DESIGN OF PI CONTROLLERS FOR MIMO SYSTEM WITH DECOUPLERS

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ABSTRACT

In this paper, Proportional Integral (PI) Controllers are designed for unstable Multi Input Multi Output system with delay based on the equivalent transfer function (ETF) model. Decoupling method is proposed for multivariable processes. The simplified decoupler is used to eliminate the process interactions between the controlled and manipulated variables. PI controllers are designed for the diagonal elements of ETFs by synthesis method meant for unstable first order plus time delay (FOPTD) systems to give satisfactory performances. Since the unstable system has large overshoot. To reduce the overshoot, a double loop PI controller is designed.

Key words: Two input two output control, Unstable system, PI controllers, Simplified decoupler, Robustness, Double loop control.

INTRODUCTION

Multi-loop proportional integral (PI) controllers, have been widely used for processes with small interactions because of their many practical advantages such as their simple control structure and fewer tuning parameters. Basically design of Multi input multi output (MIMO) controllers is more complex than single input single output (SISO) controllers because of the loop interactions. The MIMO process can be controlled by decoupled controllers. Unstable SISO systems are more difficult to control than that of the stable systems.

Control of unstable systems is given by Padmasree and Chidambaram[1]. There are several methods to design the unstable SISO Proportional Integral controllers. Luyben et al.[2] have given the design of decentralized PI controllers for a stable MIMO system. A comparative study of some multivariable PI controller tuning methods for stable systems given by Tanttu et al.[3] It requires the transfer function matrix. Only a few methods are

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available for unstable multivariable processes. Govindhakannan and Chidambaram[4] have given the method of designing multivariable PI controllers for unstable system. Decoupling is used to reduce the control loop interactions. Ideal, simplified and inverted are the three types of decupling techniques.

Ideal decoupling has the complicated decoupler elements. It is rarely used in practice, greatly facilitates the tuning of the controller transfer matrix. Simplified decoupling is by far the most popular method. Its main advantage is simplicity of its elements. Inverted decoupling, which is also rarely implemented. Ideal and inverted decoupling is sensitive to the modelling error.

The decentralized controller works well when the interaction among the loop is not large. If there is more interaction it won't give acceptable response. The decentralized PI controllers do not stabilize the system if the unstable pole is present in each of the transfer function. Centralized controller reduces the interaction better than the decentralized controller. Kumar et al.[5] have given a synthesis method to centralized PI controller for interacting multivariable processes. Design of double loop PI controllers for the unstable systems based on the Tanttu and Lieslehto method was given by Govindhakannan and Chidambaram[6]. The concept of equivalent transfer function/effective open-loop transfer functions (ETFs/EOTFs) to design the multi-loop control system[7,8]. Recently Rajapandiyan and Chidambaram[9] have given the simple decoupled equivalent transfer function method to Designing controller for MIMO processes. This method is extended to unstable multivariable system by Hazarika and Chidambaram[10]. However, assumption involved in this method is that decoupler should be stable. Simplified decoupler method gives the less interactions and better performances when compared to the ideal and inverted decoupling methods.

Methodology

Design based on ETF model

Consider the 2 × 2 control problem shown in Fig. 1. Because to completely characterize the process dynamics, there are two controlled variables and two manipulated variables, four process transfer functions are necessary.

$$\frac{y_1(s)}{u_1(s)} = G_{p,11}(s) \qquad \frac{y_1(s)}{u_2(s)} = G_{p,12}(s)$$

$$\frac{y_2(s)}{u_1(s)} = G_{p,21}(s) \qquad \frac{y_2(s)}{u_2(s)} = G_{p,22}(s) \qquad \qquad \dots(1)$$
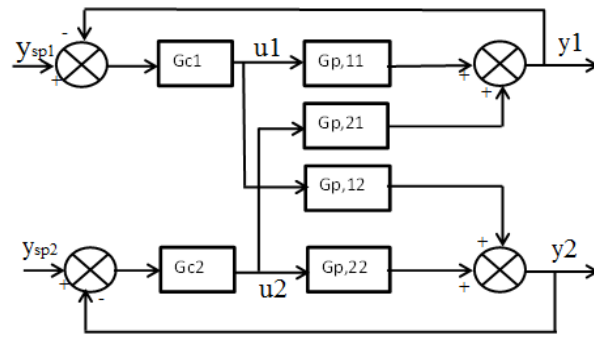
Fig. 1: Block diagram for TITO system

The transfer function in eq (1) can be used to determine the effect of a change in either u_1 or u_2 on y_1 and y_2. Simultaneous change in u_1 and u_2 have an additive effect on each controlled variable by the principle of superposition.

$$y_1(s) = G_{p11}(s)u_1(s) + G_{p12}(s)u_2(s) \qquad \qquad …(2)$$

$$y_2(s) = G_{p21}(s)u_1(s) + G_{p22}(s)u_2(s) \qquad \qquad …(3)$$

These input-output relations can also be expressed in vector-matrix notation as –

$$y(s) = G_p(s)u(s) \qquad \qquad …(4)$$

Where y(s) and u(s) are vectors with two elements

$$y(s) = \begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix}; \ u(s) = \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \qquad \qquad …(5)$$

G_p (s) is the process transfer function matrix,

$$G_p(s) = \begin{bmatrix} G_{p11}(s) & G_{p12}(s) \\ G_{p21}(s) & G_{p22}(s) \end{bmatrix} \qquad \qquad …(6)$$

The process transfer function models are expressed as FOPDT models

$$G_{ij}(s) = \frac{k_{pij}e^{-\theta_{ij}s}}{(\tau_{ij}s + 1)} \qquad \qquad …(7)$$

i = 1, 2; j = 1, 2

If the second feedback controller is the automatic mode, with $y_{r2} = 0$, then the closed-loop transfer function between y_1 and u_1.

$$\frac{y_1}{u_1} = g_{p11} - \frac{g_{p12}\, g_{c2}\, g_{p21}}{1 + g_{c2}\, g_{p22}} \qquad \ldots(8)$$

This can be written as –

$$\frac{y_1}{u_1} = g_{p11} - \frac{g_{p12}\, g_{p21}\left(g_{c2}\, g_{p22}\right)}{g_{p22}\left(1 + g_{c2}\, g_{p22}\right)} \qquad \ldots(9)$$

Similarly for the second loop,

$$\frac{y_2}{u_2} = g_{p22} - \frac{g_{p12}\, g_{p21}\left(g_{c1}\, g_{p11}\right)}{g_{p11}\left(1 + g_{c1}\, g_{p11}\right)} \qquad \ldots(10)$$

The complicated relation of eqs (9) and (10) can be simplified by assuming two assumptions: First, the perfect controller approximation for the other loop was used to simplify the eqs (9), (10) that is

$$\frac{g_{ci}\, g_{ii}}{\left(1 + g_{ci}\, g_{ii}\right)} = 1 \qquad \ldots(11)$$

$i = 1, 2$

Second, ETFs have the same structure of the corresponding open-loop model. By using the perfect controller approximation, eqs (9) and (10) can be approximated as follows:

$$g_{p11}{}^{eff} = \frac{y_1}{u_1} = g_{p11} - \frac{g_{p12}\, g_{p21}}{g_{p22}} \qquad \ldots(12)$$

$$g_{p22}{}^{eff} = \frac{y_2}{u_2} = g_{p22} - \frac{g_{p12}\, g_{p21}}{g_{p11}} \qquad \ldots(13)$$

Here, $g_{p11}{}^{eff}$ and $g_{p22}{}^{eff}$ are the effective open-loop transfer functions (EOTF). The formulation of EOTFs for the higher dimension systems is difficult. On the other, the derivation of ETFs is easy for higher dimension systems.

The basic steps required to obtain ETF for a TITO system are as follows:-

To describe the dynamic properties of a transfer function, the normalized gain (k_{Nij}) for a particular transfer function, $g_{ij}(s)$ is defined as –

$$k_{Nij} = \frac{k_{ij}}{\sigma_{ij}} = \frac{k_{ij}}{\tau_{ij} + \theta_{ij}} \qquad \text{…(14)}$$

Normalized gain matrix is expressed as –

$$k_N = K\Theta T_{ar} = \begin{bmatrix} k_{N11} & k_{N12} \\ k_{N21} & k_{N22} \end{bmatrix} = \begin{bmatrix} \dfrac{k_{11}}{\tau_{11} + \theta_{11}} & \dfrac{k_{12}}{\tau_{12} + \theta_{12}} \\ \dfrac{k_{21}}{\tau_{21} + \theta_{21}} & \dfrac{k_{22}}{\tau_{22} + \theta_{22}} \end{bmatrix} \qquad \text{…(15)}$$

Where Θ denotes Hadamard division, k is the steady state gain, and $T_{ar} = \tau_{ij} + \theta_{ij}$ is defined as the average residence time which signifies the response speed of the controlled variable y_i to manipulated variable u_j.

Hence, the RNGA (denoted as ϕ) can be obtained as –

$$\phi = k_N \otimes k_N^{-T} \qquad \text{…(16)}$$

Where

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \qquad \text{…(17)}$$

Where $\otimes$ denotes Hadamard multiplication. Relative average residence time array (RARTA), which is defined as the ratio of loop $y_i - u_j$ average residence times, when other loops are closed and when other loops are open, is given by

$$\Gamma = \phi \,\Theta\, \Lambda = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \qquad \text{…(18)}$$

Hence for an unstable system, the ETF can be expressed as –

$$\hat{g}_{pij}(s) = \hat{k}_{pij} \frac{e^{-\hat{\theta}_{ij}s}}{\hat{\tau}_{ij}s - 1} \qquad \text{…(19)}$$

$$\hat{k}_{pij} = {k_{pij}}\Big/{\Lambda_{ij}}, \quad \hat{\tau}_{ij} = \gamma_{ij}\tau_{ij}, \text{ and } \hat{\theta}_{ij} = \gamma_{ij}\theta_{ij}$$

It should be noted that the method is applicable when EOTF = ETF.

Pairing criteria

For multivariable controllers, paring is required. The problem of loop interaction can be minimized by a proper choice of input output pairings. If there is 'n' input and 'n' output in the control system, then n! ways of pairing the controlled and manipulated variables will be there. Some of these control configurations would be immediately rejected as being impractical and unworkable. Bristol's relative gain array method. It is a systematic approach. Relative Gain Array is a measure of process interactions.

$$RGA(\Lambda) = k \otimes k^{-T} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \qquad \qquad \dots(20)$$

Where k is the steady state gain.

Design of controller

Fig. 2 constituting of input variable u, output variable y, process gain g_p, decoupler element d and controller gain g_c can be modelled as follows:

The relationship between the input and process output is given by –
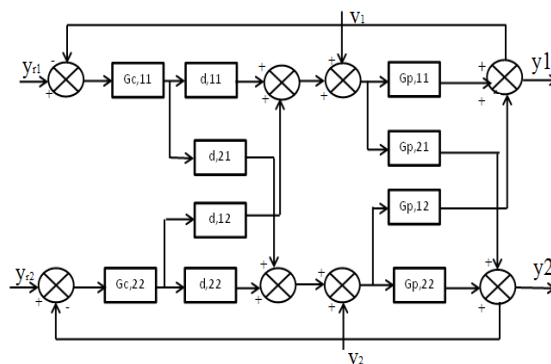
$$y(s) = g(s)d(s)u(s) \qquad \qquad \dots(21)$$

Fig. 2: TITO process with simplified decoupler ($d_{11} = 1$; $d_{22} = 1$)

The output of the TITO system with decoupler is –

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} 1 & d_{12} \\ d_{21} & 1 \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}$$

$$g(s)d(s) = \begin{bmatrix} g_{11}^* & 0 \\ 0 & g_{22}^* \end{bmatrix} \qquad \qquad …(22)$$

Design of simplified decoupler,

$$d_{11} = 1; \; d_{22} = 1$$

$$d_{12}(s) = -\frac{g_{p12}(s)}{g_{p11}(s)}; \quad d_{21}(s) = -\frac{g_{p21}(s)}{g_{p22}(s)}$$

Design of double loop control structure to reduce the overshoot

G_p, process gain; G_d, decoupler element; $G_{c,I}$ inner-loop diagonal proportional controllers; $G_{c,O}$ outer-loop diagonal proportional integral controllers; y, output variable; y_r set point values; v, load variable.
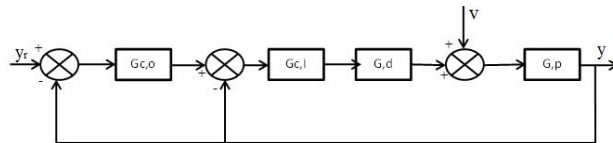

Fig. 3: Block diagram for the double loop control scheme

Design of inner loop diagonal P controller –

$$k_{pii}k_{cii} = \frac{1}{\xi_{ii}^{0.5}} \qquad \qquad …(23)$$

Design of outer loop diagonal PI controller –

$$G_{pij}' = \frac{k_{pij}' e^{-\theta_{ii}' s}}{\tau_{ii}' s + 1} \qquad \qquad …(24)$$

Here k_{pii}' is the steady state value reached, θ_{ii}' is the initial value noted, and $\tau_{ii}' = t - \theta_{ii}'$, where t is the time taken to $(0.62\ k_{pii}')$ from figure of proportional controller.

Design of outer loop diagonal PI controller –

$$k_{cii} = \frac{1}{k_{pii}'}\left(\frac{\tau_{ii}'}{\tau_{cii}} + \theta_{ii}\right) \approx \frac{0.5}{k_{pii}'} \quad \tau_{I,ii} = \tau_{ii}' \qquad \ldots(25)$$

Based on these values diagonal PI controller matrix for the outer loop can be obtained.

Synthesis method

In direct synthesis (DS) method, the controller design is based on a process model. This method is used achieve the desired closed loop transfer function for MIMO processes with multiple time delays.

$$g_{c,ii}(s) = k_{c,ii}\left(1 + \frac{1}{\tau_{I,ii}(s)}\right) \qquad \ldots(26)$$

$k_{c,ii}\ k_{p,ii} = 0.8668\ \xi_{ii}^{-0.8288}$ for $0.1 \leq \xi_{ii} \leq 0.7$

$\dfrac{\tau_{I,ii}}{\tau_{ii}} = 0.1523\ e^{7.9425\ \xi_{ii}}$ for $1 \leq \xi_{ii} \leq 0.7$

Where $\xi_{ij} = {\theta_{ii}}\big/{\tau_{ii}}$

Simulation example

Let us consider the transfer function matrix and it is expressed as –

$$G_p(s) = \begin{bmatrix} \dfrac{-2.2e^{-s}}{(7s+1)} & \dfrac{1.3e^{-0.3s}}{(7s+1)} \\ \dfrac{-2.8e^{-1.8s}}{(9.5s+1)} & \dfrac{4.3e^{-0.35s}}{(9.2s+1)} \end{bmatrix} \qquad \ldots(27)$$

From the given transfer function matrix, steady state gain,

$$k = \begin{bmatrix} -2.2 & 1.3 \\ -2.8 & 4.3 \end{bmatrix}$$

Relative gain array,

$$RGA(\Lambda) = \begin{bmatrix} 1.6254 & -0.6254 \\ -0.6254 & 1.6254 \end{bmatrix}$$

Average residence time,

$$T_{ar} = \begin{bmatrix} 8 & 7.3 \\ 11.3 & 9.55 \end{bmatrix}$$

Normalized gain,

$$k_N = \begin{bmatrix} -0.275 & 0.1781 \\ -0.2478 & 0.4503 \end{bmatrix}$$

Relative normalized gain array,

$$\phi = \begin{bmatrix} 1.5537 & -0.5537 \\ -0.5537 & 1.5537 \end{bmatrix}$$

Relative average residence time array,

$$\Gamma = \begin{bmatrix} 0.9559 & 0.8853 \\ 0.8853 & 0.9559 \end{bmatrix}$$

The ETF model matrix is given by

$$\hat{g}_{pij}(s) = \begin{bmatrix} \dfrac{-1.3535e^{-0.9559}}{6.6910s-1} & \dfrac{-2.0786e^{-0.2656}}{6.1971s-1} \\ \dfrac{4.4769e^{-1.5935}}{8.4103s-1} & \dfrac{2.6455e^{-0.3345}}{8.7940s-1} \end{bmatrix} \quad \dots(28)$$

The simplified decoupler matrix is given by –

$$d(s) = \begin{bmatrix} 1 & 0.5909\,e^{0.7s} \\ \dfrac{(5.0910s + 5.9910)e^{-1.45s}}{(9.5s+1)} & 1 \end{bmatrix} \qquad \text{...(29)}$$

The diagonal elements of PI controller is obtained by synthesis method is given by –

$$G_c(s) = \begin{bmatrix} -1.977\left(1+\dfrac{1}{3.3141s}\right) & 0 \\ 0 & 3.0305\left(1+\dfrac{1}{1.8948s}\right) \end{bmatrix} \qquad \text{...(30)}$$

Effective open-loop transfer function (EOTF) diagonal elements is obtained as

$$g_{11}^{\ eff} = \frac{y_1}{u_1} = \frac{-2.2e^{-s}}{(7s+1)} + \frac{(7.7878s + 0.8465)e^{-1.75s}}{66.5s^2 + 16.5s + 1}$$

$$g_{22}^{\ eff} = \frac{y_2}{u_2} = \frac{4.3e^{-0.35s}}{(9.2s+1)} - \frac{1.65e^{-1.1s}}{(9.5s+1)} \qquad \text{...(31)}$$

$g_{11}^{\ eff}$, $g_{22}^{\ eff}$ are the diagonal elements of EOTF. The closed loop response of ETF is same as EOTF.
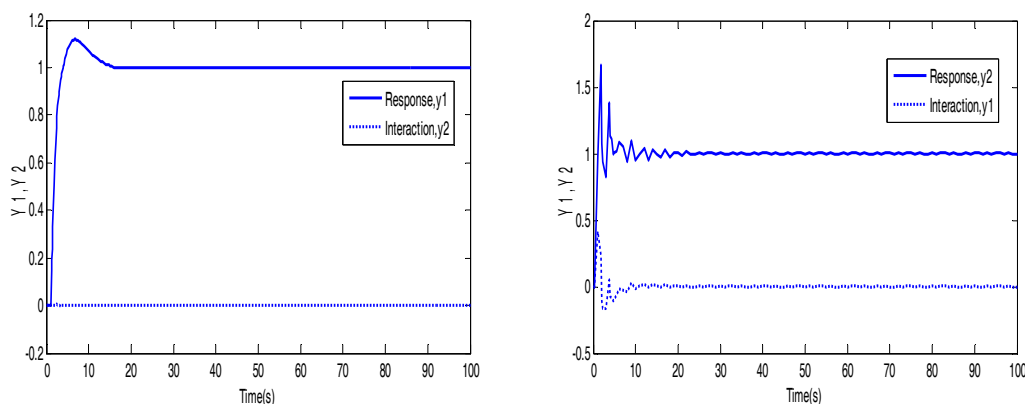

Fig. 4: Response of the PI controllers for controlled variables with interaction separately (y₁, y₂)
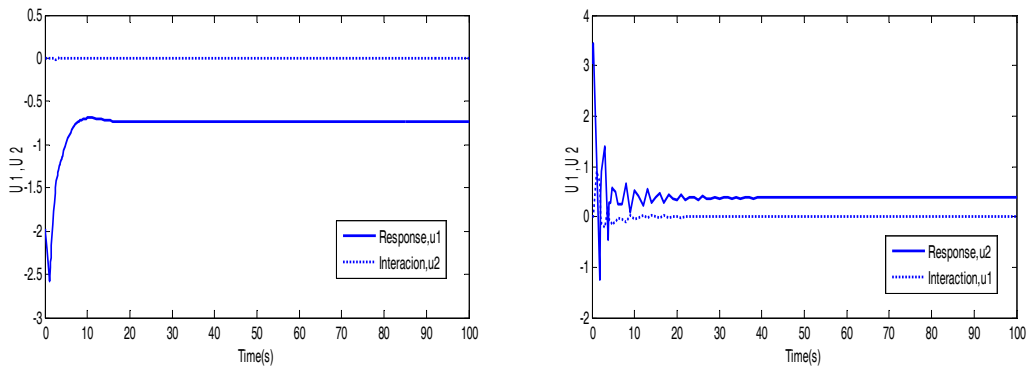
Fig. 5: Response of the PI controllers for manipulated variables with interaction separately (u₁, u₂)

To reduce the overshoot dual loop control scheme is used. Here, the inner loop proportional controller tuning parameters are $k_{c,11} = -1.2028$ and $k_{c,22} = 3.0305$. The fitted parameter are $k_{p,11}' = 0.6195$, $\tau_{11}' = 2.5$, $k_{p,22}' = 0.9999$, $\tau_{22}' = 2.5$ and $\theta_{11}' = \theta_{22}' = 1$. Which is calculated from the response of P controller with decoupler. For the stabilized FOPTD system, diagonal PI controllers by the synthesis method and placed in the outer loop.
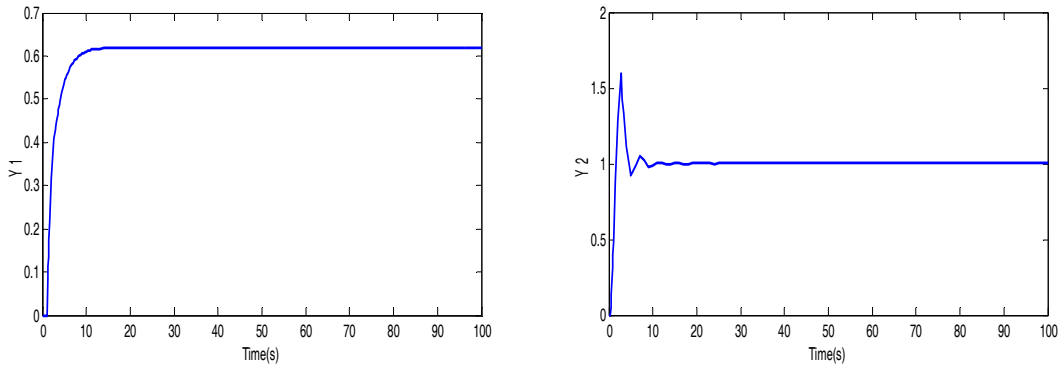

Fig. 6: Response of proportional controllers with decoupler

Hence the outer loop diagonal PI controller matrix is given by –

$$G_c(s) = \begin{bmatrix} 0.8071\left(1+\dfrac{1}{2.5s}\right) & 0 \\ 0 & 0.5005\left(1+\dfrac{1}{2.5s}\right) \end{bmatrix} \qquad \text{…(32)}$$

Fig. 7 shows that the response of the double-loop controller is much superior to that of a single loop PI controller. Fig. 8 shows the responses of single and double-loop controllers when the load variables v1 and v2 enter the system along with the inputs u1 and u2.
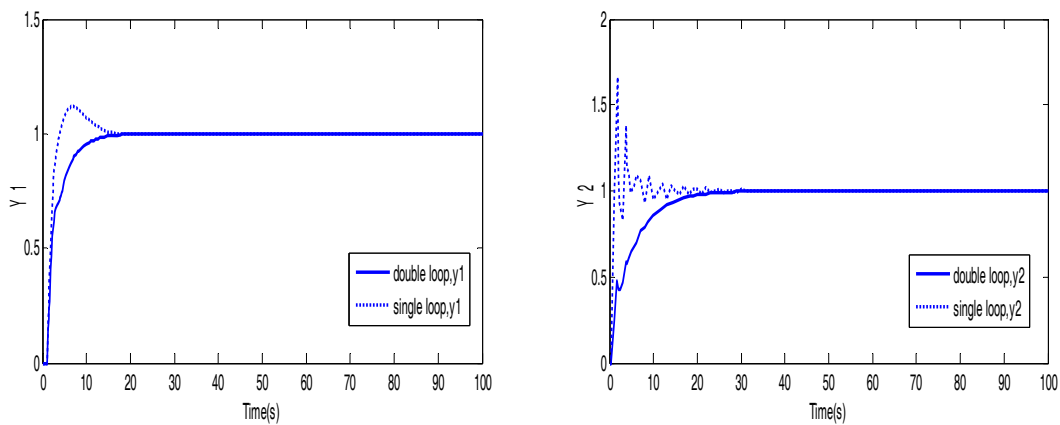

Fig. 7: Response of single-loop and double–loop two input two output PI controllers with decoupler for servo problems
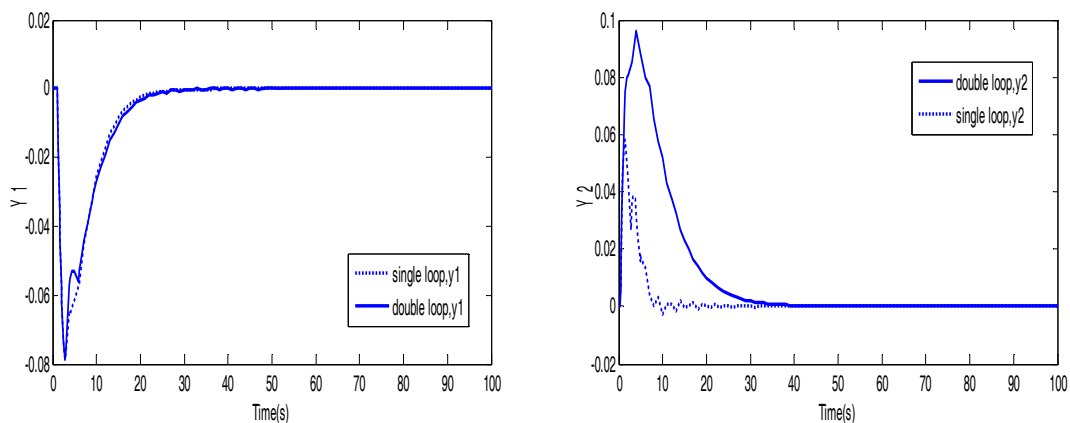

Fig. 8: Response of single-loop and double–loop two input two output PI controllers with the decoupler for regulatory problems; (left) v1 = 0.2, v2 = 0; (right) v1 = 0, v2 = 0.2

The robustness of the double-loop system is evaluated by disturbing each process gain, time delay (θ) and time constant (τ), $\pm$ 10% of the normal value in process. The robustness performance is shown in Fig. 9. Table 1 shows the IAE/ISE values for the above robustness studies for the servo and regulatory problems.
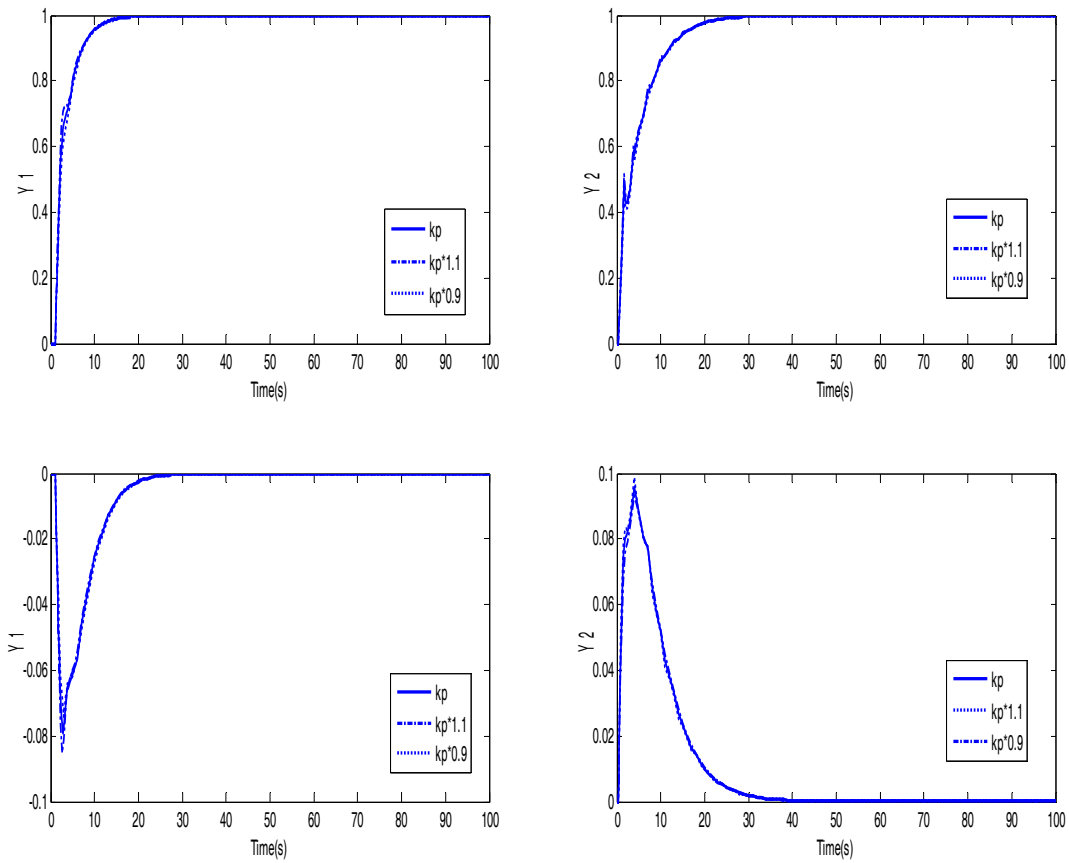
Fig. 9: Robustness of double-loop controllers for perturbation in each k_p (10%) in the process: (top panels) servo problem; (bottom panels) regulatory problem

Table 1: Comparison of IAE and ISE values for disturbance in the model parameters for the double-loop control system

Perturbation	Servo (y1)		Servo (y2)		Regulatory (y1)		Regulatory (y2)	
	IAE	ISE	IAE	ISE	IAE	ISE	IAE	ISE
k_p	3.39	1.987	4.939	2.29	0.5676	0.0234	1.022	0.0548
1.1 k_p	3.273	1.89	4.831	2.195	0.5676	0.0276	1.022	0.0631
0.9 k_p	3.533	2.106	5.071	2.402	0.5677	0.0262	1.022	0.0542
θ	3.39	1.987	4.939	2.29	0.5676	0.0234	1.022	0.0548

Cont…

Perturbation	Servo (y1)		Servo (y2)		Regulatory (y1)		Regulatory (y2)	
	IAE	**ISE**	**IAE**	**ISE**	**IAE**	**ISE**	**IAE**	**ISE**
1.1 θ	3.39	2.046	4.939	2.289	0.5637	0.0243	1.022	0.0552
0.9 θ	3.39	1.935	4.939	2.294	0.5691	0.0231	1.022	0.0566
τ	3.39	1.987	4.939	2.29	0.5676	0.0234	1.022	0.0548
1.1 τ	3.39	2.061	4.939	2.353	0.5801	0.0234	1.022	0.0558
0.9 τ	3.39	1.915	4.939	2.226	0.5746	0.0238	1.093	0.0581

CONCLUSION

Based on the equivalent transfer function (ETF) model, multivariable PI controllers are designed for unstable multivariable systems with time delay. The simplified decoupler reduces the loop interaction. The advantage of double-loop method is used to reduce the overshoots and eliminate the interactions.

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