Graph kernels and applications in protein classification

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ABSTRACT

Protein classification is a well established research field concerned with the discovery of molecule's properties through informational techniques. Graph-based kernels provide a nice framework combining machine learning techniques with graph theory. In this paper we introduce a novel graph kernel method for annotating functional residues in protein structures. A structure is first modeled as a protein contact graph, where nodes correspond to residues and edges connect spatially neighboring residues. In experiments on classification of graph models of proteins, the method based on Weisfeiler-Lehman shortest path kernel with complement graphs outperformed other state-of-art methods.   © 2014 Trade Science Inc. - INDIA

INTRODUCTION

Kernel methods are an important method which is widely used in statistical learning theory[1]. Kernels help to adapt classification regardless how classification performs. That is to say, kernels act like an interface between classification tools and data sets via Support Vector Machines[2]. Early studies on kernel methods dealt almost exclusively with vector-based descriptions of input data. This procedure, though convenient, does not always effectively capture topological relationships inherent to the data; therefore, the power of the learning process may be insufficient. Haussler[3] was the first to define a principled way of designing kernels on structured objects, the so-called R-convolution kernel. Over recent years, kernels on structured objects such as strings and trees, on nodes in graphs and on graphs have been defined. Graphs are natural data structures to model such structures, with nodes representing objects and edges the relations between them. In this context, one

often encounters two questions: "How similar are two nodes or edges in a given graph?" and "How similar are two graphs to each other?"

For instance, in protein classification, one might want to predict whether a given protein is an enzyme or not. Computational approaches infer protein function by finding proteins with similar sequence, structure, or chemical properties. A very successful recent method is to model the protein as a graph, and assign similar functions to similar graphs[4]. Generally speaking, graph kernels are based on the comparison of graph-substructures via kernels. Several different graph kernels have been defined in machine learning which can be categorized into three classes: graph kernels based on walks[5] and paths[6], graph kernels based on limited-size subgraphs[7,8], and graph kernels based on subtree patterns[5,9]. To define a graph kernel, some requirements are put forward: the kernel should be measurable on the issue of similarity for graph; second, it should be computable in an acceptable time; third, it should be

positive definite; fourth, it should be applicable widely. However, some of the kernels cannot meet all of these requirements. In this paper, we present a new graph kernel that measure similarity based on Weisfeiler-Lehman shortest path in undirected graphs, that are computable in polynomial time, that are positive semidefinite.

BASIC KNOWLEDGE

Some definitions on graph theory

We define a graph G as a triplet (V, E, l), where V is the set of vertices, E is the set of undirected edges, and $l : V \rightarrow \Sigma$ is a function that assigns labels from an alphabet Σ to nodes in the graph. The neighborhood $N(v)$ of a node v is the set of nodes to which v is connected by an edge, that is $N(v) = \{v'|(v, v') \in E\}$. We assume that every graph has n nodes, m edges, and a maximum degree of d.

The adjacency matrix A of G is defined as follows:

$$[A]_{ij} = \begin{cases} 1 & if (v_i, v_j) \in E, \\ 0 & otherwise \end{cases},$$

where v_i and v_j are nodes in G. Labels can be added on nodes or edges, these labels are referred as attributes.

A *walk w* of length $k-1$ in a graph is a sequence of nodes $v_1, v_2, \dots v_k$ where $(v_{i-1}, v_i) \in E$ for $1 < i \leq k$.

A *path p* is a walk without same nodes in the sequence.

A *cycle* is a walk with $v_1 = v_k$, a simple cycle does not have any repeated nodes except for v_1.

Suppose $G(V, E)$ is a graph with vertex set V and edge set E. Then, its complement $\overline{G}(V, \overline{E})$ is a graph with the same vertex set V, but with a different edge set $\overline{E} = V \times V \setminus E$. In other words, the complement graph is made up of all the edges missing from the original graph.

Graph isomorphism

Graph similarity or isomorphism[10] is the most essential problem for learning tasks like clustering and classification on graphs. In graph theory, an isomorphism of graphs G and H is a bijection between the vertex sets of G and H: $f : V(G) \rightarrow V(H)$, such that any two vertices u and v of G are adjacent in G if and only if $f(u)$ and $f(v)$ are adjacent in H. Graph isomorphism

problem is neither known to be polynomial-computable, nor NP-hard[11].

THE WEISFEILER-LEHMAN TEST OF ISOMORPHISM

Our method uses concepts from the Weisfeiler-Lehman test of isomorphism[12,13], more specifically its 1-dimensional variant. Assume we are given two graphs G and H and we would like to test whether they are isomorphic. The 1-dim Weisfeiler-Lehman test proceeds in iterations, which we index by i and which comprise the steps given in Algorithm 1.

The key idea of the algorithm is to augment the node labels by the sorted set of node labels of neighboring nodes, and compress these augmented labels into new, short labels. These steps are then repeated until the node label sets of G and H differ, or the number of iterations reaches n. If the sets are identical after n iterations, it means that either G and H are isomorphic, or the algorithm has not been able to determine that they are not isomorphic. See Figure 1, for an illustration of these steps.
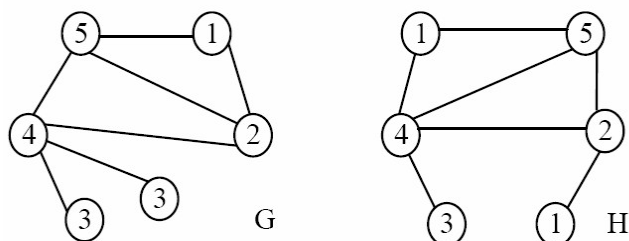
Algorithm 1

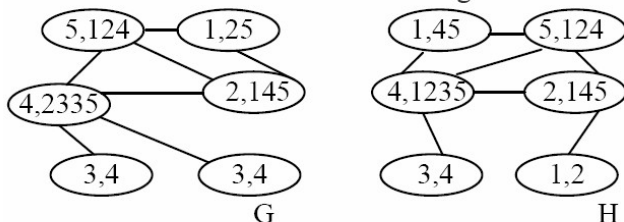One iteration of the 1-dim. Weisfeiler-Lehman test of graph isomorphism (1968)

1. Multiset-label determination
   - For $i = 0$, set $M_i(v) = l_0(v)$.
   - For $i > 0$, assign a multiset-label $M_i(v)$ to each node v in G and H which consists of the multiset $\{l_{i-1}(u)|u \in N(u)\}$.
2. Sorting each multiset
   - Sort elements in $M_i(v)$ in ascending order and concatenate them into a string $s_i(v)$.
   - Add $l_{i-1}(v)$ as a prefix to $s_i(v)$ and call the resulting string $s_i(v)$.
3. Label compression
   - Sort all of the strings $s_i(v)$ for all v from G and H in ascending order.
   - Map each string $s_i(v)$ to a new compressed label, using a function $f : \Sigma^* \rightarrow \Sigma$ such that $f(s_i(v)) = f(s_i(w))$ iff $s_i(v) = s_i(w)$.
4. Relabeling
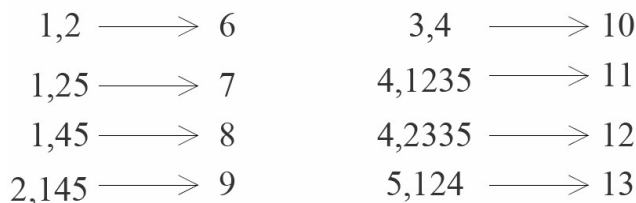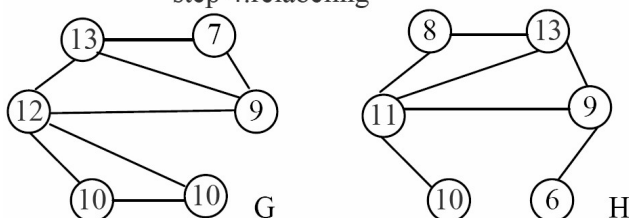   - Set $l_i(v) = f(s_i(v))$ for all nodes in G and H.

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Figure 1: Illustration of the 4 steps of one iteration of the computation of the Weisfeiler-Lehman test of isomorphism

The Weisfeiler-Lehman shortest path kernel

In each iteration i of the Weisfeiler-Lehman algorithm (see Algorithm 1), we get a new labeling $l_i(v)$ for all nodes v. Recall that this labeling is concordant in G and H, meaning that if nodes in G and H have identical multiset labels, and only in this case, they will get identical new labels. Therefore, we can imagine one iteration of Weisfeiler-Lehman relabeling as a function $r((V, E, l_i)) = (V, E, l_{i+1})$ that transforms all graphs in the same manner. Note that r depends on the set of graphs that we consider.

Weisfeiler-Lehman sequence graphs

Define the Weisfeiler-Lehman graph at height i of the graph $G = (V, E, l)$ as the graph $G_i = (V, E, l_i)$. We call the sequence of Weisfeiler-Lehman graphs,

$$\{G_0, G_1, ..., G_h\} = \{(V, E, l_0), (V, E, l_1), ..., (V, E, l_h)\},$$

where $G_0 = G$, the Weisfeiler-Lehman sequence up to height h of G. G_0 is the original graph, $G_1 = r(G_0)$ is the graph resulting from the first relabeling, and so on.

Weisfeiler-Lehman kernel with complement graphs

Let k be any kernel for graphs, that we will call the base kernel. Then the Weisfeiler-Lehman kernel with h iterations with the base kernel k is defined as:

$$k_{WL}^h(G, H) = k(G_0, H_0) + k(G_1, H_1) + \cdots + k(G_h, H_h).$$

If we take the complement graphs into consideration, we will derive the Weisfeiler-Lehman kernel with complement graphs:

$$k_{wl}^h(G, H) = k(G_0, H_0) + k(\overline{G_0}, \overline{H_0}) + k(G_1, H_1) +$$

$$k(\overline{G_1}, \overline{H_1}) + \cdots + k(G_h, H_h) + k(\overline{G_h}, \overline{H_h}),$$

where $(\overline{G_0}, \overline{G_1}, \cdots, \overline{G_h})$, $(\overline{H_0}, \overline{H_1}, \cdots, \overline{H_h})$ are complement graphs of $(G_0, G_1, ..., G_h)$, $(H_0, H_1, ..., H_h)$.

Let the base kernel k be any positive semidefinite kernel on graphs. Then the corresponding Weisfeiler-Lehman kernel k_{WL}^h is positive semidefinite.

Shortest path and Floyd-Warshall algorithm

Given an undirected graph $G = (V, E)$ the shortest path graph[14], $G_{sp}(V, E')$, which contains the same set of vertices as G and the edge between every pair of vertices is labeled with the shortest distance between them in the original graph. The transformation from G to G_{sp} can be performed by any all-pairs shortest path algorithm. Floyd–Warshall algorithm (See Algorithm2)[15] is attractive and effective because it is straightforward and has time complexity of $O(n^3)$. Then, a kernel function was used to calculate the similarity between two shortest path graphs according to the following definitions, which were first defined by Borgwardt and Kriegel[6]. It is proved to be a positive semidefinite kernel and is computable in polynomial time.

Algorithm 2

Floyd–Warshall algorithm (Graph G with n nodes and adjacency matrix A)

Floyd(G)

    for $k \leftarrow 1$ to n

      for $i \leftarrow 1$ to n

        for $j \leftarrow 1$ to n

      if(cost(i, k))+cost(k, j)<cost(i, j))

      cost(i, j)=cost(i, k)+cost(k, j)

     endif

     endfor

    endfor

   endfor

Let e_1 be the edge connecting vertices v_1 and w_1 on graph G, and e_2 be the edge connecting nodes v_2 and w_2 on graph H. A walk on an edge includes the edge and its two adjacent vertices. A walk kernel k_{walk} is used to compare the walk e_1 and e_2 as: $k_{walk}(e_1, e_2) = k_{node}(v_1, v_2) * k_{edge}(e_1, e_2) * k_{node}(w_1, w_2)$, where k_{node} is the kernel function for comparing two vertices, and k_{edge} is a kernel function for comparing two edges.

The kernel function for comparing two vertices u and v is a Gaussian kernel[16] over their respective feature vectors,

$$k_{node}(\mathbf{u}, \mathbf{v}) = \exp\left(\frac{\|\mathbf{f(u)} - \mathbf{f(v)}\|^2}{2\delta^2} \right) .$$

The kernel function for comparing two edges e and f is a Brownian bridge kernel that assigns the highest value to edges with identical weights, and 0 to all edges that differ in weight more than a constant c:

$k_{edge}(e, f) = \max(0, c - |length(e) - length(f)|).$

In this paper, we use $c = 2$.

Shortest path kernel

Given two shortest path graphs $G(V_1, E_1)$ and $H(V_2, E_2)$ the shortest path graph kernel:

$$k_{sp}(\mathbf{G}, \mathbf{H}) = \sum_{e_1 \in E_1} \sum_{e_2 \in E_2} k_{walk}(e_1, e_2),$$

where k_{walk} is a kernel function for comparing two edge walks of length 1.

Floyd-transformation requires $O(n^3)$ time. E_1 and E_2 contain $O(n^2)$ edges. The computation of the shortest-path graph kernel requires $O(n^4)$ time.

Weisfeiler-Lehman shortest path kernel with complement graphs

With the above definitions, we are ready to define Weisfeiler-Lehman shortest path kernel with complement graphs as:

$k_{WL}^h sp(G, H) = k_{sp}(G_0, H_0) + k_{sp}(\overline{G_0}, \overline{H_0}) + k_{sp}(G_1, H_1)$

$+ k_{sp}(\overline{G_1}, \overline{H_1}) + \cdots + k_{sp}(G_h, H_h) + k_{sp}(\overline{G_h}, \overline{H_h})$.

For N graphs, the runtime of WL shortest path kernel will scale as $O(N^2 n^4)$.

EXPERIMENTS

Experimental settings

We compared the performance of the random walk graph kernel[17], the shortest path kernel, the WL Shortest Kernel without complement graphs and WL Shortest Kernel with complement graphs in terms of classification accuracy of the classification on D&D[18] and ENZYMES datasets, where accuracy shows the overall percentage of correct classifications. D&D is a dataset of 691 enzymes and 487 non-enzymes. Each protein is represented by a graph, in which the nodes are amino acids and two nodes are connected by an edge if they are less than 6 Ångstroms apart. The task is to classify the protein structures into enzymes and non-enzymes. ENZYMES is a data set of protein tertiary structures obtained from Borgwardt et al (2005), consisting of 600 enzymes from the BRENDA enzyme database (Schomburg et al., 2004). In this case the task is to correctly assign each enzyme to one of the 6 EC top-level classes. Nodes are labeled in the dataset. In terms of D&D, we also analyzed the sensitivity, specificity, and Matthews correlation coefficient (MCC)[19] of the classifications in addition to accuracy (TABLE 3), where sensitivity is the percentage of enzymes that have been correctly classified as enzymes, specificity indicates the percentage of non-enzymes that have been correctly classified, and MCC shows the overlapping between the predictions and the actual distribution.

Suppose P represents positive instances N negative instances, TP the number of true positives, TN the number of true negatives, FP the number of false positives and FN the number of false negatives. Then the accuracy, sensitivity, specificity and MCC can be calculated by the following formulas,

$$accuracy = \frac{TP + TN}{P + N},$$

$$sensitivity = \frac{TP}{P} = \frac{TP}{TP + FN},$$

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$$\text{specificity} = \frac{TN}{N} = \frac{TN}{FP + TN},$$

$$MCC = \frac{TP \times TN - FP \times FN}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}}.$$

We performed 10-fold cross-validation of C-Support Vector Machine Classification using LIBSVM[20], using 9 folds for training and the rest one for testing. All parameters of the SVM were optimized on the training data set only. To exclude random effects of fold assignments, we repeated the whole experiment 10 times. We show average classification accuracies and standard deviations in TABLE 1. TABLE 2 shows the size of both data set and runtime of the methods computing on them.

TABLE 1 : The classification accuracy (%) and standard deviation of each kernel on protein data sets

Method/Data set	D&D	ENZYMES
Random Walk Kernel	70.26(±0.86)	20.14(±0.69)
Shortest Path Kernel	78.19(±0.26)	42.18(±0.43)
WL Shortest Path Kernel without complement graphs	81.27(±0.70)	62.47(±0.61)
WL Shortest Path Kernel with complement graphs	83.64(±0.92)	63.96(±0.84)

TABLE 2 : CPU runtime for kernel computation on protein classification

Data set	D&D	ENZYMES
Class size	2	6
Maximum nodes	5478	126
Average nodes	284.32	32.63
Number of graphs	1178	600
Random Walk Kernel	52days	39days
Shortest Path Kernel	25h 17min22s	38s
WL Shortest Path Kernel without complement graphs	64days	1min3s
WL Shortest Path Kernel with complement graphs	71days	2min11s

TABLE 3 : Comparison of our method with others using D&D data set

Method	Sensitivity	Specificity	MCC
Random Walk Kernel	65.28%	71.35%	0.523
Shortest Path Kernel	71.32%	79.64%	0.736
WL Shortest Path Kernel without complement graphs	78.24%	83.77%	0.821
WL Shortest Path Kernel with complement graphs	81.05%	86.13%	0.836

Results

In terms of runtime, The shortest path kernel and the WL shortest path kernel were competitive to the random walk kernel on smaller graphs (ENZYMES), but on D&D their runtime degenerated to more than 25 hours for the shortest path kernel, 64 days for the WL shortest path kernel without complement graphs and 71 days for the WL shortest path kernel with complement graphs. Using a graph to model the distribution of amino acid residues on the 3D structure, our method efficiently captures various structural determinants related to protein function. The kernels using WL method performed better than other kernel types. Furthermore, the WL shortest path kernel with complement graphs outperforms the other kernels with an accuracy of at least 83.64%, and it achieves improvements in accuracy more than 2% over the WL shortest path kernel without complement graphs. Meanwhile, considering shortest paths instead of walks increases classification accuracy significantly. For the random walk kernel, classification is the worst as with an increasing number of tottering walks, classification accuracy decreases. TABLE 3 also shows that our proposed method outperforms other methods.

CONCLUSIONS

In this paper, We propose a simple yet effective and efficient graph classification approach that is based on topological and label graph attributes. Our main idea is that graphs from the same class should have similar attribute values. On the basis of an extensive comparison with state-of-the-art graph kernel classifiers, we show that our approach yields competitive or better accuracies and has typically much lower computational times. Our conclusion is that graph attributes are effective in capturing discriminating structural information from different classes. Our new kernels based on Weisfeiler-Lehman test of isomorphism open the door to applications of graph kernels on large graphs in bioinformatics, for instance, protein function prediction via detailed graph models of protein structure on the amino acid level, or on gene networks for phenotype prediction. A challenging question for further studies will be to consider kernels on graphs with

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continuous or high-dimensional node labels and their efficient computation.

ACKNOWLEDGMENTS

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