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# Fast algorithm for image interpolation based on data dependent triangulation 

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#### Abstract

The Data Dependent Triangulation algorithm which represents image as the triangulation mesh, and then take Gouraud interpolation internal triangles. For the less operation efficiency of the Gouraud interpolation, propose a improved triangle internal interpolation method based on the barycentric coordinates properties. It reduces the operation time efficiently. At the same time, employ the edges overlap determination, improve the triangulations accuracy rating. The experimental results show that the improved algorithm is better to improve the image edge smoothness and operation speed.


## KEYwords

Data dependent triangulation; Image interpolation; Triangle internal interpolation; Edges overlap.

## INTRODUCTION

Image interpolation is one of the important contents in image processing field ${ }^{[1]}$. It also has some other names, such as image adjusted, image sampling, image scaling, resolution enhancement, etc. And under the condition of no adjustment of resolution, image interpolation can also be called image restoration techniques. Image interpolation techniques used widely. In daily life, photography enthusiasts hope to do further subtle picture processing. The images and videos which capture from the community monitoring system also need to be made scaling. The High Definition Television technology requires fast image interpolation processing. In medicine, neurosurgery doctor also need magnify the brain faultage images. And astronomical image processing system can construct high resolution astronomy images that depended on low rate of data transmission. We can find that, the image interpolation technology is implemented at all walks of life.

Image interpolation technology is varied. The traditional image interpolation technology is linear filter interpolation, including the nearest interpolation, double linear interpolation, and double cubic interpolation. Linear filter interpolation just consider the relation between the interpolation point and neighbouring points, but miss taking into account the content of the image itself. Therefore, these methods can be realization simply, but the interpolated image is fuzzy or serrated. Because the algorithm of linear filter interpolation carry with the low pass filtering properties, which makes the interpolated image details degenerated in different degree. Although these methods could get excellent effect of smoothness, but some of the high frequency information is lost directly. And the high frequency information is the data for image edges. It is an important factor for examining the image interpolation result. Therefore, in order to preserve the image edge, overcome the inherent defect of the linear filter interpolation, the image processing researchers employed various mathematical tools in the image interpolation field for the better interpolation result. Such as bilateral filter interpolation, local covariance interpolation method, interpolation method of wavelet decomposition, neural network image interpolation, the partial differential equations for image interpolation, and so on ${ }^{[2-12]}$. Compared with the traditional linear filter interpolation, these methods improved the interpolation of image edge preservation in a certain degree, but most of them are difficult to realize for their computational complexity, let alone the real time scaling.

In this paper, we do a lot of work in the image interpolation for data dependent triangulation and propose a fast algorithm which based on the image quadrilateral mesh. At the same time, we founded an improved triangulation method for resizing the accuracy rate of the edges determination. The experiments show that the new algorithm has the less complexity about the computation. And the edges overlap anticipation can improve the accuracy rate of the edges determination. So the image interpolation algorithm which is proposed in this paper is practical.

## IMAGE INTERPOLATION BASED ON DATA DEPENDENT TRIANGULATION

Triangulation is an important issue in the field of aggregate computation ${ }^{[13]}$. Triangulation divides the discrete data to the triangle nets with the aim for making a surface or a body. Classic triangulations algorithm is Delaunay triangulations and some of its optimization algorithm. Such as Watson algorithm, Lawson algorithm, scan line algorithm, etc. Delaunay triangulation is mainly used for triangular mesh extraction. In image interpolation field, there is a view that the bilinear interpolation can get the better results than the Delaunay triangulations. The Data Dependent Triangulations algorithm, hereinafter referred to as DDT algorithm, which proposed by Nira Dyn, David Levin and Samuel Rippa. They found that the algorithm is excellent in achieving a better image interpolation vision effect ${ }^{[14,15]}$.

Given a discrete points set $\mathrm{V}=\left\{\left(X_{i}, Y_{i}\right)\right\}$ on the x-y plane. DDT algorithm divides them into a number of triangle sets that degradation distributed. And the division meet the following conditions.
a). All points in the discrete point set of V must be the vertexes of the triangles, and all of the triangle vertexes must belong to point set V too.
b). Each edge of the triangles only contains two points from set V.
c). the merger of all of the triangles is a convex grid shell.
d). any group of two triangles in the triangles set can only share an edge at most.

For the gray image, the point set V is the image pixels set, and the image interpolation was equivalent to the piecewise linear interpolation on the convex grid shell ${ }^{[16]}$. The goal of the DDT algorithm and its optimization algorithm is searching the smoothest triangle mesh shell. All of these algorithms need build a smooth degree function, and take iterative comparison.

As a result of the original DDT algorithm carry with computational complexity and poor practical application performance. Dan Su and Philip Willis proposed a new DDT image interpolation algorithm which based on image quadrilateral mesh in 2004. This algorithm does not gain a better interpolated result than the original DDT algorithm, but it does not need to carry out the complex smooth degree function calculation and the repeat iteration process. So it can reduce the image interpolation operation time greatly, improve the usefulness of the DDT algorithm. The DDT algorithm which based on image quadrilateral mesh is taking the edges judgement about the image quadrilateral grids, and dividing each quadrilateral into two triangles (as shown in Figure 1). After the division, the algorithm takes linear interpolation in both of two triangles. Compared with the original DDT algorithm, the proposed algorithm is tend to linear interpolation algorithm, but gaining a better image edges saving than the general linear interpolation algorithm.


Figure 1 : Triangulation on a quadrilateral

## IMPROVED IMAGE INTERPOLATION ALGORITHM BASED ON DATA DEPENDENT TRIANGULATION

## Improved interpolation algorithm internal the triangle

Most of the triangle internal linear interpolation algorithms employ the Gouraud interpolation which interpolate along the triangular edge and the horizontal scan line separately and get the final pixel value within the triangle(as shown in Figure 2). The formula of $I_{a}, I_{b}$ and $I_{p}$ is as follows.
$I_{a}=I_{1}-\left(I_{1}-I_{2}\right) \frac{Y_{1}-Y_{p}}{Y_{1}-Y_{2}}$
$I_{b}=I_{1}-\left(I_{1}-I_{3}\right) \frac{Y_{1}-Y_{p}}{Y_{1}-Y_{3}}$
$I_{p}=I_{b}-\left(I_{b}-I_{a}\right) \frac{X_{b}-X_{p}}{X_{b}-X_{a}}$


Figure 2 : Gouraud interpolation


Figure 3 : The barycentric coordinates interpolation
Though Gouraud interpolation is commonly used for triangle internal interpolation.But the image triangle mesh is projected as the right-angle isosceles triangles on the $x-y$ plane, and the computational speed and complexity could be improved by employing the triangle barycentric coordinates interpolation. Any one point internal the triangle can be determined by weighted average of three vertexes, and the weight is called triangle barycentric coordinates. As shown in Figure 3, set $\mathrm{b} 1, \mathrm{~b} 2$, b 3 are the components of the point P , and these three values are corresponding to the ratio of child triangle area $\left(S_{\triangle P B C}, S_{\triangle P A C}, S_{\triangle P A B}\right)$ to the entire triangle area $S_{\triangle A B C}$ respectively. $I_{1}, I_{2}$ and $I_{3}$ are the values of the three of
triangle vertexes, $I_{P}$ is the value of point P . Then we get $\mathrm{b} 1+\mathrm{b} 2+\mathrm{b} 3=1, I_{p}=\left[\begin{array}{lll}b_{2} & b_{3} & b_{1}\end{array}\right]\left[\begin{array}{lll}I_{2} ; & I_{3} ; & I_{1}\end{array}\right]$. As shown in Figure 4, choose a point P internal the triangle ABC which is in three-dimensional space, we could get the value of point p from this formula: $Z_{p}=\left[\begin{array}{lll}b_{2} & b_{3} & b_{1}\end{array}\right]\left[\begin{array}{lll}Z_{2} ; & Z_{3} ; & Z_{1}\end{array}\right] . Z_{i}$ is the $Z$ coordinate value of the point in three-dimensional space, and stand for the pixel value in gray image processing. According to plane projection properties, the projected area ratio is $\operatorname{COS} \theta, \theta$ is the angle between the triangle ABC and the projection plane. Then the follow three formulas can be obtained.

$$
\begin{aligned}
& b_{2}=\frac{S_{\triangle \mathrm{PAC}}}{S_{\triangle \mathrm{ABC}}}=\frac{S_{\triangle \mathrm{pac}} / \cos \theta}{S_{\triangle \mathrm{abc}} / \cos \theta}=X_{p}-X_{1}=x \\
& b_{3}=\frac{S_{\triangle \mathrm{PAB}}}{S_{\triangle \mathrm{ABC}}}=\frac{S_{\triangle \mathrm{pab}} / \cos \theta}{S_{\triangle \mathrm{abc}} / \cos \theta}=Y_{p}-Y_{1}=y \\
& b_{1}=\frac{S_{\triangle \mathrm{PBC}}}{S_{\triangle \mathrm{ABC}}}=\frac{S_{\triangle \mathrm{pbc}} / \cos \theta}{S_{\triangle \mathrm{abc}} / \cos \theta}=1-x-y
\end{aligned}
$$

Finally, we could achieve the Results derived. $Z_{P}=\left[\begin{array}{lll}x & y & (1-x-y)\end{array}\right]\left[\begin{array}{lll}Z_{2} ; & Z_{3} ; & Z_{1}\end{array}\right]$. As shown in Figure 5, $x$ is defined as the horizontal offset that the point P projected on the $\mathrm{x}-\mathrm{y}$ plane, and y is the vertical offset. Obviously, the shape of the projection triangle on the $x-y$ plane is a right-angle isosceles triangle. The value of the components of point $p$ could be calculated fast. So the interpolation method would be better than the Gouraud interpolation in operation speed.


Figure 4 : Triangle projected in three-dimensional space


Figure 5 : The projection plane

## Improved determination algorithm

It is a key factor that making a determination of the edges orientation in the fastest time. Dan Su and Philip Willis proposed a fast algorithm. Suppose that the pixel values on a diagonal of the quadrilateral are a and c , and the other pair of pixels values are $b$ and $d$. It recognize the edge orientation just compare the absolute values between the pixels values difference on the diagonals. When $|a-c|>|b-d|$, the diagonal which contains $b$ and $d$ is supposed to be the edge. Conversely, the other diagonal is the right determination ${ }^{[17,18]}$. It is equivalent to dividing the four pixels structure into two triangles. However, although this edge judgement algorithm is operated quickly, but invalid or easy to mistake in some specific
circumstances. Such as the pixel values of a and c are 10 and 11 , b and d are 178 and 180 , it is difficult to make the judgement that both of the edges may exist. In this paper, we propose an improved edges determination algorithm to resolve this problem.

We put forward an improved edges determination algorithm based on the quadrilateral edges overlap. The original algorithm is only dependent on four pixels values, and may make some misjudgement. The visual length of the image edges is longer than four pixels in usual. As shown in Figure 6, the grid of number 5 is stand for the four pixel square for determined, we would make the edges judgement on the four bigger grids which names are 1245, 5689, 2356 and 4578. The name of the bigger grid is on behalf of its four child grid. And we store the edges orientation with 0 and 1 , if the edge slope is positive, the value is 1 , otherwise the value is 0 . Suppose $F$ is the determination function, and $F(5)$ is the direction for judgement. If $F(1245)=F(5689)=0$, that $F(5)=0$. Else if $F(2356)=F(5678)=1$, that $F(5)=1$. Else $F(5)$ is to be calculated on the original determination method.


Figure 6 : The edges overlap determination

## The time complexity analysis of the improved algorithm

We analyse the complexity of the improved method in this section. At first, it takes edges overlap judge between the bigger grids at the top left corner and the lower right corner. If the judgement result is true, obtain the edge determination. Else if the result is false, it takes the second edges overlap judge between the bigger grids at the top right corner and the lower left corner. If the second judgement result is true, obtain the edge determination. Else it gives up the overlap judgement, and takes the original method.


Figure 7 : flowchart of the algorithm
Suppose the original image I has width $n$ and height $m$, so the number of pixels is $n * m$.The original edges determination needs two subtractions and one comparison. But the improved method we proposed needs take the edges overlap judgement first. There are four big grids to be calculated, so it needs eight subtractions, four comparison and two logic and at most. We have found some result from the above-mentioned experiment, nearly 60 percent of the determination would be done at the two overlap judgement, so the actual calculated amount of edges determination is more or less $13 *(m * n)$. However, we have proposed the improved interpolation internal triangle, we will Included it in the calculation scope. The improved interpolation method needs two subtractions, two add and three multiplications, while the Gouraud interpolation needs three subtractions, three add three multiplications and one division. Moreover, the edges determination is taking place at the original image mesh, and the interpolation is at the interpolated place. So when the amplifying time is bigger, the more calculation we could reduce. As a result, though our method increases some subtractions and logic add
operation at the edges determination process, but it cut down the calculated amount at the interpolation sharply. Our improved method is closer to the bilinear interpolation in speed. The flowchart of the improved algorithm shown in Figure 7.

## INTERPOLATION EXPERIMENT AND RESULTS COMPARISION

In order to compare the calculation speed between the improved algorithm and the original one, we extract the image Peppers for half and one fourth separately for the follow experiments. It is the magnified result comparison by a factor of 2(Figure 8) and 4(Figure 9). Experimental platform: Intel dual-core CPU E4500, two Gbits of memory volume. Programming tool: MATLAB 7.6.0.

As shown in Figure 8-11, the improved method we proposed in this paper is superior to original one at the edges saving and the edges transition smoothness. And according to the table one, our method could get a better PSNR result than the original one too. The result proved that the edges overlap determination is correct and efficient. Finally, the interpolation time in table two, our proposed algorithm have advantages in speed.

(a)original image (b) original method (c)improved method

Figure 8 : Image peppers magnified by a factor of 2

(a)original image (b) original method (c)improved method

Figure 9 : Image Peppers magnified by a factor of 4

(a)original image (b) original method (c)improved method

Figure 10 : Image Lena magnified by a factor of 3

(a)original image (b) original method (c)improved method

Figure 11 : Local effect image

TABLE 1 : Comparison of the PSNR result (peppers)

| Factor | bilinear | Original method | Improved method |
| :---: | :---: | :---: | :---: |
| 2 | 29.9936 | 30.0696 | 30.0716 |
| 4 | 24.5852 | 24.6169 | 24.6180 |

## CONCLUSIONS

A Fast algorithm for Image Interpolation based on Data Dependent Triangulation is proposed in this paper. It represents an image as a data-dependent triangulation mesh. Every four-pixel square is divided into two triangles by the edges overlap determination which we proposed. Then it calculate the interpolate pixels values by the barycentric coordinates properties which reduce the operation time efficiently. We proved our method by some experiments which have shown above. So the Fast image interpolation algorithm proposed in this paper is superior and practical.

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