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Isentropic sound wave propagation in a saturated porous media

H.M.Duwairi[1,*], A.Al-Qaisiah[1], L.Batarseh[2]

[1]Mechanical Engineering Department, Faculty of Engineering & Technology, The University of Jordan, 11942 Amman, (JORDAN)
[2]Mechanical Engineering Department, College of Engineering, Phylidelphia University, Jerash, (JORDAN)
E-mail: duwairi@ju.edu.jo

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Abstract : The process of sound propagation in a saturated porous medium with ideal gas behavior was studied inside the pores under the boundary layer approximations assumptions. It is found that the main three parameters that governs the propagation process are: the Darcy number, $Da = \sqrt{K}/L$, shear wave number, $S = L\sqrt{\rho\omega/\mu}$ and the porosity, ε. The propagation process is going under isentropic condition and the sound disturbance is under mono-frequency and harmonic conditions. It is found that the increasing of porosity, ε, increases attenuations and decreases phase shift for the both the forward and backward sound waves; this is due to unfavorable acoustic wave propagation in plain medium limits. The effect of increasing Darcy number, *Da*, is to increase attenuation and increase phase shift velocities; this is also due to movement of the porous medium to plain medium limit by removing solid matrix. It is also found that as the fluid flow velocities inside solid matrix is increased the attenuation and phase shift of the forward acoustic sound waves are increased and decreased for backward acoustic sound waves.

Keywords : Sound waves; Darcy law; Porous medium.

INTRODUCTION

The using of porous media as insulation material find its place in many engineering applications especially in acoustical insulation configurations. Recently, many researches were conducted on such materials to analyze and produce material with improved insulation behaviors. The process of absorbing sound in porous media takes place due to thermal losses and viscosity in the microstructure of such materials. In many of porous media, specially the artificial one, microstructure is an indication to the fabric structure represented by fibers shape, density,… and other properties, such applications can be also found in coating of different materials.

Fedorov *et al*.[1] performs an experimental research on hypersonic boundary layer on a wall covered by a porous coating with equally spaced cylindrical blind micro-holes. Fedorov and his colleagues conclude that 30 % 0f the laminar boundary layer displacement thickness of coating porous material is a good stabilizer and these results were verified experimentally.

Other common case is the acoustic insulating metal foam which can be produced in many thicknesses and various acoustic insulation levels. Han *et al.*[2] study the acoustic absorption behavior of open-celled aluminum foam. The research was conducted to proof that the open-celled foam is better insulator than the closed-celled foam. Many pore sized samples were taken. The absorption coefficient was taken for compassion and the final results match the expectations.

In porous media, the structure it self play a significant rule in absorption process. Liu and Liu[3] investigate the effect of anisotropy of solid skeleton on the propagation characteristics of Rayleigh waves in orthotropic fluid saturated media. The results of Liu's investigation show the differences between wave's propagation in orthotropic and the isotropic porous media.

Porous media, specially, flat surfaces is an important issue in seismology science. In fact, the most important type of waves in such cases is the surface waves. Gubaidullin and Boldyreva[4] conduct a research on the propagation of acoustic waves along the free boundary of a saturated porous media. Attenuation and phase shift are results for many cases of porous saturation and pores sizes.

Other investigations on porous flat plate porous media absorption and many materials are used for such cases such as using of coconut coir fiber with porous layer backing to control noise Zulkifli *et al.*[5] or the study of interaction of acoustic waves with porous layer by Gubaidullinand *et al.*[6] and other studies.

Duwairi and Dwairi[7] study isentropic propagation of sound waves in cylindrical tube filled with a porous media. The results obtained of this research are compared to the results obtained by Duwairi and Dwairi. Even the physical systems are different; the results were close for some levels.

This study attempt to model the sound wave propagation in a fluid saturated porous media under the boundary layer approximation. A Cartesian coordinate system is used describe the infinite porous medium, the propagation of sound wave is taken in the same direction of flow. The effect of flow will be under investigation (i. e. Stationary or movable).

MATHEMATICAL FORMULATION

The physical model and the used coordinate system are illustrated in Figure 1.

Considering a porous media of length (L). A Cartesian coordinate system is used where the *x*-direction has been taken horizontal and *y*-direction is taken normal to porous medium. Considering the boundary layer approximation, the equations that govern the process of sound propagation in a saturated porous media are the continuity and momentum equations, see, Nield and Bejan[8]. Assuming zero normal velocity and applying continuity and momentum equations, one can obtain:

$$\varepsilon\frac{\partial\rho^*}{\partial t^*}+u^*\frac{\partial\rho^*}{\partial x^*}+\rho^*\frac{\partial u^*}{\partial x^*}=0 \quad (1)$$

$$\rho^*\left[\frac{1}{\varepsilon}\frac{\partial u^*}{\partial t^*}\right]=-\frac{\partial p^*}{\partial x^*}-\frac{\mu}{K}u^* \quad (2)$$

where u^* is the average of area fluid velocity component in axial direction x^*, ρ^* and p^* are the fluid density and pressure respectively. μ is the fluid absolute

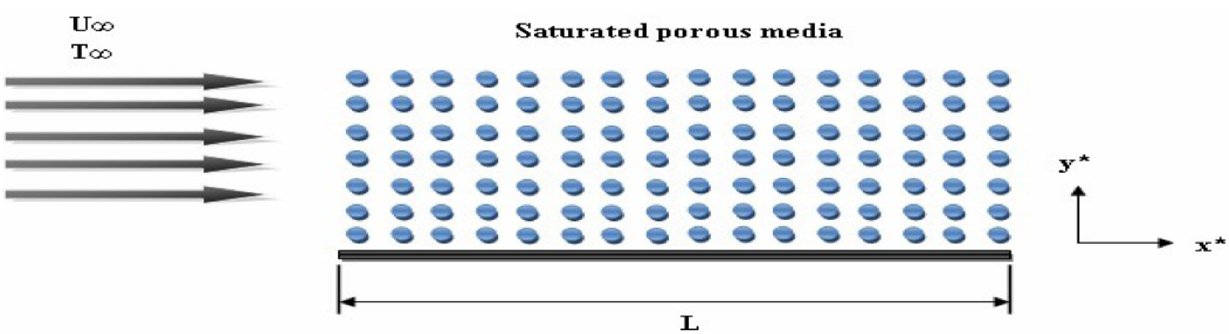

Figure 1 : Physical model and coordinate system

viscosity, ε is the porosity of porous media and K is the media permeability. The fluid is assumed to be perfect gas and the porous media is assumed to have low porosity so the Darcy law is adopted to describe the momentum and acoustic transfer between the fluid layers inside the porous matrix

Now, the flow in the porous medium is taken to be as a superposition of axial, steady and incompressible flow and a small harmonic acoustic disturbance of frequency ω. The fluid is assumed to have a constant density $\bar{\rho}$ and sound speed $\bar{a}$. Under the mono-frequency condition $\left(e^{(i\omega t^*)}\right)$. Transforming the variables to dimensionless forms, fluid variables are expanded as:

$$\rho^* = \bar{\rho}\left[1 + \alpha\,\rho(\eta) e^{\left(\Gamma\xi + i\omega t^*\right)}\right] \tag{3}$$

$$u^* = \bar{a}\left[M_o(\eta) + \alpha\,u(\eta) e^{\left(\Gamma\xi + i\omega t^*\right)}\right] \tag{4}$$

$$p^* = \frac{\bar{\rho}\,\bar{a}^2}{\gamma}\left[p_o(\xi) + \alpha\,p(\eta) e^{\left(\Gamma\xi + i\omega t^*\right)}\right] \tag{5}$$

where α<<1 and it represent a linearization factor and γ is the ratio of specific heats. Ccombining the acoustic variables ρ, u and p with steady flow variables p_o and Mach number M_o produce none-dimensional forms of fluid variables. Introduce ζ and η in the transforming process to dimensionless variables:

$$\xi = \frac{\omega}{\bar{a}} x^*, \quad \eta = \frac{y^*}{L} \tag{6}$$

where Γ is the propagation constant, in this case, Γ is assumed to have complex notation between the attenuation and phase shift, the expression of Γ is given as:

$$\Gamma = \Gamma' + i\Gamma'' \tag{7}$$

where Γ' is acoustic wave attenuation per unit distance and Γ'' is the wave phase shift at the same distance. Substituting the assumed variables from Eqs 3,4 and 5 into governing equations 1 and 2 and collecting the terms of α^0, α^1 and α^2. The terms of α^2 are canceled due to smallness of α^2 and then the governing equations become linear in order of α^1. It is found that the terms of α^0 in continuity equation are identically satisfied while the terms produced from the axial momentum equation are:

$$-\frac{\bar{\rho}\bar{a}\omega}{\gamma}\frac{dp_o}{d\xi} - \frac{\mu}{K}\bar{a}\,M_o(\eta) = 0 \tag{8}$$

solve Eq.8 for M_o to have

$$M_o(\eta) = -\frac{1}{S^2Da^2}\frac{dp_o}{d\xi} \tag{9}$$

where $Da = \frac{\sqrt{K}}{L}$ is the Darcy number and $S = L\sqrt{\frac{\bar{\rho}\omega}{\mu}}$ is the shear wave number. Equating terms of α^1 will produce the following linearized equations:

$$\left[\varepsilon\rho i + M_o\,\rho\Gamma + u\Gamma\right] = 0 \tag{10}$$

$$\frac{1}{\varepsilon} u i = -p\frac{\Gamma}{\gamma} - \frac{1}{S^2Da^2} u \tag{11}$$

One step is still to have system of solvable equations which is defining ρ, from the assumption of perfect gas and dimensionless transforming to state equation, a value of ρ is found to be:

$$\rho = p\gamma \tag{12}$$

ANALYTICAL SOLUTION

To solve Eqs. 10, 11 and 12, a reasonable assumption of the axial acoustic velocity to be constant; this assumption results from using of Darcy law, so:

$$u = C \tag{13}$$

substitute Eq.13 into Eq.11 and solve for C to have:

$$C = \frac{-p\frac{\Gamma}{\gamma}}{\frac{i}{\varepsilon} + \frac{1}{S^2Da^2}} \tag{14}$$

substitute Eqs.12 and 13 in Eq.10 and solve for C to have:

$$C = -\frac{\varepsilon\frac{p}{\gamma} i + M_o\frac{p}{\gamma}\Gamma}{\Gamma} \tag{15}$$

now, equate both values of 'C' from Eqs. 14 and 15 and rearrange to have:

$$\Gamma^2 - \left[\left(\frac{i}{\varepsilon} + \frac{Da^2}{S^2}\right)M_o\right]\Gamma - \left(\frac{i}{\varepsilon} + \frac{1}{S^2Da^2}\right)\varepsilon i = 0 \tag{16}$$

eq.16 is a second order equation in Γ and it can be solved analytically and the propagation constant can be found as:

$$\Gamma = \frac{\left(\frac{i}{\varepsilon}+\frac{1}{Da^2S^2}\right)M_o \pm \sqrt{\left[\left(\frac{i}{\varepsilon}+\frac{1}{Da^2S^2}\right)M_o\right]^2 + 4\left(\frac{i}{\varepsilon}+\frac{1}{Da^2S^2}\right)\varepsilon i}}{2} \quad (17)$$

RESULTS AND DISCUSSION

The propagation equation (17) gives two analytical values for propagation constant, Γ, one is the real part with attenuation of acoustic sound waves and the imaginary part with phase shift velocities of the acoustical sound waves. This equation shows that the propagation process depends on three main parameters; shear wave number (*S*), Darcy number (*Da*) and porosity (ε). The Steady flow Mach number (M_o) is the indication of the stationary ideal fluid or movable ideal fluid inside the solid matrix, for stationary flow, $Mo = 0$. To emphasize the effects of each parameter on attenuation and phase shift, two parameters are held constant and the last parameter is take with reasonable range. Attenuation is the real conjugate of proportion constant while phase shift is written in non-dimensional form as given in Eq.18

$$w = \left|\frac{W}{\bar{a}}\right| = \left|\frac{1}{\Gamma''}\right| \quad (18)$$

where *W* is the dimensional phase velocity and *w* is the dimensionless form of phase velocity.. In acoustic wave propagation in porous media, two waves are produced: the forward and backward acoustical sound waves. Eq.17 gives two different solutions, one for forward wave and the other for the backward wave, unless the mean Mach number (M_o) is holding zero. At this case, the problem is under stationary fluid motion and both forward and backward are the same.

The effect of Porosity ε on the acoustical sound waves is presented in Figure 2 for the stationary fluid and in Figure 3 for the movable fluid inside the solid matrix. It is found that as the porosity ε is increased the attenuation is decreased and phase shift velocities are decreased; this is due to unfavorable moving of solid matrix from the flow and moving toward plain media limit ε=1. So the using of small pores of solid matrix gives an excellent damping effect and large acoustical sound waves duration.

The effect of increasing Darcy number, *Da* is presented in Figures 4 and 5 for both a stationary and movable fluid inside the solid matrix. It is found that as the Darcy number Da is increased the attenuation is decreased and phase shift velocities are increased; this is again due to unfavorable removing of solid matrix from the flow and this means smaller acoustical sound waves duration in the solid matrix.

The effect of increasing fluid velocities inside solid matrix is presented in Figure 6, it is clear that as the Mach number M_0 is increased, both the attenuation and phase velocities are increased for the forward sound waves and both attenuation and phase velocities are decreased for the backward sound waves; this trend is due to the contribution of fluid flow in transferring forward sound waves and damping backward acoustical sound waves.

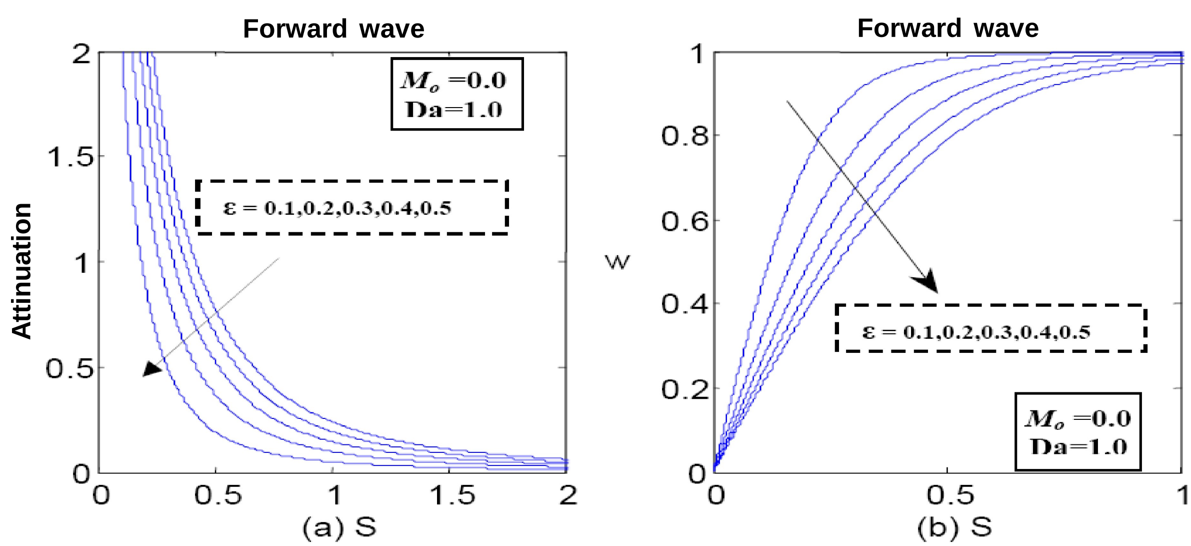

Figure 2 : Attenuation and phase shift of stationary (M_o=0) fluid in plain media (*Da*=1) at various porosity (ε): (a) attenuation (b) phase shift

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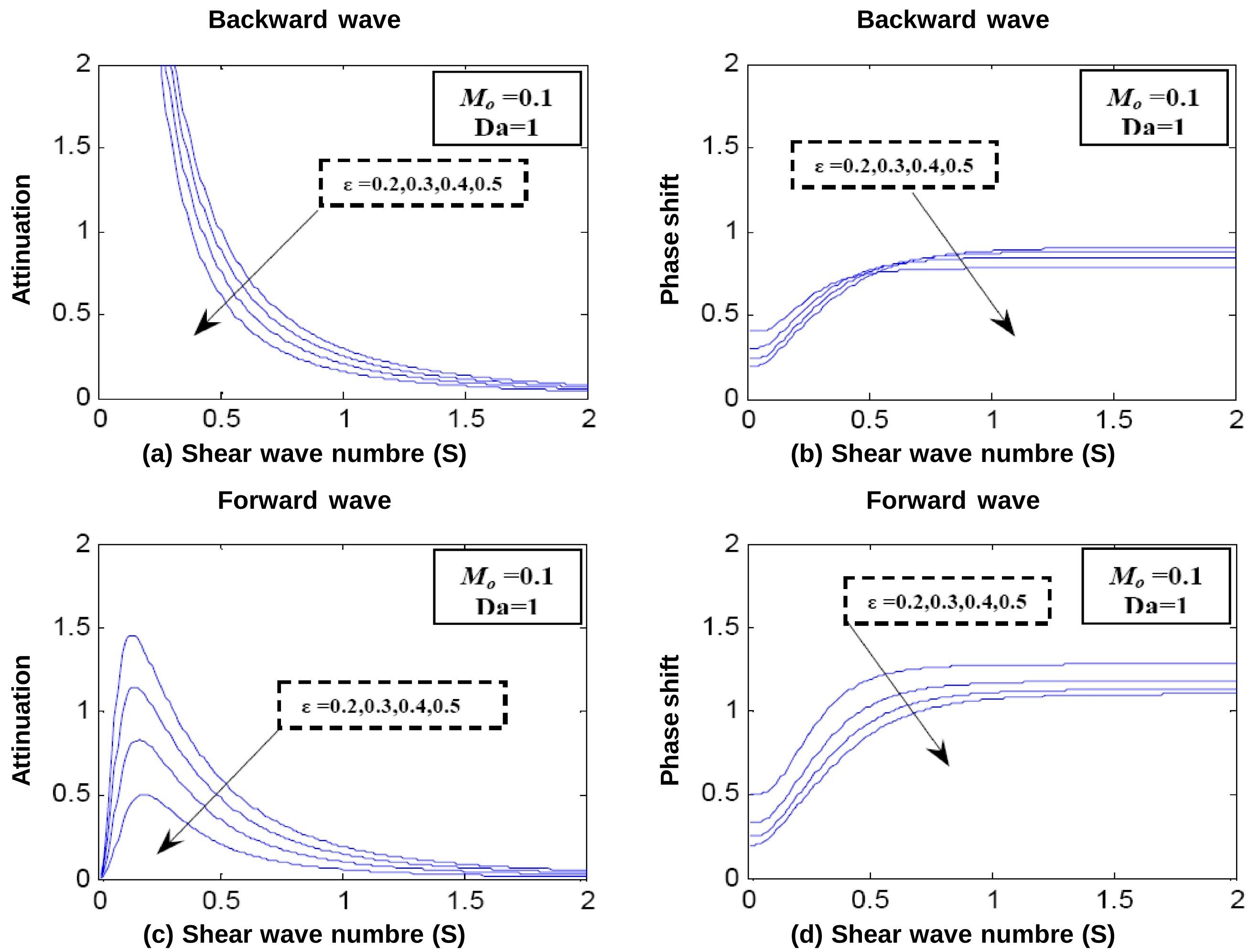

Figure 3 : Forward and backward waves attenuation and phase shift at steady state flow (M_o=0.1) and plain media (*Da*=1): (a) backward wave attenuation (b) backward wave phase shift (c) forward wave attenuation (d) forward wave phase shift

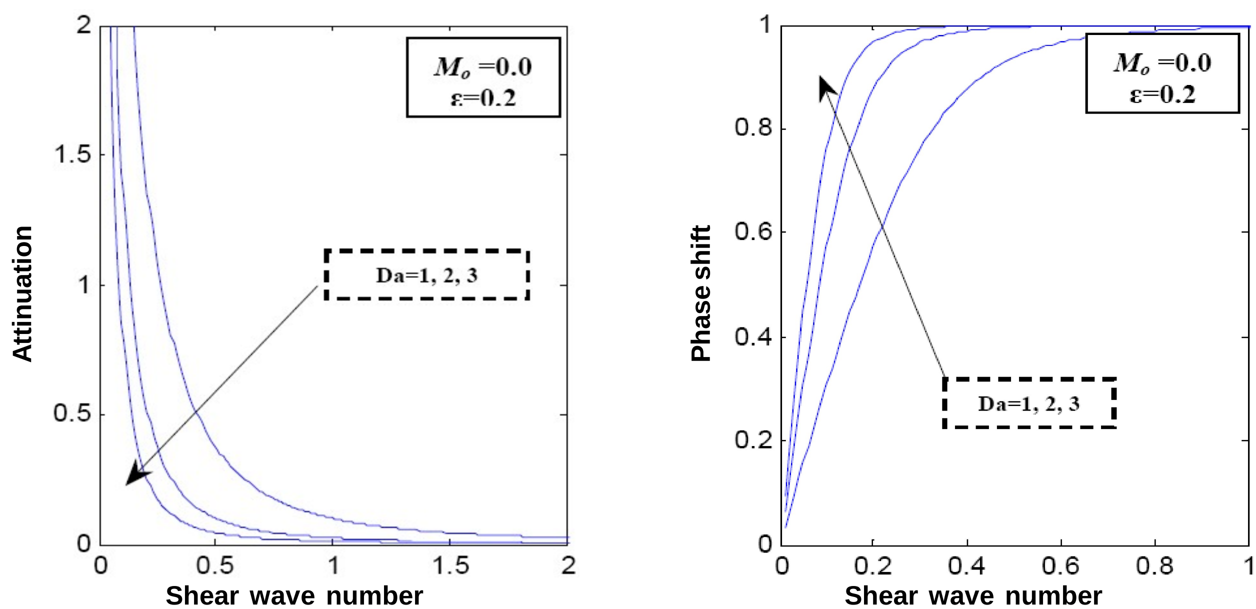

Figure 4 : Attenuation and phase shift of stationary (M_o=0) fluid porosity (ε=0.2) at various Darcy number (Da): (a) attenuation (b) phase shift

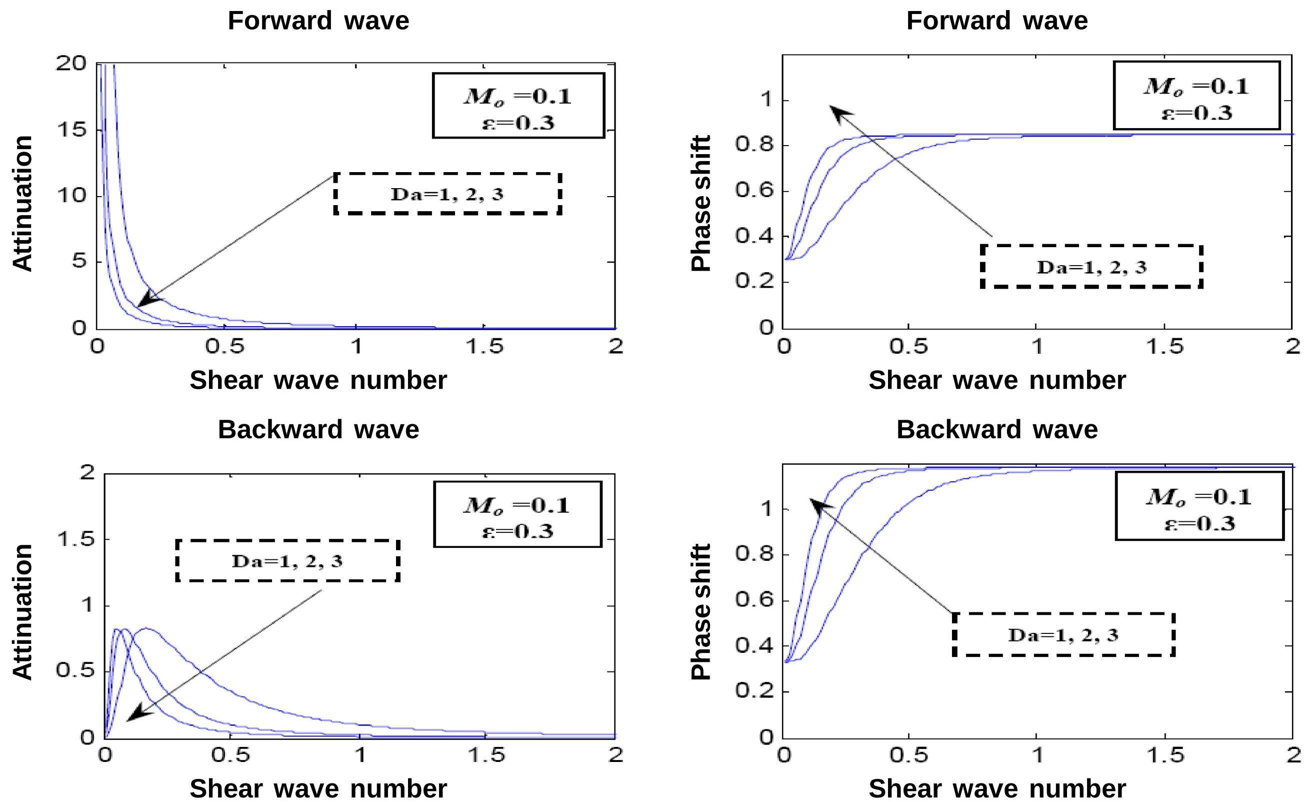

Figure 5 : Forward and backward waves attenuation and phase shift at steady state flow (M_o=0.1) and porosity (ε=0.3): (a) backward wave attenuation (b) backward wave phase shift (c) forward wave attenuation (d) forward wave phase shift

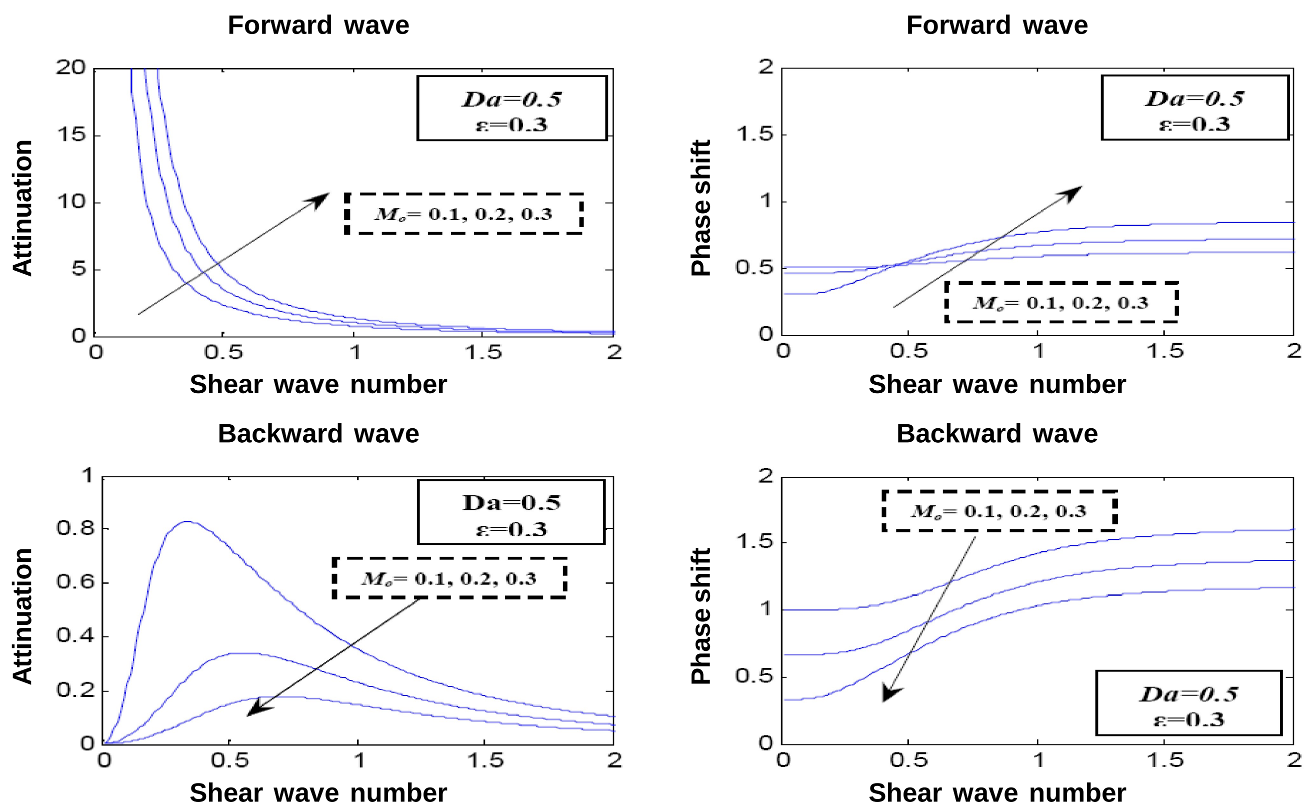

Figure 6 : Forward and backward waves attenuation and phase shift at Darcy number (Da=0.5) and porosity (ε=0.3): (a) forward wave attenuation (b) forward wave phase shift (c) backward wave attenuation (d) backward wave phase shift

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NOMENCLATURE

$\bar{a}$: mean speed of sound of steady flow
Da : Darcy Number, $Da = \sqrt{K}/L$
i : imaginary number
K : Permeability
M_o : Steady flow mach number
p : Acoustic pressure
p_o : Steady flow pressure
S : Shear wave number
t* : time
u* : velocity component in x direction
u : Acoustic velocity in x direction
x* : Axial direction
W : Phase velocity
w : Dimensionless phase velocity

Greek symbols

α : Perturbation parameter
ρ : Acoustic density
$\bar{\rho}$: Mean steady flow density
ρ* : Fluid density
ζ : Dimensionless axial coordinate
η : Dimensionless normal coordinate
γ : Ratio of specific heats
ω : Harmonic disturbance frequency
ε : Porosity
μ : Dynamic viscosity
Γ : Propagation constant
Γ' : Attenuation
Γ'' : Phase shift velocity

CONCLUSIONS

1) A theoretical model for sound waves propagation in saturated porous media is presented, the governing equations are written in dimensionless form than solved using suitable mathematical methods, the following conclusion can be laid out:
2) It is found that the increasing of porosity of solid matrix gives smaller attenuation values for both forward and backward sound waves for a certain value of Darcy number for either moving or stationary fluid inside the solid matrix; this is due to larger damping effects of both the fluid and solid matrix. It is also found that the increasing of porosity gives smaller phase shift and noise duration.
3) It is found that the increasing of the Darcy number had decreased attenuation and increased phase shift; this is due to large pore sizes or higher permabilities of the porous medium. Again the increasing of Darcy number had increased the duration of the acoustical waves propagation in the porous matrix.
4) It is found that the increasing of the fluid velocity increased the attenuation and phase shift of the forward sound waves and decreased their values for backward sound waves; this is due to retardation effects of the fluid velocity inside pores.
5) It is found that the increasing of shear wave number decreased attenuation and increased phase shift; this is due to smaller viscosity effects of the fluid filling the solid matrix or of higher actuating frequencies of the impinging sound waves.

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