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A stage-structured model of a single-species with density-dependent and birth pulses

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ABSTRACT

This paper considers a stage-structured model of a single-species with density-dependent and birth pulses. Using the stroboscopic map, we transform the mathematical model into discrete dynamical system. Furthermore, the local stability of the equilibrium is proved.

KEYWORDS

Density-dependent; Birth pulse; Stage structure; Local stability.

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INTRODUCTION

In nature world, many populations have the different characteristics in different growth process. For example, they have two life stages, immature and mature. They have different ecological characteristics and dynamic behaviors in a different stage, just as caterpillars will proceed through the different phases and become butterflies finally.

Many researchers have taken up the deep study of the stage-structured mathematical models [1,2]. Most of these models suppose that birth rate of adult population is continuous, but the reproduction of many animals has obvious seasonal and instantaneous characteristics. That is, they breed once every once in a while. By use of impulsive differential equation theory, we can describe the discontinuous dynamic phenomena [3,4,5].

This paper considers a stage-structured model of a single-species with density-dependent and birth pulses. Using the stroboscopic map, we transform the mathematical model into discrete dynamical system. Furthermore, the local stability of the boundary equilibrium and positive equilibrium is proved.

The stage-structured Model of a Single-species

Let the single-species growth model with stage is as follows

$$\dot{N}(t) = B(N)N - dN$$

Where positive d is the death rate constant. $B(N)N$ is birth rate.

$B(N)$ satisfies the following assumptions.

(1) $B(N) > 0$

(2) $B(N)$ is a continuous differentiable function and $B'(N) < 0$.

(3) $B(+\infty) < d < B(0)$

Where N is a positive number. We can discover $B(N) = be^{-N}$ from biological literature.

We suppose that the population has two life stages, immature and mature. $x(t)$ and $y(t)$ represent the numbers of immature and mature respectively. Adult population has the ability to reproduce and the growth rate of it is equal to conversion rate from laver to adult minus death rate. Juvenile population does not have the ability to reproduce and the growth rate of it is equal to reproductive rate of adult population minus death rate of juvenile population and conversion rate from laver to adult. And we suppose that the death rate of adult population is equal to the death rate of juvenile population. Then, we get the following model.

$$\begin{cases} x'(t) = be^{-[x(t)+y(t)]}y(t) - dx(t) - sx(t) \\ y'(t) = sx(t) - dy(t) \end{cases} \qquad (1)$$

Theorem1 The system (1) has a boundary equilibrium $A(0,0)$. When $R_0 = \dfrac{sb}{d(d+s)} > 1$, we can get the unique and positive equilibrium $B(x^*, y^*)$.Where

$$x^* = \frac{d}{s+d}\ln\frac{bs}{d(d+s)}, \; y^* = \frac{s}{s+d}\ln\frac{bs}{d(d+s)}$$

Theorem2 For the system (1), when $R_0 < 1$, the equilibrium A is locally asymptotically stable; when $R_0 > 1$, the equilibrium B is locally asymptotically stable.

Proof Let $P(x) = be^{-[x(t)+y(t)]}y(t) - dx(t) - sx(t)$, $Q(x,y) = sx(t) - dy(t)$,

We have

$$\frac{\partial P}{\partial x} = -be^{-[x(t)+y(t)]}y(t) - d - s \,,$$

$$\frac{\partial P}{\partial y} = -be^{-[x(t)+y(t)]}y(t) + be^{-[x(t)+y(t)]} \,,$$

$$\frac{\partial Q}{\partial x} = s \,, \quad \frac{\partial Q}{\partial y} = -d$$

Then the Jacobi matrix is

$$J = \begin{pmatrix} \dfrac{\partial P}{\partial x} & \dfrac{\partial P}{\partial y} \\ \dfrac{\partial Q}{\partial x} & \dfrac{\partial Q}{\partial y} \end{pmatrix}$$

So the characteristic polynomial is

$$\left|\lambda E-J\right|=\begin{vmatrix}\lambda-\frac{\partial P}{\partial x} & -\frac{\partial P}{\partial y}\\ -\frac{\partial Q}{\partial x} & \lambda-\frac{\partial Q}{\partial y}\end{vmatrix}=\lambda^2-(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y})\lambda+\frac{\partial P}{\partial x}\frac{\partial Q}{\partial y}-\frac{\partial P}{\partial y}\frac{\partial Q}{\partial x}$$

For the equilibrium A, the characteristic polynomial is

$\lambda^2+(2d+s)\lambda+(d+s)d-sb=0$.

When $R_0<1$, it has two negative eigenvalues, so the equilibrium A is locally asymptotically stable.

When $R_0>1$, the equilibrium A is unstable.

For the equilibrium B, the characteristic polynomial is

$\lambda^2+(k+2d+s)\lambda+k(d+s)=0$.

Where $k=\frac{d(d+s)}{s}y*$. When $R_0>1$, it has two negative eigenvalues, so the equilibrium B is locally asymptotically stable.

The model of a single-species with birth pulses

We suppose adult population can not breed at any time, but in the form of pulse propagation. That is, Adult population only breed in time of $m,m+1,m+2,\cdots$. We can establish the following impulsive differential equations.

$$\begin{cases}x'(t)=-dx(t)-sx(t), m<t<m+1\\ y'(t)=sx(t)-dy(t),\\ x(m^+)=x(m^-)+be^{-[x(t)+y(t)]}y(m^-)\end{cases}\qquad(2)$$

The meanings of s and d are the same as in the system (1).

Let

$$N(t)=x(t)+y(t)\qquad(3)$$

We solve the first equation of (2) and have

$$x(t)=x(m^+)e^{-(d+s)(t-m)}, m<t<m+1\qquad(4)$$

Adding the first equation to the second equation of the equations (2),we integrate the result and obtain

$$N(t)=N(m^+)e^{-d(t-m)}, m<t<m+1\qquad(5)$$

(5) minus (4) leaves the the following formula

$$y(t)=e^{-d(t-m)}[y(m^+)+x(m^+)(1-e^{-s(t-m)})], m<t<m+1\qquad(6)$$

In time of $m,m+1,m+2,\cdots$, we can get the difference equation as follows

$$\begin{cases}x((m+1)^+)=x(m^+)e^{-(d+s)}+be^{-e^{-d}(x(m^+)+y(m^+))}e^{-d}[y(m^+)+(1-e^{-s})x(m^+)]\\ y((m+1)^+)=e^{-d}(1-e^{-s})x(m^+)+e^{-d}y(m^+)\end{cases}\qquad(7)$$

Theorem3 Let difference equations have the boundary equilibrium $A(0,0)$.When $R^*=\frac{be^{-d}(1-e^{-s})}{(1-e^{-(d+s)})(1-e^{-d})}>1$,they have the positive equilibrium $B(x^*,y^*)$.

Where

$$x^*=\frac{1-e^{-d}}{1-e^{-(d+s)}}e^d\ln R^*, y^*=\frac{e^{-d}(1-e^{-s})}{1-e^{-(d+s)}}e^d\ln R^*=\frac{1-e^{-s}}{1-e^{-(d+s)}}\ln R^*\qquad(8)$$

Lemma1 (Jury inequalities) Suppose that the linear system of difference equations is $X_{m+1}=WX_m$,then the system is stable if the spectral radius of W is less than 1.That is, W Satisfy the following conditions.

$$\begin{cases}1-\mathrm{tr}W+\det W>0\\ 1-\mathrm{tr}W+\det W>0\\ 1-\det W>0\end{cases}\qquad(9)$$

Theorem4 For the system (2), when $R_0 < 1$, the equilibrium A is locally asymptotically stable; when $R_0 > 1$, the equilibrium B is locally asymptotically stable.

Proof For the equilibrium $A(0,0)$, the linear approximation matrix is

$$W_0 = \begin{pmatrix} e^{-(\mathrm{d+s})} + be^{-d}(1-\mathrm{e}^{-s}) & be^{-d} \\ e^{-d}(1-\mathrm{e}^{-s}) & e^{-d} \end{pmatrix}$$

By calculating, we get

$$1 - \mathrm{tr}W_0 + \det W_0 = (1-e^{-d})(1-e^{-(d+s)}) - be^{-d}(1-e^{-s})$$

$$1 + \mathrm{tr}W_0 + \det W_0 = (1+\mathrm{e}^{-d})(1+e^{-(\mathrm{d+s})}) + be^{-d}(1-\mathrm{e}^{-s}) > 0$$

$$1 - \det W_0 = 1 - e^{-(2\mathrm{d+s})} > 0$$

So when $R_0 < 1$, the equilibrium A is locally asymptotically stable.

For the equilibrium $B(x^*, y^*)$, the linear approximation matrix is

$$W^* = \begin{pmatrix} A^* & B^* \\ e^{-d}(1-\mathrm{e}^{-s}) & e^{-d} \end{pmatrix}$$

Where

$$A^* = e^{-(d+s)} + be^{-d}\frac{1}{R^*}(1-e^{-s}-y^*),\ B^* = be^{-d}\frac{1}{R^*}(1-y^*)$$

$$trW^* = A + e^{-d} = e^{-(d+s)} + be^{-d}\frac{1}{R^*}(1-e^{-s}-y^*) + e^{-d}$$

$$\det W^* = Ae^{-d} - Be^{-d}(1-e^{-s}) = e^{-d}[e^{-(d+s)} - be^{-d}\frac{1}{R^*}e^{-s}y^*]$$

When $R_0 > 1$, we substitute into Jury inequalities and can receive

$$1 - \mathrm{tr}W^* + \det W^* = \frac{be^{-d}}{R^*}y^*(1-e^{-(d+s)}) > 0$$

$$1 + \mathrm{tr}W^* + \det W^* = (1+\mathrm{e}^{-d})(1+e^{-(\mathrm{d+s})}) - (1-\mathrm{e}^{-d})(1-e^{-(\mathrm{d+s})})(1-\frac{1+e^{-(\mathrm{d+s})}}{1-e^{-(\mathrm{d+s})}}) > 0$$

$$1 - \det W^* = 1 - e^{-(2\mathrm{d+s})} + be^{-(d+s)}\frac{1}{R^*}y^* > 0$$

So when $R_0 > 1$, the positive equilibrium B is asymptotically stable.

CONCLUSIONS

This paper considers a stage-structured model of a single-species with density-dependent and birth pulses.First,we study a stage-structured mathematical model of a single-species and obtain the equilibrium(Theorem 1) and the local stability(Theorem 2).Second,we transform the mathematical model into discrete dynamical system and acquire the equilibrium(Theorem 3) by using the stroboscopic map.Furthermore,the local stability of the boundary equilibrium and positive equilibrium is proved.

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REFERENCES

[1] Aiello W G, Freedman H I. A Time Delay of Single-species Growth with Stage Structure. Math Biosci, pg.101,pg.139-153 (1990).

[2] Aiello W G, Freedman H I, Wu J. Analysis of a Model Representing Stage Structured Population Growth with State-Dependent Time Delay. SIAM Appl. Math, 52, pg.855-869 (1990).

[3] Zhang S W, CHEN L S. Stage-Structured Model of a Single-species with Density-Dependent Birth Pulses. System science and math science, 26,6, pg.752-760 (2006).

[4] YU S M. Single Population Birth Pulse Stage-Structure Model. Mathematics in Practice and Theory,36,4,pg.23-28 (2006).

[5] LI Q Y, ZHANG F Q, WANG W J. Single-species model with impulsive birth and contraception control. Journal of Shandong University. 49,6, pg.85-90 (2014).