target distance of all the decision alternatives of the same category and maximum average overall off-target distance of the decision alternatives of different categories is built and attribute weights of the model are calculated, thus resulting in a complete rank for the decision alternatives and realizing preliminary and basic category judgment for some unclassified decision alternatives. For better understanding and description of referential feature of decision-making methods as well, the research in this paper is based on real-number type cases, so it is applicable to other categories of decision alternatives.

PRE-KNOWLEDGE

Assume a multi-index decision-making issue has n evaluated objects or determined decision alternatives, $W=\{\omega_j \mid j=1,2,...,m\}$, a index set $W=\{\omega_j \mid j=1,2,...,m\}$ made up m of evaluation indexes or attributes and a real number subscript set and an interval number subscript set set as M_1 and M_2 respectively, then value of the decision matrix $X=(x_{ij})_{n\times m}$ will be:

$$X=\begin{cases} x_{ij}\,(j\in M_1, i\in N) \\ [x_{ij}^L, x_{ij}^U](j\in M_2, i\in N) \end{cases}$$

For such a decision matrix made up of areal numbers and interval numbers, its standardized processing and relevant concepts are as follows:

Standardized processing and relevant concepts of a real number type decision matrix

Definition 1[21]: In the standardized processing of real number type decision-making data, profit type decision-making data: $r_{ij}=x_{ij}/\sqrt{\sum_{i=1}^{n} x_{ij}^2}(\,i\in n, j\in m\,)$; cost type decision-making data: $r_{ij}=(\,1/x_{ij})/\sqrt{\sum_{i=1}^{n}(1/x_{ij})^2}\,(\,i\in n, j\in m\,)$; standardized decision matrix: $R=\{r_i \mid i=1,2,...,n\}$.

Definition 2[21]: Assume r_1 and r_2 are two random real numbers, then their hamming distance will be:

$$d(\,r_1,r_2\,)=|r_1-r_2| \tag{1}$$

Definition 3[22]: Assume $r_j^+=\max\{r_{ij} \mid 1\le i\le n, 1\le j\le m\}$, positive target center of the real number type decision matrix is : $R^+=\{r_1^+,r_2^+,...,r_m^+\}$; $r_j^-=\min\{r_{ij} \mid 1\le i\le n, 1\le j\le m\}$, negative target center of the real number type decision matrix is: $R^-=\{r_1^-,r_2^-,...,r_m^-\}$.

Standardized processing and relevant concepts of an interval number type decision matrix

Definition 4 [21]: In the standardized processing of interval number type decision-making data, profit type decision-making data: $r_{ij}^L=x_{ij}^L/\sqrt{\sum_{i=1}^{n} x_{ij}^U}$, $r_{ij}^U=x_{ij}^U/\sqrt{\sum_{i=1}^{n} x_{ij}^L}(\,i\in n, j\in m\,)$; cost type decision-making data: $r_{ij}^L=(\,1/x_{ij}^U)/\sqrt{\sum_{i=1}^{n}(1/x_{ij}^L)}$, $r_{ij}^U=(\,1/x_{ij}^L)/\sqrt{\sum_{i=1}^{n}(1/x_{ij}^U)}\,(\,i\in n, j\in m)$; standardized decision matrix:

$$R=\begin{bmatrix} [r_{11}^L,r_{11}^U] & [r_{12}^L,r_{12}^U] & \dots & [r_{1m}^L,r_{1m}^U] \\ [r_{21}^L,r_{21}^U] & [r_{22}^L,r_{22}^U] & & [r_{2m}^L,r_{2m}^U] \\ & & & \\ [r_{n1}^L,r_{n1}^U] & [r_{n2}^L,r_{n2}^U] & & [r_{nm}^L,r_{nm}^U] \end{bmatrix}$$

Obviously, $r_{ij}^L, r_{ij}^U \in[0,1]$ can be reached after standardization.

Assume $\tilde{a}=[a^L,a^U]=\{x \mid a^L\le x\le a^U\}$ ($\tilde{a}$ is an interval number), particularly, when a^L equals to a^U, a will change into a real number.

Definition 5[21]: Assume $\tilde{a}=[a_1,a_2]$ ($a_1\le a_2$) and $\tilde{b}=[b_1,b_2]$ ($b_1\le b_2$) are two interval numbers, then their hamming distance will be:

$$d(\tilde{a},\tilde{b})=\frac{1}{2}[|a_1-b_1|+|a_2-b_2|]\ d(\,r_1,r_2\,)=|r_1-r_2| \tag{2}$$

Obviously, the hamming distance is a real number.

Definition 6: Assume a ($\tilde{a}=[a_1,a_2]$, $a_1 \le a_2$) is an interval number and b is a real number, then their distance will be:

$$d(\tilde{a},b)=\frac{1}{2}[|a_1-b|+|a_2-b|] \tag{3}$$

Calculation of the overall off-target distance of a decision matrix made up of real numbers and interval numbers

Definition 7[22]: Assume Z is a decision alternative set, then decision alternative i will a positive target distance as below:

$$\varepsilon_i^+=\omega_1 d(r_{i1},r_1^+)+\omega_2 d(r_{i2},r_1^+)+\ldots+\omega_m d(r_{im},r_m^+)=\sum_{j=1}^{m}\omega_j d(r_{ij},r_j^+) \tag{4}$$

Decision alternative z_i will have a negative target distance as below:

$$\varepsilon_i^-=\omega_1 d(r_{i1},r_1^-)+\omega_2 d(r_{i2},r_1^-)+\ldots+\omega_m d(r_{im},r_m^-)=\sum_{j=1}^{m}\omega_j d(r_{ij},r_j^-) \tag{5}$$

Decision alternative z_i will have a off-target distance between the positive target center and negative target center as below:

$$\varepsilon_i^0=\omega_1 d(r_1^+,r_1^-)+\omega_2 d(r_2^+,r_2^-)+\ldots+\omega_m d(r_m^+,r_m^-)=\sum_{j=1}^{m}\omega_j d(r_j^+-r_j^-) \tag{6}$$

Definition 8[22]: If projection of the connecting line between positive clout and negative clout of decision alternative z_i is defined as the overall off-target distance, then:

$$\varepsilon_i^*=\frac{(\varepsilon_i^+)^2+(\varepsilon_i^0)^2-(\varepsilon_i^-)^2}{2\varepsilon_i^0} \tag{7}$$

According to document[22], distance ε_i^0 between positive clout and negative clout of each decision alternative is a constant, so the formula above can be changed into:

$$\begin{aligned}\varepsilon_i&=(\varepsilon_i^+)^2+(\varepsilon_i^0)^2-(\varepsilon_i^-)^2\\&=\sum_{j=1}^{m}\omega_j d(r_{ij},r_j^+)^2+\sum_{j=1}^{m}\omega_j d(r_j^+-r_j^-)^2-\sum_{j=1}^{m}\omega_j d(r_{ij},r_j^-)^2\\&=\sum_{j=1}^{m}\omega_j{}^2\left|d(r_{ij},r_j^+)^2+d(r_j^+-r_j^-)^2-d(r_{ij}-r_j^-)^2\right|\end{aligned} \tag{8}$$

The overall off-target distance shows quality of the effect vector: The smaller the overall off-target distance of z_i, the better the decision alternative will be; the larger the overall off-target distance of z_i, the worse the decision alternative will be.

PROBLEM DESCRIPTION

Problem description and thinking

The common multi-attribute decision-making issue is as below: When an index set $W=\{\omega_j\,|\,j=1,2,\ldots,m\}$ and a decision alternative set $Z=\{z_i\,|\,i=1,2,\ldots,n\}$ are given and index ω_j in decision alternative z_i has an attribute value of r_{ij}, decision alternatives are ranked following a decision-making rule; the decision alternatives in multi-index classified decision-making are classified into specific categories and the decision alternatives of each category have a similar nature. However, in many decision-making cases, there are many decision alternatives for evaluation, making it difficult to completely and strictly classify all of them; it is common that some decision alternatives under some indexes cannot have definite attribute values because of complicated and uncertain data statistics. To solve these problems, the sample cases can be analyzed and studied in line with the case study method and the ideas of data mining and fuzzy mathematics, thus realizing ranking of decision alternatives.

This method has a less strict requirement for the information extraction of decision markers and is more acceptable for decision makers and easier for them to submit relevant information. Meanwhile, even if sample information lacks, ranking of decision alternatives can be realized. The research in this paper has the following characteristics:

(1) Due to uncertainty of statistics and fuzzy thinking of decision makers, a decision-making attribute possibly lacks. For example, in decision alternative z_i, attribute value r_{ij} of index ω_j is possibly null.

(2) Decision makers do not classify decision alternatives strictly. For example, it is believed by some experts that decision alternative set $Z = \{z_i \mid i = 1, 2, ..., n\}$ can be classified into s categories through several decision alternatives and that there are t decision alternatives not belonging to either the s categories or any other categories.

(3) Final ranking depends on index weight determination. In the process, both the case of decision alternations with null attribute values and the case of objective classification of decision makers for all decision alternatives need to be considered, which is the key of problem analysis.

Decision-making method

(a) Fuzzy processing of decision-making data with incomplete information

Case-based reasoning is an intuition thinking way with a basic basis that similar problems have similar solutions[23]. In data statistics, massive information is usually accumulated. After summing, analysis, sorting and combination, these information form sample modes that are used as models. Thus, we can believe that incomplete decision-making data stimulate the brains of decision makers and, after processing, form corresponding models and that matching data can be found and expressed by means of fuzzy comparison with the modules, thus solving the problems in next stage.

In decision matrixes, the contents saved as accurate information are main characteristics of decision alternatives under different indexes and essences of samples. After fuzzy processing based on case-based reasoning, the most similar and matching cases can be searched from the case library according to locations of the incomplete information can be processed into intervals of similar or matching case data. In this paper, interval numbers are used for supplementing incomplete information.

Definition 9: If r_{ij} of some decision alternatives of the same category is null, it may be expressed as "-" and $\tilde{r}_{ij} \in [r_{ij}^L, r_{ij}^U]$ can be reached, $r_{ij}^L = \max\{r_{ij} \mid 1 \le i \le n, j = m\}, r_{ij}^U = \min\{r_{ij} \mid 1 \le i \le n, j = m\}$. For example, if r_{23} is incomplete in decision matrix $\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} \\ r_{21} & r_{22} & - & r_{24} & r_{25} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} \end{bmatrix}$, it may be expressed as below by interval numbers: $\tilde{r}_{23} \in [\max(r_{13}, r_{33}, r_{43}), \min(r_{13}, r_{33}, r_{43})]$, $\tilde{r}_{23} \in [r_{23}^L, r_{23}^U]$.

It is easy to be understood in practical application too. For example, experts evaluated information system investment projects 1 to 5 from income level w_1, anti-risk capability w_2, social effect w_3, market effect w_4 and technical feasibility and believed that information system investment projects 1 to 4 were of the same category. They did not give evaluation data for information system investment project 2, thinking its social benefit was difficult to be evaluated. Because information system investment projects 1 to 4 were of the same category, we can believe that the market effect of information system investment project 2 should be between numeric intervals of the market effect of information system investment projects 1, 3 and 4 and social effect data of information system investment project 2 can be made between maximum and minimum of attribute w_3 of information system investment projects 1, 3 and 4 by means of fuzzy processing of interval numbers.

(b) Correction of positive target center and negative target center of a decision matrix made up of real numbers and interval numbers

In grey target decision-making, reference point selection is the key step. For a standardized decision matrix made up of real numbers and interval numbers, the elements are standardized data; the differences between the cost type indexes and the benefit type indexes have been eliminated; each index is expected to its maximum. Thus, when building the best effects, maximum real numbers and maximum interval numbers corresponding to the indexes in the standardized decision matrix should be selected as the best effects of the indexes. In this paper, incomplete information corresponds to the interval numbers determined in line with attribute values of the decision alternatives of the same category and under the same attributes, in other words, it is made up of the interval numbers formed by the maximums and the minimums the decision alternatives of the same category and under the same attributes. Thus, in this paper, as of attribute values of interval numbers, their corresponding maximum real number and minimum real number under the same attributes are regarded as the positive target center and the negative target center.

Definition 10: In a decision matrix, if r_{ij} is a real number attribute value and $\tilde{r}_{ij}$ is an interval number attribute value, it can be expressed as below: $\tilde{r}_{ij} \in [r_{ij}^L, r_{ij}^U]$, $r_{ij}^L = \max\left\{r_{ij} \left| 1 \le i \le n, j = m\right.\right\}, r_{ij}^U = \min\left\{r_{ij} \left| 1 \le i \le n, j = m\right.\right\}$. For a decision matrix made up of real numbers and interval numbers: $r_j^+ = \max\{r_{ij} \left| 1 \le i \le n, 1 \le j \le m\right.\}$, the positive target center of the decision matrix $R^+ = \{r_1^+, r_2^+, \ldots, r_m^+\}$; $r_j^- = \min\{r_{ij} \left| 1 \le i \le n, 1 \le j \le m\right.\}$, negative target center of the decision matrix $R^- = \{r_1^-, r_2^-, \ldots, r_m^-\}$.

(c) Calculation of relative overall off-target distance of decision alternatives of the same category

Definition 11: For any two decision alternatives, their relative overall off-target distance can be expressed as absolute of difference of their overall off-target distance, i.e. $\beta_{st} = |\varepsilon_s - \varepsilon_t|$. Distance ε_i^0 between positive clout and negative clout of each decision alternative is a constant[22], this formula can be changed into formula (9) according to formula (8).

$$
\begin{aligned}
\beta_{st} &= |\varepsilon_s - \varepsilon_t| = \left|(\varepsilon_s^+)^2 + (\varepsilon_s^0)^2 - (\varepsilon_s^-)^2 - (\varepsilon_t^+)^2 - (\varepsilon_t^0)^2 + (\varepsilon_t^-)^2)\right| \\
&= \left|(\varepsilon_s^+)^2 - (\varepsilon_s^-)^2 - (\varepsilon_t^+)^2 + (\varepsilon_t^-)^2)\right| \\
&= \left|\sum_{j=1}^m \omega_j d(r_{sj}, r_j^+)^2 - \sum_{j=1}^m \omega_j d(r_{sj}, r_j^-)^2 - \sum_{j=1}^m \omega_j d(r_{tj}, r_j^+)^2 + \sum_{j=1}^m \omega_j d(r_{tj}, r_j^+)^2\right|
\end{aligned} \tag{9}
$$

If both r_{sj} and y_{tj} are real numbers, formula (9) can be simplified as below:

The relative overall target center shows similarity of decision alternatives: The larger the β_{st}, the more different the two decision alternatives will be; the smaller the β_{st}, the more similar the two decision alternatives will be.

(d) Learning model for cases that experts have incomplete categories for preference for decision alternatives

Definition 12: If experts believe the decision alternatives in decision alternative set O are similarly excellent, these decision alternatives will be regarded as a single category and recorded as $O=\{z_i^{(O)}|i=1,2,...,l\}$. The decision alternatives in O have the following order of excellenc e: $z_1^{(O)}\approx z_2^{(O)}\approx ...\approx z_l^{(O)}$, namely, any two decision alternatives in O have equivalent overall off-target distances, i.e. $\varepsilon_1^{(O)}\approx\varepsilon_2^{(O)}\approx ...\approx\varepsilon_l^{(O)}$. To realize this equivalence, the relative overall off-target distance of any two decision alternatives should be the minimum.

For example, for decision alternatives $z_1^{(O)}$ and $z_2^{(O)}$ in decision alternative set O, adjust weight $\omega_j\,(j=1,2,...,m)$ according to formula (10) to minimize the relative overall off-target distance of decision alternatives 1 and 2 in O to establish a single-target planning model as below:

$$\min\beta_{12}^{(O)}=\sum_{j=1}^{m}\omega_j^{\;2}\left|d(r_{1j}^{(O)},r_j^{+})^2-d(r_{1j}^{(O)},r_j^{-})^2-d(r_{2j}^{(O)},r_j^{+})^2+d(r_{2j}^{(O)},r_j^{-})^2\right|$$
$$s.t.\sum_{i=1}^{m}\omega_j=1,\omega_j\geq 0,\omega_j\in[\omega_j^L,\omega_j^U],\,j=1,2,...,m \tag{10}$$

It can be expressed as below:

$$\left|r_{(1-2)j}^{(O)}\right|=\left|d(r_{1j}^{(O)},r_j^{+})^2-d(r_{1j}^{(O)},r_j^{-})^2-d(r_{2j}^{(O)},r_j^{+})^2+d(r_{2j}^{(O)},r_j^{-})^2\right| \tag{11}$$

The single-target planning model can be simplified as:

$$\min\beta_{12}^{(O)}=\sum_{j=1}^{m}\omega_j^{\;2}\left|r_{(1-2)j}^{(O)}\right|$$
$$s.t.\sum_{i=1}^{m}\omega_j=1,\omega_j\geq 0,\omega_j\in[\omega_j^L,\omega_j^U],\,j=1,2,...,m$$

To make overall off-target distances of all the decision alternatives in O or, in other words, to minimize the relative overall off-target distance of any two decision alternatives in O, a multi-target decision model as below can be established.

$$\min\left[\beta_{12}^{(O)},\beta_{23}^{(O)},...,\beta_{1l}^{(O)},\beta_{23}^{(O)},...,\beta_{2l}^{(O)},...,\beta_{l-1}^{(O)},...,\beta_{l}^{(O)}\right]$$
$$s.t.\sum_{i=1}^{m}\omega_j=1,\omega_j\geq 0,\omega_j\in[\omega_j^L,\omega_j^U],\,j=1,2,...,m$$

The decision alternatives have no preference relationship but a fair competition relationship. If $\beta^{(O)}$ refers to sum of relative overall off-target distances of all the decision alternatives in O, the multi-target decision model above can be changed into planning model (M-1) below by means of isobaric processing.

$$\min\beta^{(O)}=\sum_{j=1}^{m}\omega_j^{\;2}\left|r_{(1-2)j}^{(O)}+r_{(1-3)j}^{(O)}+...+r_{(1-l)j}^{(O)}+r_{(2-3)j}^{(O)}+...+r_{(2-l)j}^{(O)}+...+r_{((l-1)-l)j}^{(O)}\right|$$
$$s.t.\sum_{i=1}^{m}\omega_j=1,\omega_j\geq 0,\omega_j\in[\omega_j^L,\omega_j^U],\,j=1,2,...,m \tag{M-1}$$

The decision matrix is of real number type, so,
$\left|r_{(1-2)j}^{(O)}+r_{(1-3)j}^{(O)}+...+r_{(1-l)j}^{(O)}+r_{(2-3)j}^{(O)}+...+r_{(2-l)j}^{(O)}+...+r_{((l-1)-l)j}^{(O)}\right|$ is obviously a real number and can be expressed as:

$$\left|r_j^{(O)}\right|^*=\left|r_{(1-2)j}^{(O)}+r_{(1-3)j}^{(O)}+...+r_{(1-l)j}^{(O)}+r_{(2-3)j}^{(O)}+...+r_{(2-l)j}^{(O)}+...+r_{((l-1)-l)j}^{(O)}\right| \tag{12}$$

M-1 can be simplified into planning model (M-2) below.

$$\min \beta^{(O)} = \sum_{j=1}^{m} \omega_j^2 \left| r_j^{(O)} \right|^*$$
$$s.t. \sum_{i=1}^{m} \omega_j = 1, \omega_j \geq 0, \omega_j \in [\omega_j^L, \omega_j^U], j = 1, 2, ..., m \qquad \textbf{(M-2)}$$

Meanwhile, experts believe the decision alternatives in decision alternative set O are similarly excellent, so arithmetic mean of overall off-target distances of all the decision alternatives is practically significant.

Definition 13: If $\gamma^{(O)}$ refers to average overall target center of all the decision alternatives in O, the following formula can be reached according to formula (8).

$$\gamma^{(O)} = \frac{1}{l} \varepsilon_i^{(O)} = \frac{1}{l} \sum_{i=1}^{l} \left(\sum_{j=1}^{m} \omega_j d(r_{ij}^{(O)}, r_j^+)^2 + \sum_{j=1}^{m} \omega_j d(r_j^+ - r_j^-)^2 - \sum_{j=1}^{m} \omega_j d(r_{ij}^{(O)}, r_j^-)^2 \right) \qquad \textbf{(13)}$$

Obviously, $\gamma^{(O)}$ is a vector expression about ω^2.

Definition 14: It is believed by some experts that decision alternative set $Z = \{z_i \left| i = 1, 2, ..., n \right.\}$ can be classified into s categories through several decision alternatives and that there are t decision alternatives not belonging to either the s categories or any other categories. If the decision alternative sets with categories for preference are set as $O_1, O_2, ..., O_S$ and the decision alternative set without categories for preference is set as $A = \{a_i^{(A)} \left| i = 1, 2, ..., t \right.\}$, the following relationship will be met.

$$\begin{cases} O_1 \cap O_2, O_1 \cap O_3, ..., O_1 \cap O_S, O_1 \cap A = \phi \\ O_2 \cap O_3, O_2 \cap O_4, ..., O_2 \cap O_S, O_1 \cap A = \phi \\ ... = \phi \\ O_{k-1} \cap O_k, O_{k-1} \cap A = \phi \\ O_k \cap A = \phi \\ O_1 \cup O_2 \cup ... \cup O_k \cup A = Z \end{cases}$$

According to definition 12, all relative overall off-target distances of all the decision alternatives in any of decision alternative sets $O_1, O_2, ..., O_k$ should be the minimums, so:

$$\min \left[\beta^{(O_1)}, \beta^{(O_2)}, ..., \beta^{(O_k)} \right]$$
$$s.t. \sum_{i=1}^{m} \omega_j = 1, \omega_j \geq 0, \omega_j \in [\omega_j^L, \omega_j^U], j = 1, 2, ..., m$$

All the decision alternative sets have a fair competition relationship, so the model above can be changed into model (M-3) below according to formula (8).

$$\min \beta^{(O)} = \sum_{j=1}^{m} \omega_j^2 \left| r_j^{(O_1)} \right|^* + \sum_{j=1}^{m} \omega_j^2 \left| r_j^{(O_2)} \right|^* + ... + \sum_{j=1}^{m} \omega_j^2 \left| r_j^{(O_k)} \right|^* = \sum_{p=1}^{k} \sum_{j=1}^{m} \omega_j^2 \left| r_j^{(O_p)} \right|^*$$
$$s.t. \sum_{i=1}^{m} \omega_j = 1, \omega_j \geq 0, \omega_j \in [\omega_j^L, \omega_j^U], j = 1, 2, ..., m, p = 1, 2, ..., k \qquad \textbf{(M-3)}$$

According to definition 13, decision alternatives $O_1, O_2, ..., O_k$ should have the maximum differences of average overall off-target distances. If $\left| \gamma^{(O)} \right|^*$ refers to sum of the absolutes of the average overall target enter distance differences of any two of decision alternative sets $O_1, O_2, ..., O_k$, it can be expressed as:

$$\begin{aligned}\left|\gamma^{(O)}\right|^* &= \left|\gamma^{(O_1)} - \gamma^{(O_2)}\right| + \left|\gamma^{(O_1)} - \gamma^{(O_3)}\right| + ... + \left|\gamma^{(O_1)} - \gamma^{(O_k)}\right| \\ &+ \left|\gamma^{(O_2)} - \gamma^{(O_3)}\right| + ... + \left|\gamma^{(O_2)} - \gamma^{(O_k)}\right| + ... + \left|\gamma^{(O_{k-1})} - \gamma^{(O_k)}\right|\end{aligned} \tag{14}$$

Target planning model (M-4) below can be established.

$$\begin{aligned}&\max\left|\gamma^{(O)}\right|^* \\ &s.t.\sum_{i=1}^{m}\omega_j = 1, \omega_j \ge 0, \omega_j \in [\omega_j^L, \omega_j^U], j = 1, 2, ..., m\end{aligned} \tag{M-4}$$

$\gamma^{(O)}$ is a vector expression about ω^2, so $\left|\gamma^{(O)}\right|^*$ is obviously a vector expression about ω^2 too.

For a decision matrix with incomplete categories for preference, planning model (M-5) can be established through minimum sum of relative overall off-target distances of all the decision alternatives in a decision alternative set of the same category and maximum sum of the absolutes of the average overall target enter distance differences of any two of decision alternative sets $O_1, O_2, ..., O_k$.

$$\begin{cases}\min \beta^{(O)} = \sum_{p=1}^{k}\ \sum_{j=1}^{m}\omega_j^2\left|r_j^{(O_p)}\right|^* \\ \max\left|\gamma^{(O)}\right|^* \\ s.t.\sum_{i=1}^{m}\omega_j = 1, \omega_j \ge 0, \omega_j \in [\omega_j^L, \omega_j^U], j = 1, 2, ..., m, p = 1, 2, ..., k\end{cases} \tag{M-5}$$

After single-target processing, model (M-5) can be changed into model (M-6) below:

$$\begin{cases}\min\left\{\eta\sum_{p=1}^{k}\ \sum_{j=1}^{m}\omega_j^2\left|r_j^{(O_p)}\right|^* + (1-\eta)\left|\gamma^{(O)}\right|^*\right\} \\ s.t.\sum_{i=1}^{m}\omega_j = 1, \omega_j \ge 0, \omega_j \in [\omega_j^L, \omega_j^U], j = 1, 2, ..., m, p = 1, 2, ..., k\end{cases} \tag{M-6}$$

In the model above, η is between 0 and 1. In consideration of fair competition of the target function, we usually set η as 0.5. Through software lingo.11, we can obtain the value of $\omega = (\omega_1, \omega_2, ..., \omega_m)$ according to model (M-6). Based on the obtained w value, the overall target enter distance of each decision can be obtained according to formula (8), thus realizing ranking for the decision alternatives.

Thus, steps of the method introduced in this paper can be summed up below:

Step 1: Supplement the incomplete decision matrix by means of case-based reasoning.

Step 2: Determine positive target center R^+ and negative target center R^- of the decision matrix according to the principle of grey target decision-making.

Step 3: Calculate of relative overall off-target distance sum β and average overall off-target distance γ of all the decision alternatives in the decision alternatives of a single category.

Step 4: Calculate sum γ^* of the absolutes of the average overall target enter distance differences of any two of the decision alternatives of different categories.

Step 5: Optimize target weight vector $\omega = (\omega_1, \omega_2, ..., \omega_m)$ in line with different categories for preference of experts for the decision alternatives and preference model (M-6).

Step 6: Overall off-target distances of the decision alternatives can be obtained on the basis of w and formula (8), thus realizing the best ranking for the decision alternatives. Compare overall target enter distances of the unclassified decision alternatives with overall target enter distances of the classified decision alternatives and reach a basic classification judgment for the unclassified decision alternatives.

NUMERICAL EXAMPLE

The case in document[24] is used for verifying feasibility of the method introduced in this paper. In the case, the statistical data about main benefit indexes of industry economy provided by sixteen provinces and municipalities in China for use in the China Industry Economic Statistical Yearbook were used as essential data for economic benefit evaluation comparison and ranking analysis. The indexes used in the evaluation included overall labor productivity (w_1), profit-tax rate of capital (w_2), profit of one hundred sales income (w_3), current fund occupied by one hundred of industrial output value (w_4) and profit and tax ratio of production (w_5). It was assumed that the statistical data are incomplete because of delay of data submission by some provinces and/or municipalities. w_4 was an extremely minor index while the other four indexes were extremely major indexes. The original data were processed by means of an extremum method. The standardized indexes are introduced in TABLE 1.

Experts made an intuitionistic judgment that Guangdong, Zhejiang, Fujian, Jiangsu, Shandong, Hubei and Tianjin are of one category while Liaoning, Hebei and Jiangxi are of another category and that each of the indexes should have an attribute weight of 0.15 to 0.25. Weight determination and ranking are done below according to the decision alternatives introduced herein.

TABLE 1: Main Benefit Indexes of Industry Economy or some Provinces and Municipalities in China in 1992 and Corresponding Evaluation Conclusions

No.	Place	w_1	w_2	w_3	w_4	w_5
1	Beijing	0.6836	1	1	0.3336	1
2	Shanghai	1	0.6989	0.5820	0.6827	0.6531
3	Guangdong	0.9675	0.3130	0.3575	0.9581	0.2177
4	Zhejiang	0.5359	0.6337	0.3722	0.8549	-
5	Fujian	0.4887	0.4957	0.3944	-	0.3864
6	Jiangsu	0.6741	0.3457	0.2053	1	0
7	Shandong	0.4602	0.1641	0.1861	0.7502	0.1579
8	Hubei	0.2441	-	0.2999	0.3871	0.4821
9	Tianjin	0.5807	0.1815	0.2260	0.4306	0.1232
10	Anhui	0.1299	0.2989	0.0384	0.6633	0.2919
11	Hunan	0.0872	0.3652	0.0443	0.3600	0.4737
12	Henan	0.1427	0.2093	0.1388	0.4065	0.4103
13	Hebei	0.1785	0.0783	0.1551	-	0.2105
14	Liaoning	0.1946	0.0293	0	0.3313	0.1962
15	Shanxi	0	0	0.3752	0	0.4617
16	Jiangxi	-	0.0913	0.0679	0.2157	0.1447

Step 1: According to the objective judgments of experts, Guangdong, Zhejiang, Fujian, Jiangsu, Shandong, Hubei and Tianjin are combined into decision alternative set O_1 while Liaoning, Hebei and Jiangxi into decision alternative set O_2. Supplement the incomplete decision matrix by means of case-based reasoning. The calculation of decision alternative set O_1 is taken for example (see items 2 to 9 in TABLE 2). Similarly, the incomplete decision matrixes of Hebei and Jiangxi can be supplemented (see items 13 and 16 in TABLE 2).

TABLE 2: Fuzzy Processing of Incomplete Data in the Decision Alternatives of the Same Categories

No.	Region	w_1	w_2	w_3	w_4	w_5
3	Guangdong	0.9675	0.3130	0.3575	0.9581	0.2177
4	Zhejiang	0.5359	0.6337	0.3722	0.8549	[0,0.4821]
5	Fujian	0.4887	0.4957	0.3944	[0.3871， 1]	0.3864
6	Jiangsu	0.6741	0.3457	0.2053	1	0
7	Shandong	0.4602	0.1641	0.1861	0.7502	0.1579
8	Hubei	0.2441	[0.1641,0.6337]	0.2999	0.3871	0.4821
9	Tianjin	0.5807	0.1815	0.2260	0.4306	0.1232
13	Hebei	0.1785	0.0783	0.1551	[0.2157， 0.3313]	0.2105
16	Jiangxi	[0.1785,0.1946]	0.0913	0.0679	0.2157	0.1447

Step 2: Determine positive target center R^+ and negative target center R^- of the decision matrix ($R^+ = \{1,1,1,1,1\}$, $R^- = \{0,0,0,0,0\}$). Calculate squares of relative positive target centers and relative negative target centers of all the decision alternatives according to formula (8) (TABLE 3).

Calculate relative overall off-target distances of the decision alternatives of the same categories. Guangdong and Zhejiang of category O_1 under attribute w_2 are taken for example.

$$\left|r_{(3-4)2}^{(O_1)}\right| = \left|d(r_{32}^{(O_1)}, r_2^+)^2 - d(r_{32}^{(O_1)}, r_2^-)^2 - d(r_{42}^{(O_1)}, r_2^+)^2 + d(r_{42}^{(O_1)}, r_2^-)^2\right|$$
$$= \left|0.4720 - 0.0980 - 0.1342 + 0.4016\right| = 0.6414$$

With the method above, the following result is reached:

$$\left|r_{(3-4)1}^{(O_1)}\right| = 0.8632, \left|r_{(3-4)3}^{(O_1)}\right| = 0.0294, \left|r_{(3-4)4}^{(O_1)}\right| = 0.2064, \left|r_{(3-4)5}^{(O_1)}\right| = 0.0468$$

According to formula (12), sum β of relative overall off-target distances of the decision alternatives of a single category is as below:

$$\beta^{(O_1)} = 9.9864\omega_1^2 + 8.2294\omega_2^2 + 4.0978\omega_3^2 + 13.5332\omega_4^2 + 8.0346\omega_5^2$$

$$\beta^{(O_2)} = 0.3892\omega_1^2 + 0.2480\omega_2^2 + 0.6204\omega_3^2 + 1.0300\omega_4^2 + 0.2632\omega_5^2$$

According to formula (13), average proximity γ of the decision alternatives of a single category is as below:

$$\gamma^{(O_1)} = 0.0139\omega_1^2 + 0.4193\omega_2^2 + 0.5596\omega_3^2 - 0.1964\omega_4^2 + 0.6833\omega_5^2$$

$$\gamma^{(O_2)} = 1.0197\omega_1^2 + 1.2007\omega_2^2 + 1.1847\omega_3^2 + 0.6532\omega_4^2 + 0.9657\omega_5^2$$

Step 4: Calculate sum $|\gamma|^*$ of the absolutes of the average proximity differences of any two of decision alternatives of different categories.

$$\left|\gamma^{(O)}\right|^* = \left|\gamma^{(O_1)} - \gamma^{(O_2)}\right| = 1.0058\omega_1^2 + 0.7814\omega_2^2 + 0.6251\omega_3^2 + 0.8496\omega_4^2 + 0.2824\omega_5^2$$

Step 5: Establish a target optimization model in line with different categories for preference of experts for the decision alternatives and preference model (M-5).

TABLE 3: Squares of Relative Positive Target Centers and Relative Negative Target Centers of Main Benefit Indexes of Industry Economy of some Provinces and Municipalities in China in 1992

No.	Place	Index and its positive target distance square $d(r_{ij}, r_j^+)^2$					Index and its negative target distance square $d(r_{ij}, r_j^-)^2$				
		w_1	w_2	w_3	w_4	w_5	w_1	w_2	w_3	w_4	w_5
1	Beijing	0.1001	0.0000	0.0000	0.4441	0.0000	0.4673	1.0000	1.0000	0.1113	1.0000
2	Shanghai	0.0000	0.0907	0.1747	0.1007	0.1203	1.0000	0.4885	0.3387	0.4661	0.4265
3	Guangdong	0.0011	0.4720	0.4128	0.0018	0.6120	0.9361	0.0980	0.1278	0.9180	0.0474
4	Zhejiang	0.2154	0.1342	0.3941	0.0211	0.5759	0.2872	0.4016	0.1385	0.7309	0.0581
5	Fujian	0.2614	0.2543	0.3668	0.4809	0.3765	0.2388	0.2457	0.1556	0.0939	0.1493
6	Jiangsu	0.1062	0.4281	0.6315	0.0000	1.0000	0.4544	0.1195	0.0421	1.0000	0.0000
7	Shandong	0.2914	0.6987	0.6624	0.0624	0.7091	0.2118	0.0269	0.0346	0.5628	0.0249
8	Hubei	0.5714	0.3613	0.4901	0.3756	0.2682	0.0596	0.1591	0.0899	0.1498	0.2324
9	Tianjin	0.1758	0.6699	0.5991	0.3242	0.7688	0.3372	0.0329	0.0511	0.1854	0.0152
10	Anhui	0.7571	0.4915	0.9247	0.1134	0.5014	0.0169	0.0893	0.0015	0.4400	0.0852
11	Hunan	0.8332	0.4030	0.9134	0.4096	0.2770	0.0076	0.1334	0.0020	0.1296	0.2244
12	Henan	0.7350	0.6252	0.7417	0.3522	0.3477	0.0204	0.0438	0.0193	0.1652	0.1683
13	Hebei	0.6749	0.8495	0.7139	0.2775	0.6233	0.0319	0.0061	0.0241	0.2239	0.0443
14	Liaoning	0.6487	0.9423	1.0000	0.4472	0.6461	0.0379	0.0009	0.0000	0.1098	0.0385
15	Shanxi	1.0000	1.0000	0.3904	1.0000	0.2898	0.0000	0.0000	0.1408	0.0000	0.2132
16	Jiangxi	0.8149	0.8257	0.8688	0.6151	0.7315	0.0095	0.0083	0.0046	0.0465	0.0209

$$\begin{cases} \min \beta^{(O)} = 10.7648\omega_1^2 + 8.5946\omega_2^2 + 5.5986\omega_3^2 + 14.8584\omega_4^2 + 9.3026\omega_5^2 \\ \max \left|\gamma^{(O)}\right|^* = \omega_1^2 + 0.7814\omega_2^2 + 0.6251\omega_3^2 + 0.8496\omega_4^2 + 0.2824\omega_5^2 \\ s.t. \sum_{i=1}^{5} \omega_j = 1, \omega_j \geq 0, \omega_j \in [0.15, 0.25], j = 1, 2, ..., 5 \end{cases}$$

Change model M-5 into a single-target optimization model below by taking 0.5 for u.

$$\begin{cases} \min(4.8795\omega_1^2 + 4.6880\omega_2^2 + 3.1119\omega_3^2 + 7.0044\omega_4^2 + 4.7925\omega_5^2 \\ s.t. \sum_{i=1}^{5} \omega_j = 1, \omega_j \geq 0, \omega_j \in [0.15, 0.25], j = 1, 2, ..., 5 \end{cases}$$

The following result is reached:

$$\omega = (0.15, 0.2, 0.25, 0.15, 0.25)$$

Step 6: Based on $\omega = (0.15, 0.25, 0.25, 0.15, 0.2)$ and formula (8), proximity of the ideal point can be obtained, thus realizing the best ranking for the decision alternatives (TABLE 4). Compare overall off-target distances of the unclassified decision alternatives and overall off-target distances of the classified decision alternatives. A preliminary judgment that Henan and Shanxi are of the same category as Guangdong, Zhejiang, Fujian, Jiangsu, Shandong, Hubei and Tianjin if viewed from their economic benefits is reached.

Due to different incomplete data processing forms (real number type or interval number type) and classification conditions (whether there is a reference point; whether there is complete classification; whether there is a complete excellence relationship), full and accurate result comparison cannot be done with existing documents (such as[12,13,18,20]); however, viewed from the analysis process, this paper has the following characteristics:

(1) It can realize easy ranking based on incomplete information. The method introduced in this paper is applicable in uncertain conditions;

(2) It can correct the attribute values (in different forms) of a same attribute by means of fuzzy mathematic processing to ensure easy understanding when the positive target center and the negative target center are determined;

(3) Viewed from the final calculation result, it can rank all statistical data by means of terminal calculation of overall off-target distances, so as to provide reference for experts to judge in the next stage.

TABLE 4: Ranking and Categories of Decision Alternatives

No.	Decision alternative	Method introduced in this paper	
		Overall off-target distance ε_i	Ranking
1	Beijing	0.0442	1
2	Shanghai	0.1280	2
3	Guangdong	0.2227	3
4	Zhejiang	0.2227	3
5	Fujian	0.2369	5
6	Jiangsu	0.2783	7
7	Shandong	0.2927	8
8	Hubei	0.2598	6
9	Tianjin	0.3000	12
10	Anhui	0.3051	13
11	Hunan	0.2932	9
12	Henan	0.2941	10
13	Hebei	0.3191	14
14	Liaoning	0.3501	15
15	Shanxi	0.2954	11
16	Jiangxi	0.3571	16

CONCLUSION

To solve the problems that decision alternatives have incomplete information and that experts have incomplete categories for preference for decision alternatives, this paper introduces an improvement method for grey target decision-making. By means of supplementation of incomplete information by interval numbers, a method for processing positive target center and negative target center of a decision matrix made up of real numbers and interval numbers is established; relative overall off-target distances of the decision alternatives of the same category and average overall off-target distance of the decision alternatives of different categories are considered comprehensively, making decision-making information more complete and comprehensive and in line with the intuitionistic judgments of decision makers. By means of case-based learning, an attribute weight determination model is established for the decision alternatives with incomplete categories for preference and preliminary and basic classification judgment is done for unclassified decision alternatives. The models introduced in this paper have clear significance and a great practical value. We will later study the categories for preference decision-making of decision alternatives under different attributes.

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Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this article.

REFERENCES

[1] Z.S.Xu; Uncertain multiple attribute decision making method and Application[M], Tsinghua University Press **(2004)**.

[2] S.H.Kim, B.S.Ahn; Group decision making procedure considering preference strength under incomplete information[J], Computers & Operations Research, **24(12)**, 1101-1112 **(1997)**.

[3] I.Fores, L.More-Lopez, R.Morales et al.; Inductive learning models with missing values[J], Mathematical and Computer Modelling, **44(9/10)**, 790-806 **(2006)**.

[4] Z.X.Su, L.Wang, G.P.Xia; Vikor Expansion Method of Interval-number Dynamic Multiattribute Decision-making [J], Control and Decision, **25(6)**, 836-841 **(2010)**.

[5] H.M.Moshkovich, A.I.Mechitov, D.L.Olson; Ordinal judgments in mult-iattribute decision analysis [J], European J of Operational Research, **137(3)**, 625-641 **(2002)**.

[6] C.Y.Liang, E.Q.Zhang; Combined Multi-attribute Group Decision-making Method Based on Inco mplete Evaluation Information[J], Chinese Journal of Management Science, **17(4)**, 126-132 **(2009)**.

[7] A.D.Pearman; Establishing dominance in multi-attribute decision making using an ordered metric method[J], J of the Operational Research Society, **44(5)**, 461-469 **(1993)**.

[8] B.S.Ahn, K.S.Park; Comparing methods for multiattribute decision making with ordinal weights[J], Computer & Operations Research, **35(5)**,1660-1670 **(2008)**.

[9] P.Sarabando, L.C.Dias; Multiattribute choice with ordinal information: A Comparison of different decision rules[J], Systems, Man and Cybernetics, Part A:Systems and Humans, IEEE Trans on, **39(3)**, 545-554 **(2009)**.

[10] K.S.Lee, K.S.Park, Y.S.Eum et al; Extended methods for identifying dominance and potential optimality in multicriteria analysis with imprecise information[J], European J of Operational Research, **134(3)**, 557-563 **(2001)**.

[11] K.S.Park; Mathematical programming models for characterizing dominance and values and weights are simultaneously incomplete[J], IEEE Trans on Systems, Man and Cybernetics, Part A, **34(5)**, 601-614 **(2004)**.

[12] A.Mateos, A.Jimenez, S.Rios-Insua; Solving dominance and potential optimality in imprecise multiattribute additive problems[J], Reliability Engineering & System Safety, **79(2)**, 253-262 **(2003)**.

[13] J.P.Li, C.Y.Yue, W.Li; A Dominance Relation Based Incomplete Information Multi-attribute Decision-making Method [J], Control and Decision, **28(2)**, 229-234 **(2013)**.

[14] Y.Chen, D.M.Kilgur, K.W.Hipel; Using a benchmark in case-based multiple-criteria ranking[J], IEEE Transactions on Systems, Man, and Cybernetics, Park A:Systems and Humans, **39(2)**, 358-368 **(2009)**.

[15] Y.Chen, D.Marc Kilgour, K.W.Hipel;; A case-based distance method for screening in multiple-criteria decision aid[J], Omega, **36(3)**, 373-383 **(2009)**.

[16] H.H.Wang, J.J.Zhu, Z.G.Fang; Language Information Grey Target Decision-making Classification Model Based on Case Analysis [J], Journal of Systems Science and Information, **33(12)**, 3172-3181 **(2013)**.

[17] Z.X.Sun, M.Han, K.H.Qiu; A Classification Method for Multi-attribute Decision-making [J], Control and Decision, **21(2)**, 171-174 **(2006)**.

[18] X.Z.Zhang, C.X.Zhu; Generalized Level Preference Ranking Method for Multiattribute Decision-making [J], Journal of Systems Science and Information, **11(1)**, 2852-2858 **(2013)**.

[19] C.Y.Liang, D.X.Gu, X.Fan et al.; Case Search Algorithm for Uncertain Multiattribute Decision-making [J], Chinese Journal of Management Science, **17(1)**, 131-137 **(2009)**.
[20] N.Jiang, X.S.Bai, J.J.Sun; Statistical Data Classified Evaluation Model for Multiattribute Decision-making [J], Acta Geodaetica et Cartographica Sinica, **36(2)**, 198-202 **(2007)**.
[21] C.H.Goh, Y.C.A.Tung, C.H.Cheng; A revised weighte sum decision model for robot selection, Computers & Industrial Engineering,1996,30(2):193-199.Chen Ting. Decision Analysis [M], Science Press **(1987)**.
[22] J.Song, Y.G.Dang, Z.X.Wang, K.Zhang; Grey Target Decision-making Model for Positive Target Center and Negative Target Center [J], Journal of Systems Science and Information, **30(10)**, 1822-1827 **(2010)**.
[23] W.D.Zhao, Q.H.Li, Z.H.Sheng; Decision-making Problem Solving based on Case-based Reasoning [J], Journal of Management Sciences in China, **3(4)**, 29-36 **(2000)**.
[24] Y.J.Guo, P.T.Yi; Multi-attribute Decision Making in View of Degree of Preference for Part of Alternatives [J], Journal of Northeastern University (Natural Science), **28(12)**, 1782-1785 **(2007)**.