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ESTIMATION OF BINDING ENERGIES AND EXCHANGE ENERGIES OF CONDENSED MATTER IN SUPERSTRONG MAGNETIC FIELD

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ABSTRACT

Binding energies and exchange energies of hydrogen, helium, carbon and oxygen matter were evaluated in a superstrong magnetic field. The evaluation is performed by theoretical formalism of Skjervold and ϕ stgaard using three adjustable parameters η, R (a_0) and l (a_0). Our theoretical results indicate that inclusion of exchange energies enhances the binding energies and this enhancement is more pronounced in lower value of z.

Key words: Binding energy, Neutron star, Exchange energies, Lattice spacing, Enlongated atoms

INTRODUCTION

In an earlier paper,[1] the method of evaluation of binding energy of hydrogen, helium, carbon and oxygen matter without exchange energy term has been presented. In this paper, the binding energies calculations including exchange energy term are presented. Without exchange energy term, the binding energy of hydrogen, helium, carbon and oxygen matter increases with the increase of the magnetic field strength B. Several workers[2–7] have reported that inclusion of exchange energy increases the binding energy substantially. The effect of enhancement is towards lower value of z, i.e. one may find exchange energies to be 5–30 % of the total binding energy for a magnetic field of 10^{12} G. In stronger fields, however, the exchange become smaller and it is also smaller in atomic calculations.[8,9]

In this paper, an analytic expression of electron exchange interaction energy in terms of M_0 and K has been obtained. The details of these parameters are given in earlier paper[1]. Then the total expression for the total energy in terms of parameters η, ξ and z were obtained.

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and the expression E_{ex} and E_{total} were numerically evaluated for hydrogen, helium, carbon and oxygen matter as function of magnetic field strength B ranging from 10^{12} to 10^{15} G. The results are shown in Tables 1, 2, 3 and 4 respectively.

Mathematical methods used in the evaluation

As discussed in earlier paper[1], the energy of system can be written as

$$E = E_F + E_{+-} + E_{++} + E_{ex} \qquad \ldots(1)$$

when E_F is the kinetic energy of the Fermi gas, E_{ij} is the potential energy because of interactions between two particles (charge i and j), E_{ex} is the exchange term in the electron–electron interaction energy. The total energy E then depends on two parameters i and M_0 (or R), where we assumed $L \rightarrow \alpha$

Hence Landau levels of orbital radius -

$$\rho_M = (M + 1/2)\hat{\rho},\ M = 0, 1, 2 \qquad \ldots(2)$$

$$\hat{\rho} = (2\hbar c/e_B)^{1/2}$$

is the cyclotron radius.

The electrons occupy Landau orbitals, where the outer orbital has the radii as -

$$R = (M_0 + 1/2)^{½}\ \hat{\rho} \cong M_0^{½}\ \hat{\rho} \qquad \ldots(3)$$

where

$$M_0 = (R/\hat{\rho})^2$$

Introducing dimensionless variables

$$\lambda = L/\hat{\rho}$$

$$K = K_F\hat{\rho}$$

$$\mu_0 B = \hbar^2/m\hat{\rho}^2 \qquad \ldots(4)$$

Here L is the length of the system in the z–direction of the field.

We have -

$$E_F = \mu_0 B K^3\ \lambda\ M_0/6\ \pi \qquad \ldots(5)$$

$E_{+-} = -(Z^2/e^2/l)\ [2\ \ln(L/\hat{\rho}) + 2\ \ln 2 - 1 - \ln M_0 - 3/2\ M_0^{-1}]$ …(6)

Similarly,

$E_{++} = -(Z^2e^2/l)\ [\ln(L/\hat{\rho}) + \ln(\hat{\rho}/2l) + \varepsilon]$ …(7)

where ε is Euler's constant.

The direct Coulomb interaction energy of the electrons can be written as -

$E_{--} = \sum_{M_1, M_2} E_{--}(M_1, M_2)$ …(8)

Where

$E_{--} = (M_1, M_2)\ (1/2)e^2 \sum_{k_1, k_2} \iint r_{12}^{-1} \left|\phi_{M_1, M_2}(r_1)\right|^2 \left|\iint r_{12}^{-1}\right| \phi_{M_1, M_2}(r_2)\big|^2 dr_1 dr_2$ …(9)

where normalized electron wave functions in cylindrical coordinates

$\phi_{KM}(\rho, z, \phi) = (\pi L M!\,\hat{\rho}^2)^{-1}\ (\rho^2/\hat{\rho}^2)^{M/2}\ x\ \exp(-\rho^2/2\hat{\rho}^2)\ \exp(ikz)\ \exp(-iM\phi)$ ...(10)

and

$$\int_0^{\infty} \rho d\rho \int_{-L/2}^{+L/2} dz \int_0^{2x} \phi^*_{KM} \phi_{KM} d\phi = 1$$ …(11)

on solving, we have

$E_{--} = (z^2e^2/l)\left[\ln(L/\hat{\rho}) + \frac{1}{2}\ln g - \frac{1}{4} - \frac{1}{2}\ln M_0 - \frac{1}{2}(\ln 2)M_0^{-1}\right]$ …(12)

where $\psi(M_0) = \ln M_0$.

Calculation of electron–exchange interaction energy

The exchange energy is

$E_{ex} = \sum_{M_1,M_2} E_{ex}(M_1, M_2)$ …(13)

Where

$E_{ex}(M_1,M_2) = \frac{1}{2}\ e^2 \sum_{k_1,k_2} E_{ex} \iint r_{12}^{-1} \varphi^*_{K_1,M_1}(r_1)\varphi^*_{K_2,M_2}(r_2)\varphi_{K_1,M_1}(r_1)\varphi_{K_2,M_2}(r_2)dr_1dr_2$ …(14)

then on solving, we have -

$$E_{ex} \cong (e^2\hat{\rho}\lambda/Ll)M_0^2[M_o^{-1}(\ln k + \ln 2 - 2.44)]$$

$$\cong (z^2e^2/l)M_0^2[M_o^{-1}(\ln k + \ln 2 - 2.44)] \quad \ldots(15)$$

where $K = \pi\hat{\rho}/l$.

Then total energy

$$E = -(2\,z^2/l)\left\{\ln(2l/R) - (\varepsilon - C_1) + \frac{1}{2}\,Z\text{-}3\eta\text{-}2[\ln(2^{3/2}lR^2z^{1/2}\eta^3)\right\}$$

$$+ (\pi^2/12l^2z^3\eta^4R^4)R_y \quad \ldots(16)$$

where

$$C_2 = 2.44 - \ln 2 = 1.75 \quad \ldots(17)$$

Minimizing the energy with respect to R and l gives

$$\text{Ln}(2l/R) = \varepsilon - C_1 + 3/2$$

$$l = 2.87\,R \quad \ldots(18)$$

Now, one has also a relation -

$$\frac{1}{2} + 2\,z^3\eta^2R^2)^{-1}[\ln(0.36 \text{ x } 2^{3/2}\,R^3z^{7/2}\,\eta^3) + 3/4] = 0.7195/(z^3\eta^4R^5) \quad \ldots(19)$$

where

$$R = 2^{-1/2}\xi\,\eta^{-1}z^{-7/6}\,a_0 \quad \ldots(20)$$

where ξ is given by the equation -

$$\xi^5 + 6z^{-2/3}\,\xi^2\,(\ln\xi + 0.2945) = 8.14\,\eta\,z^{5/6} \quad \ldots(21)$$

The total energy is given by the parameters, ξ, η and z i.e.

$$E = 2.475\,z^{19/6}\eta\,\xi^{-1}[1.5\,\xi + 3\,\xi^{-2}Z^{-2/3} \times \ln(\xi + 06279)] + 5.035\,\xi^{-6}\,\eta^2z^2E_H \quad \ldots(22)$$

where

$E_H = e^2/2a_0 = 13.6$ …(23)

Equation (21) and (22) have been solved numerically and the results are shown in Tables 1, 2, 3 and 4 for hydrogen, helium, carbon and oxygen, respectively. The ground state energy for an atom in a superstrong magnetic field, when exchange terms are included, has been obtained by Thomas Fermi–Dirac method and is given by -

$E = [-153.47 - 22.37(B(10^{12}G])^{-1/5}z^{-2/5}] \times B[10^{12}G]^{2/5}z^{9/5})$ eV …(24)

i.e. the binding energy of an atom in matter, when exchange terms are included, is given by equation (22) and (24).

Table 1: Binding energy and exchange energy of hydrogen matter in superstrong magnetic field

B(10^{12}G)	η	ξ	R(a_0)	*l*(a_0)	$-E_{ex}$(KeV)	–E(KeV)
1	10.3	2.03	0.139	0.399	0.13	0.69
5	23.1	2.44	0.074	0.212	0.20	1.25
10	32.7	2.64	0.057	0.164	0.23	0.1.63
50	73.1	3.17	0.031	0.089	0.34	3.00
100	103.4	3.43	0.023	0.066	0.41	3.92
500	231.2	4.11	0.013	0.037	0.60	7.38
600	251.9	4.18	0.0125	0.035	0.63	7.78
700	264.6	4.25	0.0118	0.033	0.67	8.20
800	282.8	4.32	0.0114	0.032	0.69	8.58
900	301.6	4.39	0.0108	0.031	0.70	8.96
1000	327.0	4.44	0.0103	0.029	0.71	9.52

Table 2: Binding energy and exchange energy of helium matter in superstrong magnetic field

B(10^{12}G)	η	ξ	R(a_0)	*l*(a_0)	–E_{ex}(KeV)	–E(KeV)
1	3.7	1.93	0.93	0.479	0.13	0.69
5	8.2	2.31	0.089	0.255	0.20	0.25
10	11.6	2.50	0.068	0.195	0.23	0.63
50	25.9	2.98	0.036	0.0103	0.34	3.00
100	36.6	3.22	0.028	0.080	0.41	3.92
500	81.7	3.84	0.015	0.043	0.60	7.38
600	92.8	3.96	0.13	0.040	0.63	7.78
700	102.9	4.05	0.12	0.038	0.67	8.20
800	108.6	4.10	0.011	0.035	0.69	8.58
900	112.8	4.12	0.010	0.033	0.70	8.96
1000	115.6	4.14	0.011	0.032	0.71	9.52

Table 3: Binding energy and exchange energy of carbon matter in superstrong magnetic field

B(10^{12}G)	η	ξ	R(a_0)	*l*(a_0)	–E_{ex}(KeV)	–E(KeV)
1	0.7	1.76	0.219	0.628	0.50	4.5
5	1.6	2.09	0.116	0.291	0.77	8.4
10	2.2	2.25	0.088	0.193	0.93	11.0
50	5.0	2.67	0.047	0.135	1.40	20.6
100	7.0	3.88	0.036	0.103	1.67	27.0
500	15.7	3.41	0.019	0.055	2.49	50.7
600	17.6	3.47	0.18	0.053	2.55	52.9
700	18.9	3.52	0.017	0.050	2.67	58.6
800	20.5	3.60	0.016	0.047	2.78	60.8
900	21.8	3.62	0.015	0.044	2.84	64.5
1000	22.2	3.67	0.014	0.040	2.96	66.7

Table 4: Binding energy and exchange energy of oxygen matter in superstrong magnetic field

B(10^{12}G)	η	ξ	R(a_0)	*l*(a_0)	–E_{ex}(KeV)	–E(KeV)
1	0.5	1.71	0.234	0.671	0.72	7.4
5	1.0	2.03	0.124	0.356	1.16	14.0
10	1.4	2.18	0.094	0.270	1.34	18.2
50	3.2	2.59	0.050	0.143	2.03	34.2
100	4.6	3.79	0.038	0.109	2.42	44.9
500	10.2	3.30	0.020	0.057	3.63	84.6
600	10.9	3.33	0.18	0.055	3.87	92.5
700	11.2	3.36	0.017	0.050	3.98	100.8
800	12.5	3.42	0.0168	0.048	4.12	105.6
900	13.0	3.48	0.0158	0.045	4.22	108.7
1000	14.5	3.55	0.0150	0.043	4.31	111.1

RESULTS AND DISCUSSION

In this paper, we have evaluated the binding energies and exchange energies of hydrogen, helium, carbon and oxygen matter in the presence of strong magnetic field. The evaluation has been performed on the basis of theoretical formalism of Skjervold and ϕ stgaard.[8,9] Our theoretical result indicates that exchange energy increases with increase of magnetic field in all the four matters. Our theoretical results also indicate that total energy (binding energy) also increases with increase of magnetic field B in all the four hydrogen, helium, carbon and oxygen matter. But this increase is much faster than the increase of the exchange energy. This proves the fact of the other workers that the inclusion of exchange energies does enhance the binding energy and this enhancement is much more pronounced in the lower value of z. In the stronger field, the exchange energy becomes smaller. For helium matter, we obtain exchange energies of 0.13 KeV for a magnetic field of 12^{12}G and 0.20 KeV for a field of 5×10^{12}G, which are in good agreement with Müller corresponding result of 0.16 and 0.26 KeV, respectively. The main difference from earlier work is in the atomic dimensions i.e. for the lattice spacing or distance between the nuclei in the chain *l* (a_0) or *l* (R), we obtain *l* = 2.87 R, which indicates more elongated atoms in the earlier works by Ruderman,[10] Constanteiniscu and Rahak,[11] Chen et al.[12] Glasser and Kaplan,[13] Hillerbrandt and Müller[14] and Flowers *et al.*[15] Our results are very sensitive to the value for the constant C_1. The energy of linear chain of nuclei with charge Ze, lattice spacing l, radius R and uniform electron density has been calculated by Ruderman but with different C_1. Different workers have used the different values of C_1 i.e. $C_1 = 0.75$ by Ruderman[10] and Flowers *et.al.*[15] or $C_1 = 1.25$ by Glasser and Kaplan.[13] The independent minimization with respect to R and l then gave $l = 1.88$ R or $l = 1.14$ R. Ruderman's calculations included the first four terms of the right hand side of equation -

$$E = (E_F + E_{+-} + E_{--} + E_{++})$$

and he assumed that the electrons sheath is uniformly charged cylinder. This was improved upon latter by others by including electron exchange and an the quantized structure of the electron gas due to the magnetic field. The condensed matter in superstrong magnetic fields is assumed to consist of atoms of linear nuclear charges, where the corresponding length or interval l contains a charge Ze. The electrons are correspondingly, approximated as a one–dimensional Fermi gas, where M_0 electrons fill Landau levels and $(Z–M_0)$ electrons are quantized in the direction of the field. Recently, Bouhassouns *et al.*[16] presented a theoretical study of the binding energy of an exciton in a cylindrical quantum well wire subject to an external magnetic field. Calculations were performed using a variational approach ω_i in the effective mass approximation.

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