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Analyzing of transmission spectra of the Fibonacci layered structures containing single negative metamaterials by using effective medium theory

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Abstract

We investigate the transmission properties of the Fibonacci quasiperiodic layered structures consisting of epsilon-negative (ENG) and mu-negative (MNG) metamaterials. It is found that there exist the polarization-dependent transmission gaps which are invariant with a change of scaling and insensitive to incident angles. Also, it is found that by decreasing generation number to the low values the photonic band gap property of structure will disappear while, in the high number of N we have desirable gaps regardless to N is odd or even number. Analytical methods based on transfer matrices and effective medium theory has been used to explain the properties of transmission gap of ENG-MNG Fibonacci multilayer structures.

Key Words

Transmission spectra; Fibonacci quasiperiodic structure; Single-negative matamaterials.

INTRODUCTION

Recently, the metamaterials that exhibit simultaneously negative permittivity ε and permeability μ in a frequency band have attracted intensive studies due to their unique electromagnetic (EM) properties. They are also called the double-negative (DNG) materials or left-handed materials because the electric field, the magnetic field and the wave vector of an EM wave propagating in such a medium form a left-handed triplet[1-7]. In addition to the DNG materials, another metamaterial called the single-negative (SNG) material in which only one of the material parameters is negative deserve special attention. The SNG materials consist of the mu-negative (MNG) materials with $\mu < 0$ but $\varepsilon > 0$, and the epsilon-negative (ENG) materials with $\varepsilon < 0$ but $\mu > 0$[8].

Most of previous works on the metamaterials focused on the certain unusual properties of wave propagation in a photonic crystal. It was shown that a one-dimensional photonic crystal (1DPC) composed of alternating slabs of ordinary double-positive (DPS) and DNG media can have a type of photonic band gap (PBG) corresponding to zero averaged refractive index[9-12]. Moreover, it is well known that a 1DPC constituted by a periodic repetition of MNG and ENG layers can possess another type of photonic gap with effective phase (φ_{eff}) of zero[13-15]. When the periodicity of photonic crystal structure is broken, wave propagation is not described by Bloch states. The opposite extreme of a periodic system is a fully random structure. In the random systems waves undergo a multiple scattering process and are subject to unexpected interference effects[16]. Multiple waves scattering in disordered materials shows many similarities with the propagation of electrons in semiconductors[17]. One of the first phenomena studied in this context was coherent backscattering or weak localization of wave[18]. Knowledge on the propagation of waves in completely ordered and disordered structures is now rapidly improving; little is known about the behavior of waves in the huge intermediate regime between total order and disorder. This intermediate regime is valid in quasiperiodic structures.

Quasiperiodic structures are nonperiodic structures that are constructed by a simple deterministic generation rule. In a quasiperiodic system two or more incommensurate periods are superimposed, so that it is neither aperiodic nor a random system and therefore can be considered as intermediate the two[19,20]. In other words, due to a long-range order a quasiperiodic system can form forbidden frequency regions called pseudo band gaps similar to the band gaps of a photonic crystal and simultaneously possess localized states as in disordered media[21]. Among the various quasiperiodic structures, the Fibonacci binary quasiperiodic structure has been the subject of extensive efforts in the last two decades. The Fibonacci multilayer structure is the well-known 1D quasiperiodic structure, its electronic properties has been well-studied since the discovery of the quasi-crystalline phase in 1984[22]. Wave through a structure in the Fibonacci sequence had also been studied in past decade, and recently the resonant states at the band edge of a photonic structure in the Fibonacci sequence are studied experimentally[23]. Studies of various aspects of wave propagation in the Fibonacci quasiperiodic structures carried out in Refs.[24-31] have considerably improved our understanding of wave transport in the Fibonacci quasiperiodic structures.

In this paper, we investigate the photonic transmission spectra in the Fibonacci quasiperiodic layered structures consisting of single negative metamaterials. We study the Fibonacci quasiperiodic layered structure of ENG-MNG, with dispersive and lossless multilayer stacks. In these structures, with the help of transfer matrix method and effective medium theory, we show the transmission spectra of TE and TM waves for both normal and oblique incidences and for different layer scaling. It is shown that for both TE and TM polarizations with normal and oblique incidences, there exist the transmission gaps which are invariant with a change of scale and insensitive to the incident angles.

THEORETICAL MODEL

Quasiperiodic photonic structures are defined by simple mathematical rules which generate non-periodic structures. The Fibonacci sequence is the chief example of long-range order without periodicity, and can be constructed from juxtaposing two building blocks A and B, according to the following deterministic generation rule: $S_{N+1} = \{S_{N-1}S_N\}$ for $N \geq 1$, with $S_0 = \{B\}$ and $S_1 = \{A\}$, and the generation rule is repeatedly applied to obtain: $S_2 = \{BA\}$, $S_3 = \{ABA\}$, $S_4 = \{BAABA\}$, etc. The number of layers is given by F_N, where F_N is the Fibonacci number obtained from recursive relation $F_N = F_{N-1} + F_{N-2}$, with $F_0 = F_1 = 1$. Geometrical arrangement of 1D the Fibonacci multilayer structure, which is embedded in air, is shown in Figure 1. In this multilayer structure, the thicknesses of A and B are supposed to be d_A and d_B, respectively.

We intend to investigate the transmission properties of 1D Fibonacci multilayer structures constituted by the multilayers of MNG and ENG materials. Generally, the metamaterials are dispersive, i.e., ε and μ are frequency dependent. These materials have different expressions of ε and μ accordingly. For MNG material, we suppose that ε and μ can be expressed as[9]:

$$\varepsilon = 1 \quad \text{and} \quad \mu(\omega) = 1 + \frac{3^2}{0.902^2 - \omega^2}, \tag{1}$$

where ω is frequency in GHz. Similarly, we can take ε and μ for ENG material as,

$$\varepsilon(\omega) = 1 + \frac{5^2}{0.9^2 - \omega^2} + \frac{10^2}{11.5^2 - \omega^2}, \quad \text{and} \quad \mu = 1. \tag{2}$$

Figure 2 shows the optical constants (permittivity and permeability) of MNG and ENG materials. As one can see from Figure 2, in the frequencies range 0.9–3.2 GHz, μ is negative and in the frequencies range 0.9–3.9 GHz, ε is negative. Also, for the frequencies greater than 3.9 GHz, both ε and μ are positive. In this work, since MNG and ENG metamaterials are considered in microwave frequency region, these layers are often electrically thin, i.e.,

$$\begin{aligned} |kA|d_A &= \left| d_A \sqrt{\frac{\omega^2}{c^2}(\varepsilon_A \mu_A - \sin^2\theta)} \right| << 1 \\ |kA|d_B &= \left| d_B \sqrt{\frac{\omega^2}{c^2}(\varepsilon_B \mu_B - \sin^2\theta)} \right| << 1 \end{aligned} \tag{3}$$

where c is the velocity of light in the vacuum. Also, ε_A and μ_A, likewise ε_B and μ_B are permittivity and permeability of two building blocks A and B, respectively. As a consequence, in the long-wavelength limit, we adopt effective medium approximation by introducing effective permittivity ε_{eff} and permeability μ_{eff} to study wave propagation

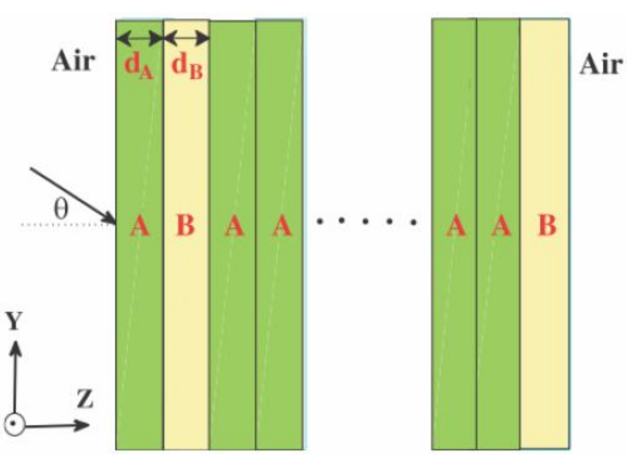

Figure 1 : Schematic drawing of the one-dimensional quasiperiodic Fibonacci structure, which is embedded in air. The thicknesses of A and B are supposed to be d_A and d_B, respectively.

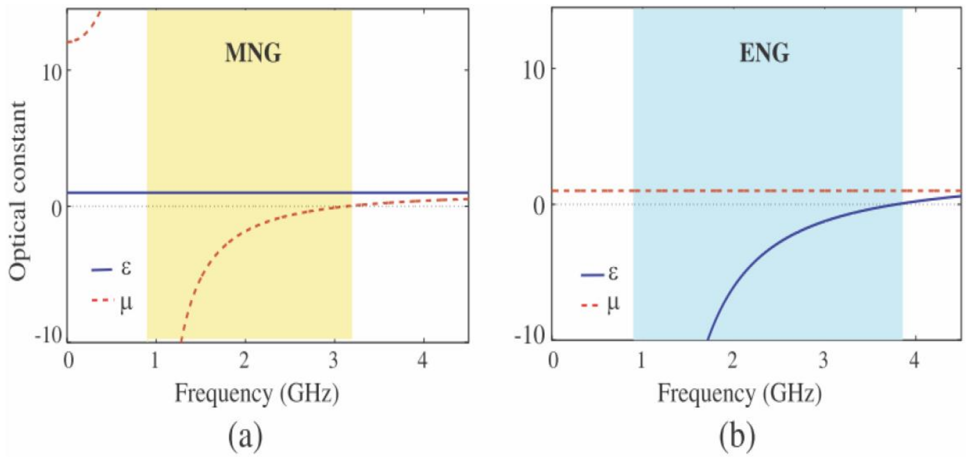

Figure 2 : The permittivity ε (solid line) and the permeability μ (dashed line) of (a) MNG and (b) ENG materials as function of frequency.

in this Fibonacci multilayer structure. ε_{eff} and μ_{eff} of this structure are given by[32]

$$\mu_{eff} = \frac{N_A d_A}{d}\mu_A + \frac{N_B d_B}{d}\mu_B,$$

$$\varepsilon_{eff} = \frac{N_A d_A}{d}\varepsilon_A + \frac{N_B d_B}{d}\varepsilon_B + \sin^2\theta\left(\frac{N_A d_A}{d}\frac{1}{\mu_A} + \frac{N_B d_B}{d}\frac{1}{\mu_B}\right) \quad (4)$$

$$+\sin^2\theta\left(\frac{1}{\frac{N_A d_A}{d}\mu_A + \frac{N_B d_B}{d}\mu_B}\right),$$

for TE polarization and

$$\varepsilon_{eff} = \frac{N_A d_A}{d}\varepsilon_A + \frac{N_B d_B}{d}\varepsilon_B,$$

$$\mu_{eff} = \frac{N_A d_A}{d}\mu_A + \frac{N_B d_B}{d}\mu_B - \sin^2\theta\left(\frac{N_A d_A}{d}\frac{1}{\varepsilon_A} + \frac{N_B d_B}{d}\frac{1}{\varepsilon_B}\right) \quad (5)$$

$$-\sin^2\theta\left(\frac{1}{\frac{N_A d_A}{d}\varepsilon_A + \frac{N_B d_B}{d}\varepsilon_B}\right),$$

for TM polarization. In Eqs. (4) and (5), $d = N_A d_A + N_B d_B$ where N_A and N_B are the number of A-type and B-type slabs, respectively. Eqs. (4) and (5) indicate that multilayer structure is anisotropic in essence because ε_{eff} and μ_{eff} depend on the incident angle θ.

We take a certain level of the Fibonacci sequence as a 1D deterministic disorder structure to calculate the transmission spectra. For this purpose, we assume that a monochromatic TE plane wave be incident from air with angle onto the Fibonacci multilayer structure. The electric component and magnetic component can be related via a transfer matrix[13]

$$M_j(\Delta z,\omega) = \begin{pmatrix} \cos(k_z^j \Delta z) & j/q_j \sin(k_z^j \Delta z) \\ jq_j \sin(k_z^j \Delta z) & \cos(k_z^j \Delta z) \end{pmatrix} \quad (6)$$

where $q_j = \sqrt{\varepsilon_j}/\sqrt{\mu_j}\sqrt{1-\sin^2/\varepsilon_j\mu_j}$, $k_z^j = (\omega/c)\sqrt{\varepsilon_j}\sqrt{\mu_j}\sqrt{1-\sin^2/\varepsilon_j\mu_j}$ and c is the speed of light in vacuum, also j = A, B denote ENG and MNG layers, respectively. The treatment of TM waves is similar to that for a TE waves.

RESULTS AND DISCUSSION

Here, we investigate the transmission properties of Fibonacci multilayer structure composed of MNG-ENG dispersive and lossless materials. We find some band gaps whose properties are studied in this section in detail.

In the following calculation, we choose $d_A = 8$ mm, $d_B = 4$ mm and the Fibonacci generation number N = 14. The frequency dependence of the effective permittivity ε_{eff} (solid line) and the effective permeability μ_{eff} (dashed line) of considered structure are plotted in the Figure 2 for both TE (Figure 2(a) and 2(b)) and TM (Figure 2(c) and 2(d)) waves corresponding to the incident angles 0°, 30°. The influence of the incident angle on the effective parameters ε_{eff} and μ_{eff} in the effective medium theory is introduced by expression $\sin^2\theta$ in Eqs. (4) and (5) for TE and TM modes, respectively. It is seen from Figure 3 that the μ_{eff} and ε_{eff} are sensitive to the incident angle for both TE and TM polarizations. It should be noted that the effective response of ENG-MNG structure in the normal incidence is identical for TE and TM waves.

The transmission spectra of TE and TM polarizations in ENG-MNG are represented for different incident angles of θ = 0°, 30° and 45° and for different thickness scales as d_A:d_B = 6:3, 8:4 and 12:6 mm in Figure 4. It is clear from Figures 4 that there are two band gaps in the transmission spectra of TE and TM waves. The first gap exists in the frequencies where the effective permeability μ_{eff} of structure is negative, whilst, the second gap is occurred in the frequencies where the effective permeability μ_{eff} of the structure is positive (see Figure 3(a)). One can see from Figures 4(a) and 4(b) that for TE and TM polarizations, the spectral width of the first gap in this structure is invariant with a change in the incident angles, whilst, the spectral width of the second gap increases with the incident angle keeping left edge constant. Also, the second gap is found only for the oblique incidence and it disappears for the normal incidence. In other words, the second gaps appeared in the transmission spectra which are highly sensitive to angle of incidence. From Figures 4(c) and 4(d), we can see that for both TE and TM polarizations if the thicknesses of two building block $d_A = 8$ mm and $d_B = 4$ mm are scaled by 3/4 and 3/2, the position and width of the transmission gap are nearly invariant.

It is necessary to mention that in the ENG-MNG structures, we studied the transmission spectra in 1D Fibonacci multilayer structures against generation number N. We found that there is a minimum generation number

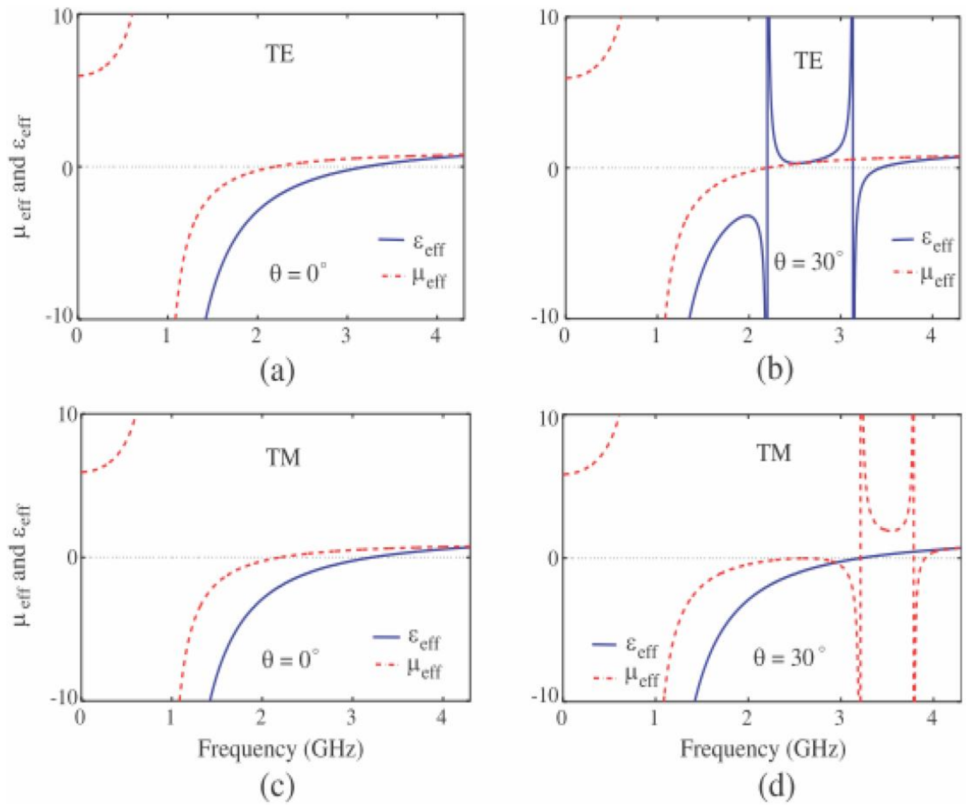

Figure 3 : The effective permittivity ε_{eff} (solid line) and the effective permeability μ_{eff} (dashed line) of ENG-MNG Fibonacci structure for TE and TM waves, corresponding to the incident angles 0°, and 30°.

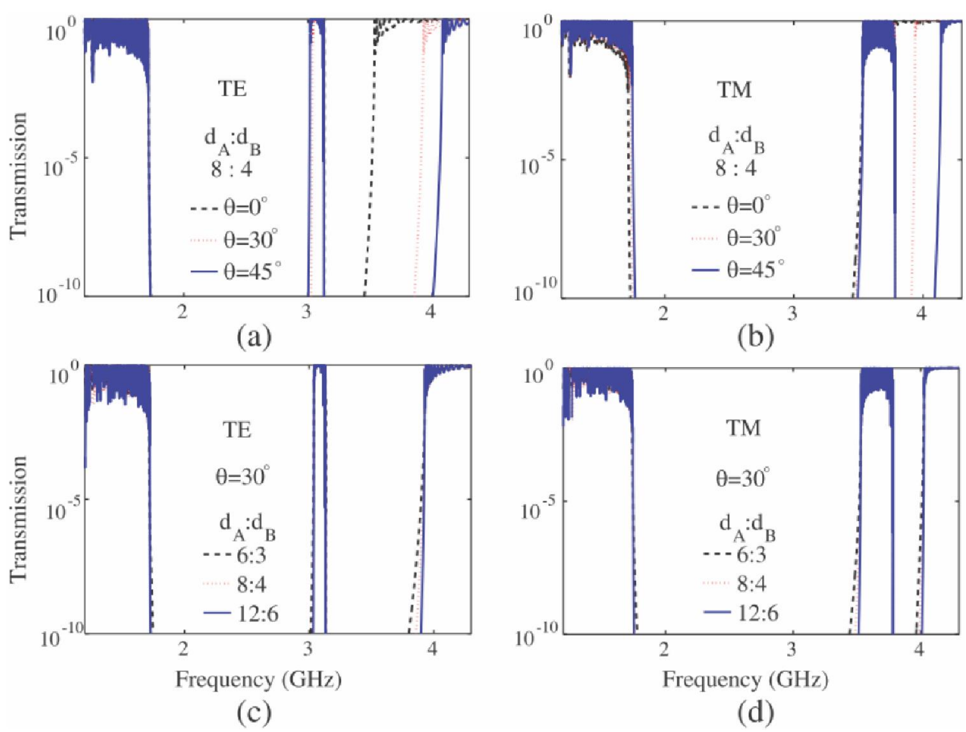

Figure 4 : Transmission spectra of ENG-MNG Fibonacci structure for 14th Fibonacci level as a function of incident frequency for TE and TM waves corresponding to the incident angles 0°, 30° and 45°, and three thicknesses d_A:d_B=6:3, 8:4 and 2:6 mm.

N = 9 in which the Fibonacci structure yield the photonic band gap with sharp gap edges. But by decreasing generation number to the low values the photonic band gap property of structure will disappear while, in the high number of N we have desirable gaps regardless to N is odd or even number.

CONCLUSION

In conclusion, based on the transfer-matrix method and effective medium theory, we have theoretically investigated the transmission spectra of quasiperiodic Fibonacci layered structures consisting of dispersive and lossless MNG and ENG materials. In ENG-MNG Fibonacci layered structure for both TE and TM waves, it is shown that there exist the transmission gaps which are invariant with a change of layer scale and insensitive to the incident angle. Moreover, for both TE and TM waves we have shown that, there is a gap which is only found at the oblique incidence, i.e., it disappears at the normal incidence. Also, we found that by decreasing generation number to the low values the photonic band gap property of structure will disappear while, in the high number of N we have desirable gaps regardless to N is odd or even number.

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