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Initial analysis on definite integral methods in solving practical problems

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ABSTRACT

Definite integral methods are widely used in solving practical problems. The methods of solving practical problems in geometry, physics, economics, and so on are discussed in this paper. Mastering some certain integral calculation methods will certainly help to solve some practical problems in life. From these few simple examples, we can see that to solve the practical problems of definite integral, the most important thing is to digitize the problem, and then writing out the formula by using the mathematical theory, and finally calculating the results by using integral principle.

KEYWORDS

Definite integral; Differential element method; Application.

INTRODUCTION

Definite integral is one of the main parts of integral. It is the result of highly abstract of the problems in mathematics, physics, engineering, technology and other areas. The problems of total amount of inhomogeneous distribution can be solved by using definite integral method. [1]Definite integral is not only a basic concept of mathematics, but also a sort of mathematical thinking. It contains one of the most important mathematical ideas to solve practical problems, transforming curve into straight[2]. It is widely applied to solve various practical problems.

The concept of the definite integral came from calculating the areas of the plane figures and solving some other practical problems. It is even early than differential concept and can be traced back to the time of ancient Greece. For example, Greek mathematician, Eudoxus, developed and perfected Antiphon's exhaustive method; Archimedes discovered a quadrature formula named Balance Method. The modern idea of integral was implied in his studies of calculating Arch form area of parabola, the area of spherical cap and sphere. [3]In China, Liuhui put forward Cyclotomic Method and Volume Theory in 263 AD. Both them were also the early idea of integral. Italian mathematician Cavalieri elicited a formula, making early integral calculus breakthrough volume calculation of real prototype and transition to the general algorithm. In the second half of the 17th century, until Newton-Leibniz formula has been established, the definite integral theory was established and developed rapidly then[4]. The Newton-Leibniz formula, reveals the internal relation between indefinite integral and definite integral, given a general, simple and applicable method of calculating definite integral. It also made definite integral to be a powerful tool of solving practical problems, and promote the great development of integral. The concept and the formulas of differential and integral are important innovations not only in the history of mathematics, but also in the history of scientific thought[5].

A lot of literature both in home and abroad such as "mathematical thinking and mathematics philosophy" wrote by Zhou Shuqi, "The Historical Development of the Calculus" wrote by C.H. Edward, "The History of Mathematics" wrote by Scotts introduced the history of development about integral in detail. Domestic research about definite integral is mostly introduced in teaching materials. These materials expatiate on the concepts and nature of definite integral and some simple applications. At present, some textbooks also includes the applications of integral in geometry, physics, biology, economics and so on.

THE DEFINITE INTEGRAL METHODS OF SOLVING PRACTICAL PROBLEMS

Definite integral methods are very practical mathematical methods. A lot of problems in natural science, engineering and technology can turn into mathematical models of Definite Integration, such as, calculating volume of revolution such as parts processed by machines, estimating water pressure on the gate of reservoirs, calculating the minimum costs and maximum profits in economics, calculating the area of irregular figure areas, estimating the volume of composition of organization by slice, and so on. This kind of problems are all additive, geometric or physical additive quantitys can be caculated by definite integral methods[6].

The process of differential element methods includes segmentation and approximation, summation and limitation. The concrete steps of the process of differential element methods are: drawing and figuring out intersection, determining the integration interval, selecting the integral variables, finding out its micro elements, turning the micro elements into definite integral and doing the calculation. The following five examples demonstrate the methods of solving practical problems by establishing the definite integral mathmetic models using differential element methods.

Definite Integral methods are widely used in solving practical problems in life. Here are some very common examples in our daily lives. The practical applications of definite integration in geometry, physics, biology, and economics are demonstrated by using the following examples, to t.

In Geometric: Calculating the area of irregular figure areas and the volume of revolving bodies

Example 1: There is a flower bed designed by a bureau of parks and woods, it is a graphic bounded by two curves, $g(x) = x^2$ and $h(x) = \sqrt{x}$, please calculate its area. Solution: the graphic bounded by the two curves is shown in Figure1, the intersections of the two curves are （0, 0）and (1, 1), so the micro element of the area is $ds = (\sqrt{x} - x^2)\cdot dx$. The integration interval is $[0,1]$, Figure1. So the asked area is: $s = \int_0^1 (\sqrt{x} - x^2)dx = \frac{1}{3}$

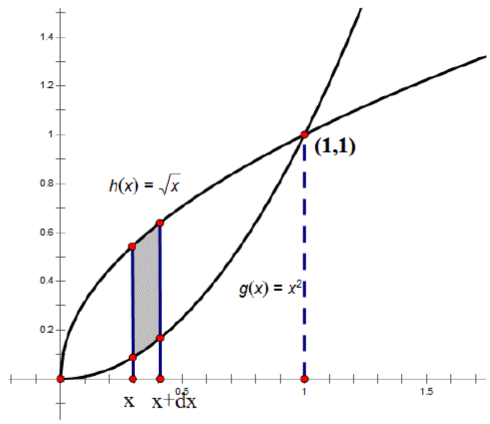

Figure 1

Example 2: There is a part, a rotating solid (Figure 2), made by a lathe. It is a form formed by a Curved trapezoid which was bounded by a curve $y^2 = x$ and three lines, $x=1, x=4$ and the x axis rotated one circle by the x axis, please calculate its volume.

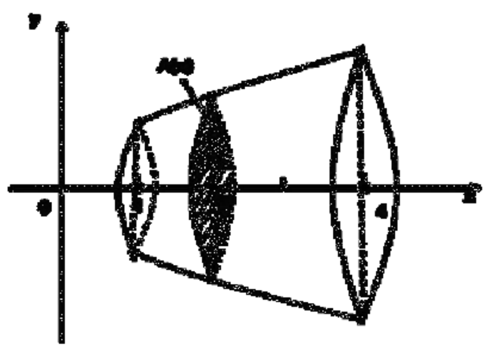

Figure 2

Solution: The cross-sectional area of the rotating solid on the x axis A (x) is

$$A(x) = \pi y^2(x)$$

The volume micro element of the solid is:

$$dv = A(x)dx = \pi y^2 dx$$

The integration interval is $[1,4]$

The asked volume is : Figure 2

$$v = \int_1^4 \pi y^2 dx = \pi\int_1^4 x dx = \frac{15\pi}{2}$$

The two above examples are very common in life. It is made in example one that a brief analysis on calculating the area of irregular figure areas. In practical, the processes of solving this kind of problems are: first, establishing mathematical models, then approximating the graphical element, finding out the approximate function about the graphic, writing out the equation by using the mathematical theory, and finally calculating the results by using the integral principle. The calculating process in example two is very similar to that in example one. In a word, the methods of solving the above problems are widely used in solving problems in geometric.

In Physics: solving the problems of work done by variable forces and lateral pressure of liquor and so on

Example 3: There is a gate of a reservoir, its form and size are both shown in Figure 3, the height of the surface of the water to the top side of the gate is 2m, please calculate the water pressure on the gate.

Solution: To establish a coordinate like Figure 3, they axis of the coordinate is on the top line of the gate, the x axis of the coordinate is plummeting. Draw a rectangle whose bottom width is 2m and perpendicular to the x axis on point x. The area of the rectangle is : $ds = 2 \cdot dx$. The pressure on the rectangle is approximate to the lateral pressure when the rectangle is perpendicular to the liquor surface and is in the depth at point x. so the micro pressure element is: $dp = \rho gxds = \rho gx \cdot 2dx$.

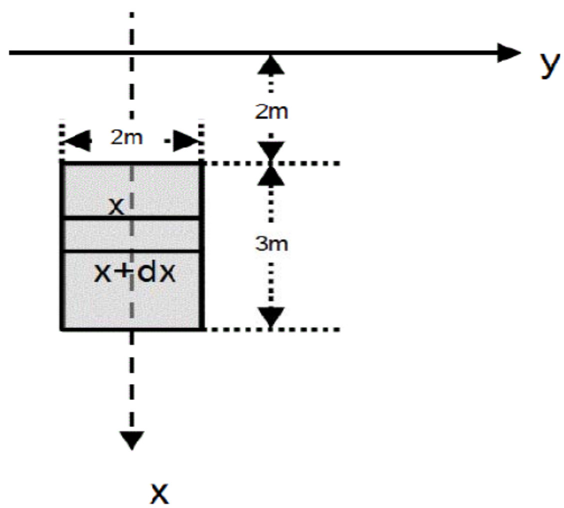

Figure 3

The integration interval is : [2,5] Figure 3, The pressure is:

$$p = \int_2^5 dp = \int_2^5 \rho gx \cdot 2dx$$

$$= 2\rho g \int_2^5 xdx = 2.06 \times 10^5 (N) \quad (\rho = 10^3 (kg/m^3), \quad g = 9.8 m/s^2)$$

Example 3 is very common in life and it is also very simple. Moreover, the integral methods are not commonly used in practical because they are relatively a little complicated. We use simpler methods. However; the integral methods are more standard methods. This example is the integral application in physics. In fact, the integral method is originated from physics. Definite integral is first put forward by Newton, a well-known physicist, and later a mathematician called Leibniz put forward the calculation method of integral in mathematics, and the formula put forward by them named the Newton- Leibniz Formula.

In Biology

Studying the structure of organization, estimating the volume of composition of organization by slice (usually less than a few microns), calculating the average velocity of blood in the intersection of the blood vessel, calculating the growth weight of creature in some period, and so on.

Example 4: In some feeding bacteria circumstance, the rate of the number of bacteria growth is $\frac{dN}{dt} = 4 + 6t \ (5 \le t \le 10)$, please calculate the total number of the bacteria produced in this period.

Solution: According to the question, the number of the bacteria is the definite integration of the given function on $[5,10]$

$$N = \int_5^{10} (4+6t)dt = 245$$

Example 4 is the integral application in biology. The approach method applied in this example is similar to that in example one. Although integral can be applied in biology, but in practical, we must pay attention to the application range of these kind digital models. Only when the variables of problems conform to the requirements of the function and when the data is within the range of operation, we can establish the digital models and use mathematic methods to solve this kind of problems in biology.

In Economics: Calculating and accounting the total output, total cost, total profit, etc

Example 5: In a company, the marginal revenue and marginal cost of a product are

$R'(t) = q - t^{\frac{1}{3}}$ （10,000 yuan ／y） $C'(t) = 1 + 3t^{\frac{1}{3}}$ (10,000 yuan／y).

Please calculate the best operation period and the total profit in this period (the fixed cost is 100,000yuan and q is a real number).

Solution: （1） the total cost is : $c(t)$ =the fixed cost +variable cost

$$c(t) = c(0) + \int_0^t (1 + 3t^{\frac{1}{3}})dt = 10 + t + \frac{9}{4}t^{\frac{4}{3}}$$

The total revenue is: $R(t) = \int_0^t (q - t^{\frac{1}{3}})dt = qt - \frac{3}{4}t^{\frac{4}{3}}$

The total profit is: $L(t)$ =R (t)-C (t)

$$L(t) = (qt - \frac{3}{4}t^{\frac{4}{3}}) - (10 + t + \frac{9}{4}t^{\frac{4}{3}}) = (q-1)t - 3t^{\frac{4}{3}} - 10$$

（2） $L'(t) = q - 1 - 4t^{\frac{1}{3}}$, let $L'(t) = 0$, then the only is stagnation point： $t = (\frac{q-1}{4})^3$, it is the best operation time.

When $t = \frac{(q-1)^3}{64}$ ▪ , $L(t)$ get its maximum value.

$$L(t) = (q-1) \cdot [\frac{(q-1)}{4}]^3 - 3[\frac{(q-1)}{4}]^{3 \cdot \frac{4}{3}} - 10$$

$$=\frac{(q-1)^4}{256}-10$$

Example five is the integral application in economics, and also very common. And the integral method used in this example is similar to the above examples.

CONCLUSIONS

The five examples are all from practical life and they are all very common. This kinds of integral method used in the above examples are the simplest and most widely used type of integral in practical. This does not include curved surface integration method, curve integration method, double integration method and triple integration method. But the above examples illustrate that definite integral methods are very commonly used in solving practical problems. Therefore, mastering some certain integral calculation methods will certainly help to solve some practical problems in life. From these few simple examples, we can see that to solve the practical problems of definite integral, the most important thing is to digitize the problem, and then writing out the formula by using the mathematical theory, and finally calculating the results by using integral principle.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this article.

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