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Safety water depth model of cliff diving based on MATLAB simulation

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ABSTRACT

In the cliff diving competition, whether the athletes should head first or feet first into the water and safety water depth issues has always been an issue of concern. This paper combines with computer and uses MTALAB software for numerical simulation, and obtains the mathematical model of cliff diving safety water depth. It uses the estimation formula of impact pressure peak on compressible water surface by the rigid plate based on VonKarman unitary collision theory and calculates the maximum pressure dropping from height 28m; according to the maximum force of the human head, it can be seen that the head cannot first enter into the water. In the study of the safety water depth issue, the athlete's movement is divided into three processes: the process from diving tower to the water surface, the process from barely touching surface to completely into water, the process from completely into water to the bottom of the pool; it uses MTALAB to simulate three processes, finally solves the safe water depth and provides theoretical support for safety competitions, training and extreme challenges.

KEYWORDS

Matrix laboratory (MTALAB);
Cliff diving;
Collision;
Fluid mechanics.

INTRODUCTION

Diving is an important aquatic event, but also modern athletics of a mass character. Diving pool must have sufficient depth to ensure diver's safety. If only for safety, of course, the deeper the pool water is the better, but the corresponding construction costs and operation and management costs will be higher when the water is deeper, which will result in meaningless waste. Generally, assuming normal athlete's weight is in between 40-80kg, platform's diving area needs to be more than 5 meters depth; it is safe only when platform diving area's water depth is more than 3m.

In recent years a new game called Red Bull Cliff Diving World Cup rising up in the world, which is a very dangerous, challenging the limits of human event. The contest regulates that man's diving tower height is 23-28m, and 18-23m for women. Guan Zhi-shan of Fujian Liancheng once has held such a game, where the man's platform height is 28m, woman's is 20m. Currently research literature on the cliff diving safety is much less; people are more concerned about corresponding technical and security problems.

This paper uses physics, theoretical mechanics and differential equation to establish a mathematical model and simulate using MTALAB, better ensures the safety of athletes during the race, and provides a theoretical basis for the training and the ultimate challenge.

THE BASIC ASSUMPTION OF THE MODEL

Assuming diver's diving status is in the shape of a cylinder with uniform mass distribution, in which the height of the diver is the height of the cylinder;

Assuming the height, mass and shoulders of both men and women divers meet international standards of 20-25 years old divers;

Assuming when the speed of athlete in the water is $1m/s$, the depth is the safety depth;

Assuming athletes have no loss of energy throughout the diving process;

Assuming wind velocity and water velocity is zero, which does not affect the athlete's whereabouts;

Assuming ignoring the pedal process of diver players;

Assuming that athletes' head or feet first enter into the water;

MATHEMATICAL MODELING AND SOLUTION OF THE PRESSURE WHEN THE BODY ENTERING INTO THE WATER

Athletes' entering into the water is a typical fluid-structure interaction problem. When athletes enter into the water, the body will stimulate the movement of the surrounding fluid medium, which in turn fluid media will apply reaction load force to the human body. Early into the water, the head collision with the water will form a compression wave in the water, and the water surface of the head will have a impact pulse of high value, and this load pulse may cause head violently shock; once there have diver that caused a concussion due to head into the water. Therefore, this paper will study and analyze the size of the impact pressure when the head into the water and describe their degree of danger.

Model building

For situation that the human body vertically into the water, the hit water instant is a collision phenomenon; the early hit water period can be assumed that a plate strikes compressible water. Based on VonKarman unitary collision theory, the estimation formula of impact pressure peak on compressible water surface by the rigid plate is:

Wherein: the density of water, the water sound velocity, the entering into water speed.

However, due to the human will do different actions in the body entering into the water phase, the angle into the water will have certain distinction. Analyze under different angles, the perpendicular pressure is the maximum. Now analyze the pressure situations under different angles: It can be seen from the following table that the force is completely different with different angles; however, the angle athletes into the water is probably in betweenand, it is necessary to obtain the force at. Now from the simulation experiment of torpedo into the water, obtain the data of object at different angles shown in TABLE 1.

Use Matlab to simulate, and finally get the maximum pressure peakwhen.

Solution of the model

1) The solution of the initial velocity entering into the water

Ignore air resistance and assume that one only suffers the action of gravity during the whereabouts process, according to the kinetic energy theorem we have:

$$mgh = \frac{1}{2} \cdot mv_0^2$$

Wherein: Athletes $m = 75kg$ mass, acceleration of gravity, speed entering into the water, diving height. $h = 28m$ Based on the above equation we have: $v_0 \approx 23.6m/s$

2) The solution of maximum anti impact force F_{max}

According to the solution obtain the speed entering into the water and continue solving the vertical impact pressure peak P_{max}. Through inquiring data we can see the force area when touching the water surface.

The maximum pressure when vertically entering into water: $P_{max} = \rho_w c_w v_0 = 35.6MPa$, $F_{max} = P_{max} s = 281\text{kN}$

The maximum pressure when entering into water by 75°: $P'_{max} = \frac{P_{max} + 2.8}{2} = 19.2MPa$, $F'_{max} = P'_{max} s = 151.7\text{kN}$

Now by the Xi'an Bureau of Transportation, the maximum force of head is $22.8kN$, if it is more than this number one is likely to die. According to the calculation

TABLE 1 : The maximum pressure when entering into the water with different angles

The velocity entering into water(m/s)	The angle entering into water(degree)	Maximum axial force(KN)	Maximum normal force(KN)	P_{max} (MPa)
12	15	12	6	3
	30	10	5.5	3.2
	45	15.5	5.2	3.5
	60	8.5	3.3	2.6
	75	15.9	2.5	3
	90	41.5	0	3.5
20	15	21	10	6
	30	15.2	8	10
	45	27.7	6.8	5.7
	60	17.2	5.6	6.6
	75	40.6	4.9	7.8
	90	154.1	0	12.4
29.2	15	50	13	7
	30	27.5	12	12.9
	45	68.1	9.7	9.6
	60	33	9	7.5
	75	77.6	9	14
	90	340.9	0	25.5

data, it shows that the biggest pressure F'_{max} that athletes can withstand is greater than the maximum pressure that athlete's head can withstand. Therefore, when athletes dive from 28m altitude, you cannot into the water with head first, and be sure to let feet first enter into the water.

THE SAFETY WATER DEPTH MODEL OF DIVING PROCESS

The problem needs to determine the depth of the pool, and compares whether it need to establish pools of different depths for athletes of different weights. Now divide the process of athletes from the cliff to the bottom into three processes: First, the process from diving tower to the water surface, and consider the two cases with air resistance and without air resistance; process two the process from barely touching surface to completely touching water surface, it is more complex when barely touching water surface; in order to obtain numerical solutions, simplify the process two, it is assumed that only considering gravity, buoyancy and water resistance, resistance and velocity are related; process three, from completely into the water to the bottom, at this stage as long as the athlete's speed will be reduced to $1 m/s$ when reaching the bottom, it will touchdown safely.

Model building

Process 1: From diving tower to the water surface, as shown in Figure 1:

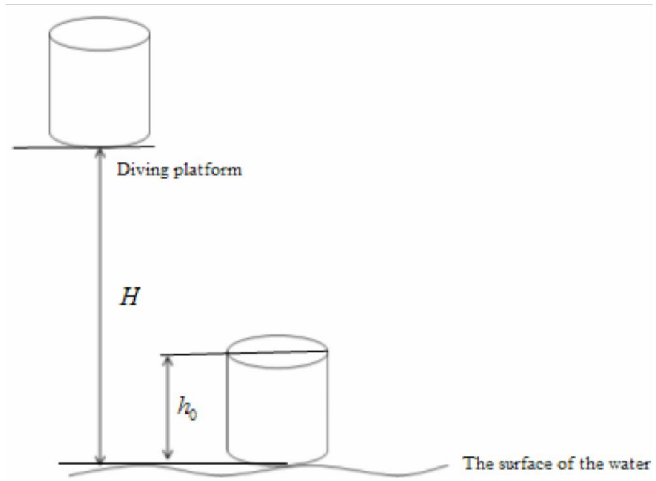

Figure 1 : The process from diving tower to the water surface

The case without considering air resistance:

$$mgH = \frac{1}{2}mv^2$$

After Simplification the following can be obtained:

$$v = \sqrt{2gH}$$

Consider air resistance. Athlete's movement process in the air: Athletes' speed when vacated the highest is zero; due to the affection of air resistance on the whereabouts process of people is small, which can be negligible, the athlete's movement is equivalent to free fall motion. So there is:

$$\frac{1}{2}mv^2 = mgh - \int_0^h f_{air}dh$$

By the formula of air resistance: $f_{\text{气}} = kv^2$

In the formula $k = 0.3$, this refers to the air resistance when freely falling.

Conduct the derivation on h of both sides in formula one and obtain: $mv\frac{dv}{dh} = mg - f_{air}$

By further simplification the following can be obtained: $h = \frac{m}{2k}(\ln g - \ln(g - \frac{k}{m}v^2))$

It translates into the form of v : $v = \sqrt{\frac{mg}{k}(1 - e^{-\frac{2hk}{m}})}$

Process 2: In this procedure cylindrical portion is from barely touching surface to completely into water, the distance of movement is height h_0, wherein $0 \le h \le h_0$, the initial velocity is v_1; establish the relational equation of underwater depth h and the movement speed v of people shown in Figure 2:

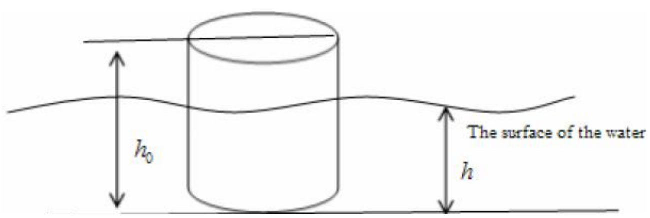

Figure 2 The process of athletes from the air into water

According to the conservation law of mechanical energy obtain: $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + mgh - \int_0^h f_{water}dh - \int_0^h F_{buoyancy}dh$

By the Archimedes' principle and water resistance formula of the hydrodynamics we have:

$$F_{buoyancy} = \rho_{water}gV \text{ , } f_{water} = \frac{1}{2}c_{water}\rho_{water}s_{water}v^2$$

And: $V = \pi R^2 h, s_{water} = \pi R^2$

Conduct the derivation on of both sides in formula two and obtain:

Then the speed can be obtained when, which can be taken as the initial velocity in the next process.

Process 3: Athletic movement after completely into the water exercise, shown in Figure 3. At this time we have: $f_{water} = \frac{1}{2}c_{water}\rho_{water}s_{water}v^2$

Similarly, we have:

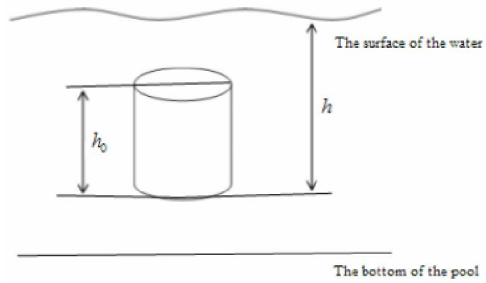

Figure 3 : The process during athletes from completely into water to the bottom of the pool

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_2^2 + mgh - \int_{h_0}^h f_{water}dh - F_{buoyancy}h$$

Conduct derivation of both sides and obtain:

$$mv\frac{dv}{dh} = mg - \frac{1}{2}c_{water}\rho_{water}\pi R^2 v^2 - \rho_{water}g\pi R^2 h_0$$

Solution of the model

(1) The relationship between weight and height. There are a lot of information giving the relationship between body weight and height, weight = height -105. But the diver's body differs from the general population; therefore this paper collects some height weight materials of China's famous diver's. Through observation, the data is approximately in linear relationship; use Matlab to linear fit and obtain relation equation: G_{man}=88.4h-113.7 G_{woman}=101.0h−113.7

According to this formula, we can get both men and women diver's weight with height 160cm 170cm and 180cm (see TABLE 2).

TABLE 2 : Diver's height and weight

Height/cm	Gender	Weight/kg
160	Man	52.5
	Woman	47.6
170	Man	61.4
	Woman	57.7
180	Man	71.2
	Woman	67.8

(2) The determination of the projection radius of human body model in the vertical direction. People's density is slightly larger than water, the average value is: men $1.05\times10^3 kg/m^3$, women $1.04\times10^3 kg/m^3$, and there is little difference under the standard body. Through density equation $\rho_{\text{人}} = m/V$ and the volume of the sphenoid $V = \pi R^2 h_0$, the following can be obtained: $R = \sqrt{\frac{m}{\pi h_0 \rho_{\text{人}}}}$

(3) Model solution: We now take a set of data to

solve, select 180cm boys to solve, the initial data is $h_0 = 1.8m$, $m = 71.2kg$, $H = 28m$,

$\rho_{人} = 1005\,kg/m^3$, $\rho_{水} = 1000\,kg/m^3$, $g = 9.8\,m/s^2$

Radius can be calculated: $R = \sqrt{\dfrac{m}{\pi h_0 \rho_{人}}} = 0.11193m$

The speed after Process one: $v = \sqrt{\dfrac{mg}{k}(1 - e^{-\frac{2Hk}{m}})} = 22.11\,m/s$

Process 2: The process from barely touching surface to completely touching water surface; Obtain Figure 4 using Matlab to solve:

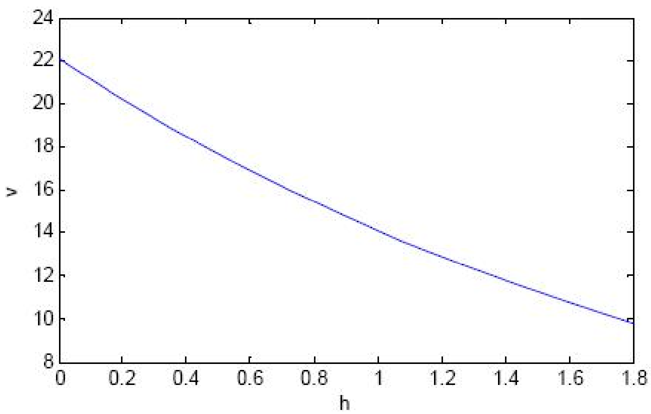

Figure 4 : The velocity curve during process two

As can be seen from the figure, when h arrives at 1.8m, we have $v_2 = 9.7718\,m/s$.

Process 3: Using Matlab to solve and the results are in following Figure 5:

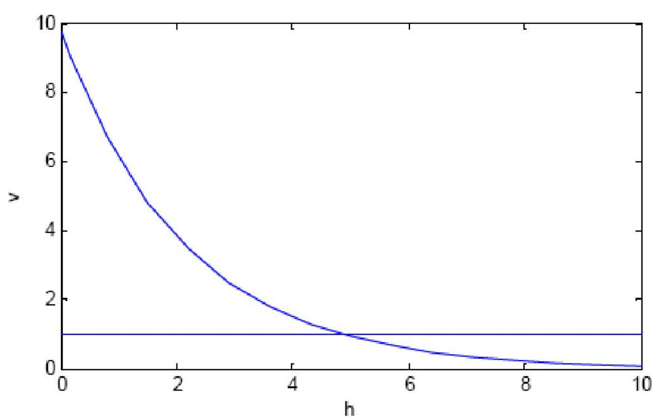

Figure 5 : The velocity curve when athlete reaches $1\,m/s$

As can be seen from the figure, when $h_1 = 5.016$ the speed has verged to $1\,m/s$ substantially reaching a safe speed, the total underwater depth of athletes can be calculated $l = h_0 + h_1 = 6.816m$.

Finally according to the method described above measure the height and weight for each athlete, the results are in TABLE 3:

TABLE 3 : Pool depth corresponding to different height

Height/cm	gender	Pool depth without air resistance/m	Pool depth with air resistance /m
160	man	6.064	6.045
	woman	6.045	6.039
170	man	6.345	6.333
	woman	6.322	6.310
180	man	6.823	6.816
	woman	6.80	6.793

From the results analysis, when two people with different weight dive, the one with bigger height and weight needs deeper water.

SYMBOL DESCRIPTION

P_{max} :Crash pressure peaks by rigid plate on compress ible surface perpendicularly.

P'_{max} :Crash pressure peaks by rigid plate on compress ible surface with 75°.

F_{max} : thletic maximum pressure when vertically entering into the water.

F'_{max} :Athletic maximum pressure when entering into the water with.

k : Air resistance coefficient $k = 0.3$.

H : Height of the diving tower.

v_2 : Athletic depth in the water at certain time.

h_0 : Athletic height.

h_1 : Athletic declining height when fully dropped into the water.

v : Athletic speed in a moment.

v_1 : Athletic speed when just touching the water surface.

v_2 : Athletic speed when fully entering the water.

R : Radius of the cylinder.

f_{air} : Air resistance.

f_{water} : Water resistance.

CONCLUSIONS

This model established in this paper fully used physics, theoretical mechanics and differential equations. It analyzed the specific physical processes of cliff diving, solved the relationship changes between physical speed and height in each process, displayed them in graphical form both accurate and beautiful, and solved the reasonable setting of pool depth. When the athlete's weight changes, according to the relationship between the three variables body mass, height of the cylinder and the cylinder radius, it can be determined that the pool is deeper when the weight is greater.

The accuracy of the model is better, which can be applied to the pool depth configuration issues of actual cliff diving; through the analysis of the problem, we can better ensure the safety of athletes during the race.

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