ISSN : 0974 - 7435 Volume 10 Issue 12

2014

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 10(12), 2014 [6416-6425]

Economic production quantity for two-stage imperfect production system with random machine unavailability and learning effect

Zhou Xideng*
Business Administration College, Nanchang Institute of Technology, Nanchang 330022, (CHINA)
E-mail : chowxd@foxmail.com

ABSTRACT

This article studies the learning effect of the unit production time for a two-stage imperfect production system with machine unavailability. The production rates are finite; preventive maintenance of unreliable machines follows some stochastic processes with known probability distributions. Due to different preventive maintenance times for two stages, two cases for shortage cost are investigated, and two cost-minimization models are formulated. The total cost includes production cost, shortage cost, rework cost, maintenance cost and inventory carrying cost. Numerical examples are provided to illustrate the proposed models.

KEYWORDS

EPQ; Imperfect production process; Inventory; Learning rate; Machine unavailability.

INTRODUCTION

In an imperfect manufacturing process, the product quality is usually depends on the state of the production process. Production models with unreliable machines have been considered in many papers. Porteaus (1986) and Rosenblatt and Lee (1986) proposed models assuming imperfect quality[1,2]. They considered the production process to function perfectly at the start of production. At some random time, the process shifts to an out-of-control state and starts to produce a percentage of non-conforming products. They found that the economic production quantity (EPQ) is smaller than the EPQ in the classical model. Kim and Hong have done an extension of Rosenblatt and Lee by considering that an elapsed time until process shift is randomly distributed[3]. Chung and Hou further extended Kim and Hong to investigated allowable shortages in the imperfect production system[4]. Lo et al. (2007) further extended Chung and Hou's model to consider inflation, and developed a manufacturer- retailer integrated inventory model[5]. Biswas and Sarker extended the above research to consider a lean production system with in-cycle rework and scrap[6]. Liao et al. developed an EPQ model by taking scrap, rework and machine breakdown maintenance into account. They introduced two types of preventive maintenance: perfect and non-perfect preventive maintenance[7].

Most of the research above focuses on machine breakdown in the production uptime period, but they do not give attention to machine unavailability. In reality, this circumstance often occurs because of some machine needing to be maintained, while others may breakdown randomly. There have been several intensive studies on production inventory model with preventive maintenance. Abboud et al. assumed random machine unavailability, no machine breakdown in the production period and allowable shortages[8]. Chung et al. extends the work of Abboud et al. by introducing EPQ for a deteriorating item model with stochastic unavailability of machine[9]. In the other hand, in practice, products are manufactured through multi-stage production process. A few authors have developed various multi-stags models in the literatures and consider that the two-stage models can be also used to approximate more complicated multi-stage production systems. Szendrovits proposed several two-stage production-inventory models where smaller lots are produced at one stage and one larger lot is processed at the other stage[10]. Kim's investigated two-stage production lot sizing problems considering various lot sizing depending on batch transfer and finite production rates between stages[11]. Hill extended Kim's model to provide an alternative way of solving the analysis which is easier to understand[12]. Pearn et al. examined a two-stage production system with imperfect processes and shortages. Chang et al. considered a two-stage assembly system with imperfect processes[13]. The former is an automatic stage in which the required components are manufactured. The latter is a manual stage which deals with taking the components to assemble the end product.

Historically, the learning curve theory has been applied to a diverse set of management decision areas such as production planning, inventory control and quality improvement. The earliest learning curve representation is a geometric progression that expresses the time required to perform a task decreases as repetitions increase. The learning curve equation introduced by Wright is commonly used because it has a simple and applicable mathematical form[14]. Jaber and Bonney studied the effect of interruptions on the economic production lot size under the condition of learning and/or forgetting phenomenon[15]. Some of the recent works that incorporated the learning curve into the production inventory problem are those of Jaber and Bonney[16], Jaber and Bonney[17], Jaber and Guiffrida[18], Chen et al.[19], Jaber and Saadany[20] and Tsai et al.[21].

In this paper, we investigate the learning effect of the unit production time for the two-stage production system with imperfect processes and machine unavailability. In a two-stage production system, products move from the first stage 1 to the final stage 2. According to instability and unreliability of facilities, defective items are produced in each stage. Defective items are reworked immediately in a parallel manufacturing system. Preventive maintenance is conducted at the end of each stage, and the maintenance time is assumed to be random. When demand is greater than the stock during machine maintenance time, shortage will occur. Depending on different machine preventive maintenance time of each stage, we investigate two cases for shortage cost, and develop two cost-minimization models. The remainder of this paper is organized as follows. Mathematical notation and basic assumptions are presented in Section 2. In Section 3, we formulate the proposed problem as a cost-minimization model. Examples are given in Section 4. Conclusions are drawn in Section 5.

NOTATION AND ASSUMPTIONS

In this section, mathematical notation and the relevant assumptions are presented.

Notation

Q production quantity (decision variable).

K_i setup cost in stage i, $i = 1, 2$.

C_i production cost per item in stage i, $i = 1, 2$.

v_i labor cost per unit time.

β_i learning rate in stage i, $i = 1, 2$.

L_i learning slope in the Wright's formulation for the learning curve in stage i, $L_i = -\log \beta_i / \log 2$, $i = 1, 2$.

$t_i(Q)$ time to produce Q units in stage i.

t_{12} time period when there is no production and the inventory depletes in stage 1.

t_3 time period when there is no production and the inventory depletes in stage 2.

t_{ri} preventive maintenance time in stage i.

T_{ij} production time for the j item in stage i.

T_i total replenishment time.

$\hat{p}_i$ average production rate for stage i in producing Q units.

D demand rate.

C_{hi} holding cost per unit per unit time in stage i.

C_{0i} maintenance cost per unit time in stage i.

C_s shortage cost per unit per unit time.

R_{ci} rework cost per unit in stage i.

τ_i a random variable which is the elapsed time for the production process to shift to ''out- of-control'' in stage i.

$f_i(\tau_i)$ probability density function for τ_i and is assumed to be exponentially distributed with the parameter u_i.

N_i number of nonconforming items in i stage.

θ_1 probability of nonconforming items when the production process is in "in-control" state in stage 1; and $0 < \theta_1 < 1$.

θ_2 probability of nonconforming items when the production process is in "out-of-control" state in stage 1; and $0 < \theta_1 < \theta_2 < 1$.

θ_3 probability of nonconforming items when the production process is in "in-control" state in stage 2; and $0 < \theta_3 < 1$.

θ_4 probability of nonconforming items when the production process is in "out-of-control" state in stage 2; and $0 < \theta_3 < \theta_4 < 1$.

t_{ri} time required for repairing the machine in stage i.

$\phi_i(t)$ probability density function for t_{ri} and is assumed to be exponentially distributed with the parameter λ_i.

Assumptions

In addition, the following assumptions are used throughout this paper:

(1) Imperfect (non-conforming) items are reworked immediately in a parallel manufacturing system.

(2) The production system is separated into two stages. The average production rates of the two stages satisfy the condition $\hat{p}_1 > \hat{p}_2 > d$.

(3) The process quality of two stages is independent.

(4)Shortages are allowed, but the backlogging of unsatisfied demand is not permitted.

Under the above assumptions and notations, the graphic representation of the inventory behavior for the two-stage imperfect production system with learning consideration can be shown as in Figure 1 and Figure 2.

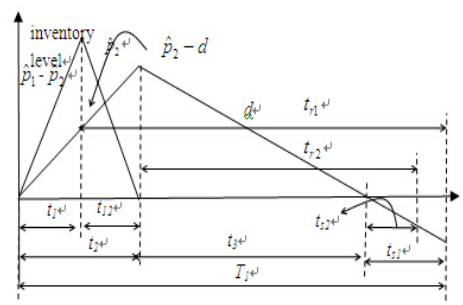

Figure 1 : Inventory level for $E(t_{r1}) \geq E(t_{r2})$

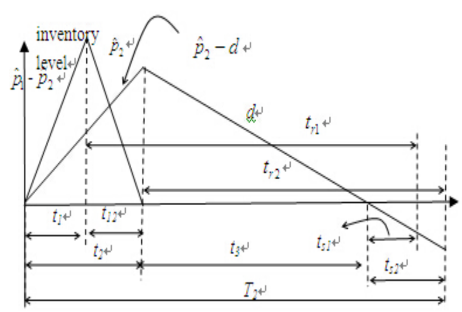

Figure 2 : Inventory level for $E(t_{r1}) < E(t_{r2})$.

FORMULATION OF THE MATHEMATICAL MODEL

We note that, in this paper, the learning effect on the per unit production time follows Wright's power function formula (1936) and can be expressed

$$T_{ij} = T_{i1} j^{-L} \quad \textbf{(1)}$$

Under the basic assumptions and notation presented in Section 2, we can formulate the two-stage imperfect production process with machine unavailability and learning effect as a cost-minimization problem. Specifically, the total cost for the two-stage imperfect process discussed in this paper includes production cost, inventory holding cost, maintenance cost, shortage cost and reworks cost. The formulations of these four costs are described in detail as follows.

The production cost (*PC*) per cycle is composed of the setup cost K_i, the material cost $C_i Q$ and labor cost $v_i t_i(Q)$. Hence,

$$PC(Q) = \sum_{i=1}^{2} (K_i + C_i Q + v_i t_i(Q)) \quad \textbf{(2)}$$

We note that $t_i(Q)$ is the cumulative time to produce Q units in stage i and $t_i(Q)$ can be obtained by summing up all of the production time in stage i.

Namely,

$$t_i(Q) = \frac{T_{i1} Q^{1-L_i}}{1-L_i}, i = 1, 2. \quad \textbf{(3)}$$

If Q is divided by $t_i(Q)$, then we can have the average production rate $\hat{p}_i$ for stage i as follows:

$$\hat{p}_i(Q) = \frac{1-L_i}{T_{i1}} Q^{L_i} \quad \textbf{(4)}$$

From Figure 1 and Figure 2, the following parameters can be obtained directly.

$$t_{12} = \frac{Q}{\hat{p}_2} - t_1(Q) \quad \textbf{(5)}$$

$$t_3 = \frac{Q}{d} - t_2(Q) \quad \textbf{(6)}$$

Now, we proceed to formulate the inventory holding cost each stage for our model.

The maximum inventory level during stage 1 is

$$(\hat{p}_1 - \hat{p}_2) t_1(Q). \quad \textbf{(7)}$$

Then, inventory holding cost in stage1 is

$$EHC_1 = \frac{1}{2} C_{h1} (\hat{p}_1 - \hat{p}_2) t_1(Q) \frac{Q}{\hat{p}_2} \quad \textbf{(8)}$$

Similarly, the maximum inventory level during stage 2 is equal to

$$(\hat{p}_2 - d) t_2(Q) \quad \textbf{(9)}$$

Then, inventory holding cost in stage 2 is

$$EHC_2 = \frac{1}{2}\frac{C_{h2}(\hat{p}_2 - d)t_2(Q)Q}{d} \quad \textbf{(10)}$$

Therefore, the total inventory holding cost (*EHC*) in two-stage is obtained as

$$EHC = \frac{1}{2}C_{h1}(\hat{p}_1 - \hat{p}_2)t_1(Q)\frac{Q}{\hat{p}_2} + \frac{1}{2}C_{h2}(\hat{p}_2 - d)t_2(Q)\frac{Q}{d} \quad \textbf{(11)}$$

Before formulating the rework cost for two stage process (ERC), we have to find out the expected number of non-conforming items two-stage production process. The exponentially distributed random variable τ_i denotes the elapsed time for the production process to shift to the 'out-of- control' state in two-stage production. The number of non-conforming items *N* can be expressed as follows:

$$\sum_{i=1}^{2} E[N_i] = \theta_2\hat{p}_1 t_1(Q) + (\theta_2 - \theta_1)\hat{p}_1\frac{1}{u_1}(1 - e^{-u_1 t_1(Q)}) + \theta_4\hat{p}_2 t_2(Q) + (\theta_4 - \theta_3)\hat{p}_2\frac{1}{u_2}(1 - e^{-u_2 t_2(Q)}) \quad \textbf{(12)}$$

Therefore rework cost can be obtained as follows:

$$ERC = R_{c1}\theta_2\hat{p}_1 t_1(Q) + R_{c1}(\theta_2 - \theta_1)\hat{p}_1\frac{1}{u_1}(1 - e^{-u_1 t_1(Q)}) + R_{c2}\theta_4\hat{p}_2 t_2(Q) + R_{c2}(\theta_4 - \theta_3)\hat{p}_2\frac{1}{u_2}(1 - e^{-u_2 t_2(Q)}) \quad \textbf{(13)}$$

From Figure.1, when the machine unavailability time t_{r1} for first stage is longer than the production down-time period t_{12}+ t_3, shortage occurs and the duration with shortage is t_{s1}; or when the machine unavailability time t_{r2} for second stage is longer than the production down-time period t_3, shortage takes place and shortage period is t_{s2}. Hence, comparing excepted shortage period $E[t_{s1}]$ and $E[t_{s2}]$, there are two cases to be discussed. Excepted shortage period $E[t_{s1}]$ and $E[t_{s2}]$ can be obtained as:

$$E(t_{s1}) = \frac{1}{\lambda_1}e^{-\lambda_1(\frac{Q}{d} - t_1(Q))} \quad \textbf{(14)}$$

And

$$E(t_{s2}) = \frac{1}{\lambda_2}e^{-\lambda_2(\frac{Q}{d} - t_2(Q))} \quad \textbf{(15)}$$

Case I : $E(t_{s1}) \geq E(t_{s2})$.

The shortage cost (ESC_1) for two-stage imperfect production system is dependent on $E(t_{s1})$, and can be obtained as

$$ESC_1 = \frac{1}{\lambda_1} C_s d e^{-\lambda_1 (\frac{Q}{d} - t_1(Q))} \quad \textbf{(16)}$$

The expected maintenance cost EMC can be expressed as

$$EMC = C_{01} \frac{1}{\lambda_1} + C_{02} \frac{1}{\lambda_2} . \quad \textbf{(17)}$$

By summing up the production cost, inventory holding cost, shortage cost maintenance cost and rework cost, we can have the following formulation for expected total cost:

$$\begin{aligned} ETC &= PC + EHC + ESC_1 + ERC + EMC \\ &= K_1 + K_2 + Q(C_{m1} + C_{m1}) + v_1 t_1(Q) + v_2 t_2(Q) + \frac{C_{01}}{\lambda_1} + \frac{C_{02}}{\lambda_2} + \frac{1}{\lambda_1} C_s d e^{-\lambda_1 (\frac{Q}{d} - t_1(Q))} \\ &\quad + \frac{1}{2} C_{h1} (\hat{p}_1 - \hat{p}_2) t_1(Q) \frac{Q}{\hat{p}_2} + \frac{1}{2} C_{h2} (\hat{p}_2 - d) t_2(Q) \frac{Q}{d} + R_{c1} \theta_2 \hat{p}_1 t_1(Q) \\ &\quad + R_{c1} (\theta_2 - \theta_1) \hat{p}_1 \frac{1}{u_1} (1 - e^{-u_1 t_1(Q)}) + R_{c2} \theta_4 \hat{p}_2 t_2(Q) + R_{c2} (\theta_4 - \theta_3) \hat{p}_2 \frac{1}{u_2} (1 - e^{-u_2 t_2(Q)}) \end{aligned} \quad \textbf{(18)}$$

The total replenishment time includes the positive inventory time $t_1 + t_2$ in stage 1, production down time t_3 in stage 2 and machine unavailability probability time in stage 1. The expected total replenishment time can be formulated as:

$$E(T_1) = \frac{Q}{d} + \frac{1}{\lambda_1} e^{-\lambda_1 (\frac{Q}{d} - t_1(Q))} \quad \textbf{(19)}$$

The corresponding total cost per unit time $ETCUT_1$, can be obtained from Eq (19) by dividing by the expected cycle length $E_1(T)$. Namely,

$$ETCUT_1(Q) = \frac{\left[\begin{aligned} & K_1 + K_2 + Q(C_1 + C_2) + \frac{v_1 T_{11} Q^{1-L_1}}{1-L_1} + \frac{v_2 T_{21} Q^{1-L_2}}{1-L_2} + C_{01} \frac{1}{\lambda_1} + C_{02} \frac{1}{\lambda_2} + (\frac{(1-L_1)Q^{L_1}}{T_{11}} - \frac{(1-L_2)Q^{L_2}}{T_{21}}) \frac{C_{h1} T_{11} T_{21} Q^{(2-L_1-L_2)}}{2(1-L_1)(1-L_2)} \\ & + \frac{1}{2} C_{h2} (\frac{Q^2}{d} - \frac{T_{21} Q^{2-L_2}}{(1-L_2)}) + \frac{1}{\lambda_1} C_s d e^{-\lambda_1 (\frac{Q}{d} - \frac{T_{11} Q^{1-L_1}}{1-L_1})} + R_{c1} \theta_2 Q + R_{c1} (\theta_1 - \theta_2) \frac{(1-L_1) Q^{L_1}}{T_{11} u_1} (1 - e^{-u_1 \frac{T_{11} Q^{1-L_1}}{1-L_1}}) \\ & + R_{c2} \theta_4 Q + R_{c2} (\theta_3 - \theta_4) \frac{(1-L_2) Q^{L_2}}{T_{21} u_2} (1 - e^{-u_2 \frac{T_{21} Q^{1-L_2}}{1-L_2}}) \end{aligned} \right]}{Q/d + 1/\lambda_1 e^{-\lambda_1 (Q/d - \frac{T_{11} Q^{1-L_1}}{(1-L_1)})}} \quad \textbf{(20)}$$

And $\hat{p}_1(Q) = \frac{1-L_1}{T_{11}} Q^{L_1}$, $\hat{p}_2(Q) = \frac{1-L_2}{T_{21}} Q^{L_2}$, $t_1 = \frac{T_{11} Q^{1-L_1}}{1-L_1}$,and $t_2 = \frac{T_{21} Q^{1-L_2}}{1-L_2}$.

The optimal Q value can be derived when the following equation is satisfied:

$$\frac{dETCUT_1(Q)}{dQ}=0 \quad \textbf{(21)}$$

Case II: $E(t_{s1})<E(t_{s2})$.

Similarly, the expected shortage cost ESC_2 for two-stage production system is obtained as

$$ESC_2=\frac{1}{\lambda_2}C_s de^{-\lambda_2(\frac{Q}{d}-t_2(Q))} \quad \textbf{(22)}$$

The total replenishment time includes the positive inventory time in stage 2 and machine unavailability probability time in stage 2. The expected total replenishment time can be formulated as:

$$E(T_2)=\frac{Q}{d}+\frac{1}{\lambda_2}e^{-\lambda_2(\frac{Q}{d}-t_2(Q))} \quad \textbf{(23)}$$

Similarly, the total expected cost per unit produced is

$$ETCUT_2(Q)=\frac{\begin{bmatrix} K_1+K_2+Q(C_{m1}+C_{m1})+v_1t_1(Q)+v_2t_2(Q)+\frac{C_{01}}{\lambda_1}+\frac{C_{02}}{\lambda_2}+\frac{1}{2}C_{h1}(\hat{p}_1-\hat{p}_2)t_1(Q)\frac{Q}{\hat{p}_2} \\ +\frac{1}{2}C_{h2}(\hat{p}_2-d)t_2(Q)\frac{Q}{d}+\frac{R_{c1}(\theta_2-\theta_1)\hat{p}_1(1-e^{-u_1t_1(Q)})}{u_1}+\frac{R_{c2}(\theta_4-\theta_3)\hat{p}_2(1-e^{-u_2t_2(Q)})}{u_2} \\ +R_{c1}\theta_2\hat{p}_1t_1(Q)+R_{c2}\theta_4\hat{p}_2t_2(Q)+\frac{1}{\lambda_2}C_s de^{-\lambda_2(\frac{Q}{d}-t_2(Q))} \end{bmatrix}}{Q/d+1/\lambda_2\, e^{-\lambda_2(\frac{Q}{d}-t_2(Q))}} \quad \textbf{(24)}$$

The optimal Q value can be calculated when the following equation is satisfied:

$$\frac{dETCUT_2(Q)}{dQ}=0 \quad \textbf{(25)}$$

The optimal Q value can be obtained by solving Equation (21) and (25), using the bisection or Newton's search method. The cost functions in Equations (20), and (24) are nonlinear equations, and taking the second derivative with respect to Q is extremely complicated. This means the optimal solution cannot be guaranteed. However, using empirical experiments, one can indicate that Equations (20) and (24) are convex for a small value of Q. Here we generate a graph of the function for some parametric values. The graph is shown in Figure 3, which has a clear minimum point.

NUMERICAL EXAMPLE

In this section, we provide two numerical examples to illustrate the features of the proposed model. The values of the parameters are shown in TABLE 1.

TABLE 1 : The values of the parameters.

K_1	**700**	P_2	**60**	C_{01}	**10**	C_{h2}	**0.2**
K_2	500	C_s	0.5	C_{02}	10	θ_1	0.05
C_1	1	R_{c1}	0.2	u_1	0.1	θ_2	0.65
C_2	2	R_{c2}	0.5	u_2	0.2	θ_3	0.05
D	50	C_{h1}	0.2	P_1	70	θ_4	0.65

Example 1. In order to illustrate the above Case I, let λ_1=0.01 and λ_2=0.12, it is obvious $E(t_{s1}) > E(t_{s2})$. Then we can obtain the optimal solution Q^*=1801. The corresponding $t_1(Q^*)$, $t_2(Q^*)$, $\hat{p}_1(Q^*)$, $\hat{p}_2(Q^*)$ and $ETCUT_1(Q^*)$ are 9.456269, 18.91254, 190.46, 95.23and 371.8 respectively. This solution is a global minimum (see Figure 3, $\beta = 93\%$). On the other hand, without learning consideration (i.e., L_1=0 and L_2=0), the optimal solutions of Q^* $t_1(Q^*)$, $t_2(Q^*)$, $\hat{p}_1(Q^*)$, $\hat{p}_2(Q^*)$ and $ETCUT_1(Q^*)$ are 1698, 16.98, 33.96, 100, 50 and 424.8 respectively. It can be found that the effects of learning are evident. This comparison reveals a decrease in the optimal expected total cost of about 12.47%((371.8-424.8)/ 432.4×100%).

Example 2. We consider the Case II, and let λ_1=0.12 and λ_2=0.01, it is easy to obtain $E(t_{s1}) < E(t_{s2})$. The optimal solutions of Q^* $t_1(Q^*)$, $t_2(Q^*)$, $\hat{p}_1(Q^*)$, $\hat{p}_2(Q^*)$ and $ETCUT_2(Q^*)$ are 1765, 9.28, 18.57, 190.07, 95.03 and 375.9 respectively. This solution is a global minimum. If L_1=0 and L_2=0, one obtains the optimal solutions of Q^*=1771, $t_1(Q^*)$=17.71, $t_2(Q^*)$=33.96, $\hat{p}_1(Q^*)$=100, $\hat{p}_2(Q^*)$=50 and $ETCUT_2(Q^*)$=432.4 respectively. The percentage change in expected total cost is about 13.06%((375.9-432.4)/ 432.4×100%). From this comparison, we note that the learning effect may have a significant impact on the results of the proposed problem. If this cost difference percentage is ignored, the production system maybe leads to a high percentage error in costs.

CONCLUSION

In this paper, we investigate the learning effect of the unit production time for a two-stage imperfect production system with machine unavailability. Two cases for shortage incurred by different machine unavailability have been discussed. We minimize the expected total cost of the production system through optimal determination of the production quantity. Numerical examples are provided to illustrate the features of the model and sensitivity analyses are performed to examine the impact of the key parameters' changes on the decision variable and objective function. The expected total cost is highly sensitive with respect to demand rate, learning rate, shortage cost, rework cost, and the mean time of machine preventive maintenance. The optimal quantity is slightly sensitive to demand rate, learning rate, shortage cost and the mean time of machine maintenance. The optimal production time is slightly sensitive to demand rate, shortage cost, learning rate, the time to produce the first unit and the mean time of machine maintenance. In practice, management should to improve production system reliability, avoid shortage, low rework cost, reduce the time to produce the first unit and the mean time of machine maintenance. It is also found that ignoring the effects of learning may result in a high percentage of error in costs. Further research can also be done to consider deteriorating items and random machine breakdown in same model.

ACKNOWLEDGMENTS

This work was supportedby Technology Project of Jiangxi Education Department under grant numbers (GJJ14232).

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