Int. J. Chem. Sci.: 8(4), 2010, 2089-2098

BINDING ENERGIES ESTIMATION OF CONDENSED MATTER IN SUPERSTRONG MAGNETIC FIELD WITHOUT INCLUDING EXCHANGE ENERGY TERM

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ABSTRACT

Binding energies of hydrogen, helium, carbon and oxygen matter were evaluated without taking exchange energy term in a superstrong magnetic field. The evaluation has been performed by theoretical formalism of Skjervold and ϕ Stgaard using three adjustable parameters, η, R(a_0) and l(a_0). Our results are in good agreement with those of the other workers.

Key words: Magnetic distorted atoms, Quadrupole–quadrupole interaction, Neutron star, Exchange energy term.

INTRODUCTION

The behaviour of atoms in superstrong magnetic field of the order of 10^{12}–10^{13} G has been investigated.[1] Such magnetic fields are assumed to exist in neutron stars.[2] Since the magnetic fields are stronger than the coulomb's forces between electrons and atomic nuclei; therefore, the properties of matter at the surface of the neutron star will be very different from those of ordinary matter. This is because of the fact that the energies associated with the magnetic motion of electrons become much larger than the Coulomb's energies.

If one considers the interaction between two magnetically distorted atoms in superstrong magnetic fields, the cylindrical shapes give rise to enormous electric quadrupole moments and the highly directional quadrupole–quadrupole interaction is extremely stronger and favour bonding along the cylinder axis in the direction of magnetic fields. The resulting molecule is extremely tightly bound and a series of atoms is expected to stick together like a tent pole to form a polymer chain. This chain will be much more stable than an isolated atom.

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One polymer chain will attract a neighbouring one even in the absence of Vander Waal's force because of the nature of the quadrupole–quadrupole interaction. The equilibrium configuration will then be a close–packed lattice that would be millions times stronger than terrestrial steel.

The properties of matter in superstrong magnetic fields have been studied by several workers.[3–13] Atoms are, for instance, treated in the Thomas–Fermi or Hartree–Fock approximation. Then these are coupled into long molecular chains and possibly aggregated to form a solid crust in a neutron star. Ruderman[4] first presented a variational calculation specifying the structure of linear chains of nuclei and surrounding non–interacting electrons, which should form the basic component of the neutron star crust. Glasser and Kaplan[9] also calculated the various contributions to the total energy of a linear chain of nuclei surrounded by an axisymmetric distribution of electrons.

Flowers *et al.*[11] then carried out the most detailed calculations of the binding energy of heavy atoms, such as iron and corresponding condensed matter, in superstrong magnetic field by a variational method. Lai and Salpeter[14] studied the energy levels of H_2 molecules in a superstrong magnetic field ($B \geq 10^{12}$ G), typically found on the surfaces of neutron stars. The interatomic interaction potentials were calculated by a Hartree–Fock method with multiconfigurations assuming that the electrons are in the ground Landau state. Both the aligned configurations and arbitrary orientations of the molecular axis with respect to the magnetic field axis were considered. Different types of molecular excitation were then studied. Electronic excitations, aligned (along the magnetic axis) vibrational excitation and transverse vibrational excitation (a constrained rotation of the molecular axis around the magnetic field line). Similar results for molecular ion were also obtained and compared with vibrational calculations.[15-19] Both numerical results and analytical fitting formula were obtained for a wide range of field strength. In contrast to the zero field case, it was found that the transverse vibrational excitation energies were larger than the vibration excitation and both were larger than the electronic excitation. Lai[20] reported that the properties of the matter are drastically modified by strong magnetic fields ($B\ 2.35 \times 10^9$G). In such strong magnetic fields, the Coulomb force on the electron acts as a small perturbation compared to magnetic force. The strong field condition can also be mimicked in laboratory semiconductor. Because of the strong magnetic confinement of electrons perpendicular to the field, atom attains a much greater binding energy compared to the zero–field case and various other bound states becomes possible including molecular chains and three–dimensional condensed matter.

In this paper, we have evaluated the binding energies of hydrogen, helium, carbon and oxygen matter in the presence of superstrong magnetic field without including the

exchange energy term. Our evaluation indicates that binding energies of all the four matter increases with the increase of the magnetic field strength B. We have taken atomic dimensions as η, R (a_0) and *l* (a_0) in the evaluation. The value of η increases with B, whereas the value of R (a_0) and *l* (a_0) both decreases with the increase of B. These atomic dimensions give the values of the lattice spacing or distance between the nuclei in the chain. For 10^{12} G, the values of binding energy are 0.16 eV, 0.56 eV, 4.0 eV and 6.7 eV for hydrogen, helium, carbon and oxygen matter. These results are in good agreement with the earlier work done by Müller[13] and Ruderman[4].

Mathematical methods used in the evaluation

One now wants to investigate the behaviour of condensed matter in superstrong magnetic fields. One considers two regions -

$$\eta = 10^{-5} (B/Z^3)^{1/2} > 1 \qquad ...(1)$$

which is most likely for condensed matter of light atoms and

$$0.1 < \eta < 1 \qquad ...(2)$$

which is most likely for condensed matter of heavier atoms in neutron stars. One assumes that a magnetic field B in the Z–direction will dominate in the plane perpendicular to the field.

$$\text{i.e. } B = B\hat{\rho} \qquad ...(3)$$

The electrons will correspondingly occupy Landau levels of orbital radius.

$$\rho M = (M + 1/2)^{1/2} \hat{\rho} \qquad ...(4)$$

where

$$\hat{\rho} = (2\hbar c/e_B)^{1/2} \qquad ...(5)$$

is the cyclotron radius. (M = 0, 1, 2 .....)

The normalized electron wave functions can be assumed to be separable in circular cylindrical coordinates. i.e.

$$\phi_{KM} (\rho, z, \phi) = (\pi LM! \hat{\rho}^2)^{1/2} (\hat{\rho}^2/\hat{\rho})^{M/2} \exp(-\rho^2/2\hat{\rho}^2) \exp(ikz) \exp(-iM\phi) \qquad ...(6)$$

and

$$\int_0^{\infty} \rho d\rho \int_{-L/2}^{L/2} dz \int_0^{2x} \phi_{KM}^2 \phi_{KM} d\phi = 1 \qquad ...(7)$$

where L is the length of the system in the z–direction of the field, i.e. we assume

$$L \to \infty \text{ for } B \to \infty \qquad ...(8)$$

In region (1), the electrons occupy Landau orbital, where the outer orbital has the radius[10,13]

$$R = (M_0 + 1/2)^{1/2}\ \hat{\rho} = M_0^{1/2}\hat{\rho} \qquad ...(9)$$

where

$$M_0 = (R/\hat{\rho})^2 \qquad ...(10)$$

One now considers a one dimensional chain of atoms in the direction of magnetic field, i.e. a linear chain of nuclear charges, where L is the length of the chain and *l* is lattice spacing or distance between the nuclei. The electrons are assumed to be distributed in such a way that one has cylindrical symmetry and one dimensional Fermi gas in Z–direction of the magnetic field. The number of electrons needed to keep charge conservation (neutrality) is (Lz/*l*), the density of momentum states (L/2π) and the volume of the one dimensional Fermi sphere is $2K_F$ i.e.

$$2K_F = M_0 (L/2\pi) = L_z/l \qquad ...(11)$$

or

$$K_F = \pi z / lM_0 \qquad ...(12)$$

This gives a charge density -

$$\sigma = Z/M_0 = K_F l/ \pi \qquad ...(13)$$

which defines the number of electrons per Landau levels, i.e.

$$\sum_{M_0=0}^{M_0=1} \sigma_M = z \qquad ...(14)$$

The energy of the system can be written as -

$$E = E_F + E_{+-} + E_{++} + E_{--} + E_{ex} \quad ...(15)$$

where E_F is the kinetic energy of the Fermi gas, E_{ij} is the potential energy because of interaction between two particles (charges) i and j; and E_{ex} is the exchange term in the electron–electron interaction energy. The total energy E then depends on two parameters *l* and M_0 (or R), when one assumes $L \to \infty$ and one wants to minimize the energy.

The assumption of one–dimensional Fermi gas gives a one dimensional kinetic energy, i.e.

$$E_F = \sum_{\hbar k} \frac{\hbar k^2}{2m}$$

$$(L/2\pi) M_0 \int_{-K_F}^{K_F} \left(\frac{h^2 k^2}{2m} \right) dk$$

$$= L\hbar^2 K_F^3 M_0/6\pi m$$

$$= (Z^3\pi^2\hbar^2 h^2/6mF)\,(\hat{\rho}/R)^4\,(L/\lambda) \quad \ldots(15)$$

Introducing dimensionless variables -

$$\left.\begin{aligned} \lambda &= L/\hat{\rho} \\ K &= K_F\hat{\rho} \\ \mu_0 B &= \hbar^2/m\hat{\rho}^2 \end{aligned}\right\} \quad \ldots(16)$$

one gets

$$E_F = \mu_0 B K^3\, \lambda\, M_0/6\pi \quad \ldots(17)$$

Now the Coulomb interaction energy between the electrons and the nuclei is given by -

$$E_{+-} = -\rho \sum_{MK} \int \left|\phi_{MK}(r)\right|^2 V(p, z)\, d^3 r$$

and in dimensionless form, we have -

$$E_{+-} = -(z^2e^2/\rho)\,[2\ln(L/\hat{\rho}) + 2\ln 2 - 1 - \ln(M_o) - 3/2\,M_o^{-1}] \quad \ldots(18)$$

The Coulomb interaction energy of the these nuclei can also be estimated classically (where one neglects boundary effects) and is given by -

$$E_{++} = (L/2l)E_0 \quad \ldots(19)$$

and in dimensionless -

$$E_{++} = 2(z^2e^2/1)\,[\ln(L/\hat{\rho}) + \ln(\hat{\rho}/21) + \varepsilon] \quad \ldots(20)$$

which ε is Euler's constant.

Now the direct Coulomb's interaction energy of the electrons can be written as -

$$E_{--} = -(z^2e^2/1)\,[\ln(L/\hat{\rho}) + 1/2\ln g - 1/4 - 1/\ln M_o - 1/2(\ln z)M_o^{-1}] \quad \ldots(21)$$

Where

$$\left.\begin{aligned} &\psi(M_o) = \ln M_o \\ &\text{and } g_2/g_1 = \ln g,\ 1 < g < 2 \\ &\text{for } g = 2,\ B \to \infty \end{aligned}\right\} \quad \ldots(22)$$

where

$$g_1 = \int_0^1 (\chi^{M_1} + \chi^{M_2})(1+\chi)^{-(M_1+M_2+2)} d\chi = M_1\,!\,M_2\,!/(M_1 + M_2 + 2) \quad \ldots(23)$$

and

$$g_2 = \int_0^1 (\chi^{M_1} + \chi^{M_2})(1+\chi)^{-(M_1+M_2+2)} \ln(1+\chi)\, d\chi \quad \ldots(24)$$

The total energy of the system without exchange term is given by -

$$E = E_F + E_{+-} + E_{++} + E_{--}$$

Now taking equation (17), (18), (20) and (21), one arrives on the result -

$$E = -\left(\frac{z^2e^2}{1}\right)\left[\ln\left(\frac{21}{R}\right) = (\varepsilon - C_1)\right] + (z^3\pi^2\hbar^2/6\,mL^2)\left(\frac{\hat{\rho}}{R}\right)^4 \quad \ldots(25)$$

where

$$C_1 = 2 \ln z - 3/2 M_o^{-1} - 1/2 \ln g) - 3/4 + 1/2 M_o^{-1} \ln 2 = 0.33 \quad \ldots(26)$$

Now, minimizing with respect to *l* and R, one gets -

$$\ln\left(\frac{21}{R}\right) = (\varepsilon - C_1 + 1) = 2Z\pi^2\hbar^2/6\rho^2 \, mL\left(\frac{\hat{\rho}}{R}\right)^4 \frac{1}{2} = 2Z\pi^2\hbar^2/6e^2 \, ml\left(\frac{\hat{\rho}}{R}\right)^4 \quad \ldots(27)$$

which can be combined to give -

$$n(2l/R) = \varepsilon - C_1 + 3/2 \quad \ldots(28)$$

$$l = 2.87 \, R$$

We have numerically evaluated the binding energies of hydrogen, helium, carbon and oxygen matter without the exchange term. The result are shown in Tables 1, 2, 3 and 4, respectively.

Table 1: Dimensions and energies for hydrogen matter in superstrong magnetic field without exchange terms

$B(10^{12}G)$	η	$R(a_0)$	$l(a_0)$	–E(KeV)
1	10.3	0.166	0.426	0.16
5	23.1	0.087	0.250	0.30
10	32.7	0.066	0.189	0.40
50	73.1	0.035	0.100	0.76
100	103.4	0.026	0.075	1.01
500	231.2	0.024	0.040	1.92
600	251.9	0.019	0.036	2.00
700	264.6	0.016	0.033	2.16
800	282.8	0.014	0.030	2.28
900	301.6	0.012	0.026	2.37
1000	327.0	0.010	0.029	2.53

Table 2: Dimensions and energies for helium matter in superstrong magnetic field without exchange terms

$B(10^{12}G)$	η	$R(a_0)$	$l(a_0)$	–E(KeV)
1	3.7	0.191	0.547	0.56
5	8.2	0.100	0.287	1.06
10	11.6	0.076	0.218	1.40
50	25.9	0.040	0.114	2.66
100	36.6	0.030	0.087	3.51
500	81.7	0.016	0.046	6.68
600	92.8	0.0152	0.045	6.98
700	102.9	0.0145	0.043	7.05
800	108.6	0.0140	0.039	7.27
900	112.8	0.0130	0.037	7.86
1000	115.6	0.012	0.035	8.81

Table 3: Dimensions and energies for carbon matter in superstrong magnetic field without exchange terms

$B(10^{12}G)$	η	$R(a_0)$	$l(a_0)$	–E(KeV)
1	0.7	0.238	0.682	4.0
5	1.6	0.125	0.358	7.6
10	2.2	0.095	0.271	10.1
50	5.0	0.050	0.143	19.2
100	7.0	0.038	0.108	25.3
500	15.7	0.020	0.057	48.2
600	17.6	0.019	0.055	53.9
700	18.9	0.017	0.052	55.6
800	20.5	0.016	0.049	58.5
900	21.8	0.014	0.045	60.2
1000	22.2	0.015	0.043	63.7

Table 4: Dimensions and energies for oxygen matter in superstrong magnetic field without exchange terms

$B(10^{12}G)$	η	$R(a_0)$	$l(a_0)$	–E(KeV)
1	0.5	0.252	0.722	6.7
5	1.0	0.132	0.379	12.8
10	1.4	0.100	0.287	16.9
50	3.2	0.053	0.151	32.2
100	4.6	0.040	0.114	42.5
500	10.2	0.021	0.060	81.0
600	10.9	0.020	0.058	90.8
700	11.2	0.019	0.055	97.2
800	12.5	0.018	0.052	100.5
900	13.0	0.017	0.048	103.2
1000	14.5	0.016	0.046	106.8

RESULTS AND DISCUSSION

In this paper, the binding energies of hydrogen, helium, carbon and oxygen matter without including exchange energy term have been evaluated and also the binding energies for magnetic field strengths ranging from 10^{12} and 10^{15} G. For magnetic field of 12^{12}G, binding energies for atoms in condensed matter of 0.16 KeV for hydrogen, 0.56 keV for helium, 4 KeV for carbon and 6.7 KeV for oxygen without including exchange energy term have been obtained.

In all the four matters, the binding energies increases as the strength of magnetic field increases. The main difference with the earlier work is in the atomic dimensions i.e. for the lattice spacing or distance between the nuclei in the chain i.e. the value of $l(a_o)$ and $R(a_o)$. In all the evaluated four matters, the value of η increases with increase in magnetic field B. However the value of $R(a_o)$ and $l(a_o)$ decreases with increase of magnetic field B. Our evaluated results are in good agreement with those of the other workers[21–22].

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Revised : 16.06.2010 *Accepted : 20.06.2010*