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Extended F-expansion method for (2+1)-dimensional B-type Kadomtsev-Petviashvili equation

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ABSTRACT

Extended F-expansion method is proposed to seek exact solutions of B-type Kadomtsev-Petviashvili equation. Many new travelling wave solutions are obtained, including Jacobi elliptic function solutions, soliton-like solutions and trigonometric function solutions.

KEYWORDS

Extended F-expansion method;
Nonlinear partial differential equations;
Nonlinear physical phenomena;
(2+1)-dimensional B-type Kadomtsev-Petviashvili equation.

INTRODUCTION

The well-known Kadomtsev-Petviashvili (KP) equation was first written in 1970 by Soviet physicists Boris B. Kadomtsev and Vladimir I. Petviashvili[1]. The KP equation can be used to model water waves of long wavelength with weakly non-linear restoring forces and frequency dispersion[2-4] and ion-acoustic waves in plasmas[5]. As a subclass of the KP equation is the B-type Kadomtsev-Petviashvili (BKP) equation which describes the processes of interaction of exponentially localized structures. It is one of a hierarchy of integrable systems emerging from a bilinear identity related to a Clifford algebra which is generated by two neutral fermion fields[6]. BKP equation can be written in the form

$$\mathbf{u_t = u_{xxx} + u_{yyy}\text{þ} + 6(uv)_x + 6(uw)_y,} \\ \mathbf{u_x = v_y, \quad u_y = w_x,} \qquad (1)$$

where $u(x, y, t)$, $v(x, y, t)$ and $w(x, y, t)$ are dependent variables with respect to x, y and t. The KP equation is associated with an A-type group and the BKP equation with a B-type group which can be used in Bosonic Picture and Fermionic fields[7,8].

It is known that many physical phenomena are often described by nonlinear evolution equations (NLEEs). Integrable systems and NLEEs have recently attracted much attention of mathematicians as well as physicists. Many methods for obtaining explicit travelling solitary wave solutions to NLEEs have been proposed. Among these are the tanh methods[9-11], Jacobi elliptic function (JEF) expansion methods[12-14], Hirota's bilinear methods[15-17], Exp-function method[18-19], the inverse scattering transform[20-22] and so on. Recently F-expansion method[23-29] was proposed to obtain periodic wave solutions of NLEEs, which can be thought of as a concentration of JEF expansion since F here stands for everyone of JEFs.

In this paper, we extend the extended F-expansion (EFE) method with symbolic computation to Eq. (1) for constructing their interesting Jacobi doubly periodic wave solutions. It is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. In addition the algorithm that we use here also a computerized method, in which generating an algebraic system.

EXTENDED F-EXPANSION METHOD

In this section, we inmtroduce a simple description of the EFE method, for a given partial differential equation

$$G(u,u_x,u_y,u_z,u_{xy},\ldots)=0. \quad (2)$$

We like to know whether travelling waves (or stationary waves) are solutions of Eq. (2). The first step is to unite the independent variables x, y and t into one particular variable through the new variable

$$\zeta = x + y - \nu t, \quad u(x,y,t) = U(\zeta)$$

where ν is wave speed, and reduce Eq. (2) to an ordinary differential equation (ODE)

$$G(U,U',U'',U''',\ldots)=0. \quad (3)$$

Our main goal is to derive exact or at least approximate solutions, if possible, for this ODE. For this purpose, let us simply U as the expansion in the form,

$$u(x,y,t) = U(\zeta) = \sum_{i=0}^{N} a_i F^i + \sum_{i=1}^{N} a_{-i} F^{-i}, \quad (4)$$

where

$$F' = \sqrt{A + BF^2 + CF^4}, \quad (5)$$

the highest degree of $\frac{d^pU}{d\zeta^p}$ is taken as

$$O\left(\frac{d^pU}{d\zeta^p}\right) = N + p, \qquad p = 1,2,3,\cdots, \quad (6)$$

$$O\left(U^q \frac{d^pU}{d\zeta^p}\right) = (q+1)N + p, \quad q = 0,1,2,\cdots, p = 1,2,3,\cdots. \quad (7)$$

Where A, B and C are constants, and N in Eq. (3) is a positive integer that can be determined by balancing the nonlinear term (s) and the highest order derivatives. Normally N is a positive integer, so that an analytic solution in closed form may be obtained. Substituting Eqs. (2)- (5) into Eq. (3) and comparing the coefficients of each power of $F(\zeta)$ in both sides, to get an over-determined system of nonlinear algebraic equations with respect to ν, a_0, a_1, ... Solving the over-determined system of nonlinear algebraic equations by use of Mathematica. The relations between values of A, B, C and corresponding JEF solution $F(\zeta)$ of Eq. (4) are given in TABLE 1. Substitute the values of A, B, C and the corresponding JEF solution $F(\zeta)$ chosen from TABLE 1 into the general form of solution, then an ideal periodic wave solution expressed by JEF can be obtained.

Where $\mathrm{sn}(\zeta)$, $\mathrm{cn}(\zeta)$ and $\mathrm{dn}(\zeta)$ are the JE sine function, JE cosine function and the JEF of the third kind, respectively. And

$$\mathrm{cn}^2(\zeta) = 1 - \mathrm{sn}^2(\zeta), \qquad \mathrm{dn}^2(\zeta) = 1 - m^2\mathrm{sn}^2(\zeta), \quad (8)$$

with the modulus m $(0 < m < 1)$.

When $m \to 1$, the Jacobi functions degenerate to the hyperbolic functions, i.e.,

$$\mathrm{sn}\zeta \to \tanh\zeta, \quad \mathrm{cn}\zeta \to \mathrm{sech}\zeta, \quad \mathrm{dn}\zeta \to \mathrm{sech}\zeta,$$

when $m \to 0$, the Jacobi functions degenerate to the triangular functions, i.e.,

$$\mathrm{sn}\zeta \to \sin\zeta, \quad \mathrm{cn}\zeta \to \cos\zeta \quad \text{and} \quad \mathrm{dn} \to 1.$$

B-TYPE KADOMTSEV-PETVIASHVILI EQUATION

We consider the BKPE (1)

$$\begin{aligned} &u_t = u_{xxx} + u_{yyy}þ + 6(uv)_x + 6þ(uw)_y \\ &u_x = v_y, \quad u_y = w_x, \end{aligned} \quad (9)$$

if we use $\zeta = xþ + y + \nu t$ carries Eq. (9) into the system of ODEs

$$\begin{aligned} &2U''' - \nu U' + 6(UV)' + 6(UW)' = 0, \\ &U' = V', \quad U' = W', \end{aligned} \quad (10)$$

where by integrating once we obtain, upon setting the constant of integration to zero,

$$\begin{aligned} &2U'' - \nu U + 6UV + 6UW = 0, \\ &U = V, \quad U = W, \end{aligned} \quad (11)$$

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if we use the second and third equations in (11) into the first one, we find

$$2U'' - \nu U + 12U^2 = 0. \quad (12)$$

Balancing the term U'' with the term U^2 we obtain $N = 2$ then

$$U(\zeta) = a_0 + a_1 F + a_{-1}F^{-1} + a_2 F^2 + a_{-2}F^{-2},$$
$$F' = \sqrt{A + BF^2 + CF^4}. \quad (13)$$

Substituting Eq. (13) into Eq. (12) and comparing the coefficients of each power of F in both sides, to get an over-determined system of nonlinear algebraic equations with respect to ν, a_i, $i = 1, -1, -2, 2$. Solving the over-determined system of nonlinear algebraic equations by use of Mathematica, we obtain three groups of constants:

$$a_{-1} = a_{-1} = 0,\ a_0 = \frac{-B + \sqrt{B^2 + 12AC}}{3},\ a_2 = -C,\ a_{-2} = -A \text{ and } \nu = \pm 8\sqrt{B^2 + 12AC}, \quad (14)$$

$$a_{-1} = a_{-1} = a_{-2} = 0,\ a_0 = \frac{-B + \sqrt{B^2 - 3AC}}{3},\ a_2 = -C, \text{ and } \nu = \pm 8\sqrt{B^2 - 3AC}, \quad (15)$$

$$a_{-1} = a_{-1} = a_2 = 0,\ a_0 = \frac{-B + \sqrt{B^2 - 3AC}}{3},\ a_{-2} = -A \text{ and } \nu = \pm 8\sqrt{B^2 - 3AC}, \quad (16)$$

the solutions of Eq. (9) are:

$$w_1 = v_1 = u_1 = \frac{1}{3(1 + m^2 + \sqrt{12m^2 + (1+m^2)^2})} - m^2 sn^2(x + y \pm \sqrt{12m^2 + (1+m^2)^2}\,t) - ns^2(x + y \pm \sqrt{12m^2 + (1+m^2)^2}\,t), \quad (17)$$

$$w_2 = v_2 = u_2 = \frac{1}{3(1 + m^2 + \sqrt{12m^2 + (1+m^2)^2})} - m^2 cd^2(x + y \pm \sqrt{12m^2 + (1+m^2)^2}\,t) - dc^2(x + y \pm \sqrt{12m^2 + (1+m^2)^2}\,t), \quad (18)$$

$$w_3 = v_3 = u_3 = \frac{1}{3(1 - 2m^2 + \sqrt{(2m^2-1)^2 - 12m^2(1-m^2)})} + m^2 cn^2(x + y \pm \sqrt{(2m^2-1)^2 - 12m^2(1-m^2)}\,t) - (1-m^2)nc^2(x + y \pm \sqrt{(2m^2-1)^2 - 12m^2(1-m^2)}\,t), \quad (19)$$

$$w_4 = v_4 = u_4 = \frac{1}{3(m^2 - 2 + \sqrt{(2-m^2)^2 - 12(-1+m^2)})} + dn^2(x + y \pm \sqrt{(2-m^2)^2 - 12(-1+m^2)}\,t) - (m^2-1)nd^2(x + y \pm \sqrt{(2-m^2)^2 - 12(-1+m^2)}\,t), \quad (20)$$

$$w_5 = v_5 = u_5 = \frac{1}{3(1 - 0.5m^2 + \sqrt{0.75m^4 + (-1+0.5m^2)^2})} - \frac{m^2}{4}\left(sn(x + y \pm \sqrt{(-1+0.5m^2)^2 - \frac{3m^4}{16}}\,t) + ics(x + y \pm \sqrt{(-1+0.5m^2)^2 - \frac{3m^4}{16}}\,t)\right)^2, \quad (21)$$

$$w_6 = v_6 = u_6 = \frac{1}{3(-2 + m^2 + \sqrt{12(1-m^2) + (2-m^2)^2})} - cs^2(x + y \pm \sqrt{12(1-m^2) + (2-m^2)^2}\,t) - (1-m^2)sc^2(x + y \pm \sqrt{12(1-m^2) + (2-m^2)^2}\,t), \quad (22)$$

$$w_7 = v_7 = u_7 = \frac{1}{3(1 - 2m^2 + \sqrt{12m^2(m^2-1) + (1-2m^2)^2})} - ds^2(x + y \mp \sqrt{12m^2(m^2-1) + (1-2m^2)^2}\,t) + m^2(1+m^2)sd^2(x + y \pm \sqrt{12m^2(m^2-1) + (1-2m^2)^2}\,t), \quad (23)$$

$$w_8 = v_8 = u_8 = \frac{1}{3(m^2 - 0.5 + \sqrt{0.75 + (0.5 - m^2)^2})} - \frac{1}{4}(ns(x + y \pm \sqrt{0.75 + (0.5 - m^2)^2}t) + cs(x + y \pm \sqrt{0.75 + (0.5 - m^2)^2}t))^2 - \frac{1}{4}(ns(x + y \pm \sqrt{0.75 + (0.5 - m^2)^2}t) + cs(x + y \pm \sqrt{0.75 + (0.5 - m^2)^2}t))^{-2}, \quad (24)$$

$$w_9 = v_9 = u_9 = \frac{1}{3(\sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2} - 0.5 - 0.5m^2)} - (0.5 - 0.5m^2)(nc(x + y \pm \sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2}t) + sc(x + y \pm \sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2}t))^2 - (0.25 - 0.25m^2)(nc(x + y \pm \sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2}t) + sc(x + y \pm \sqrt{12(0.5 - 0.5m^2)(0.25 - 0.25m^2) + (0.5 + 0.5m^2)^2}t))^{-2}, \quad (25)$$

$$w_{10} = v_{10} = u_{10} = \frac{1}{3(1 - 0.5m^2 + \sqrt{0.75m^2 + (0.5m^2 - 1)^2})} - \frac{m^2}{4}(ns(x + y \pm \sqrt{0.75m^2 + (0.5m^2 - 1)^2}t) + ds(x + y \pm \sqrt{0.75m^2 + (0.5m^2 - 1)^2}t))^2 - \frac{1}{4}(ns(x + y \pm \sqrt{0.75m^2 + (0.5m^2 - 1)^2}t) + ds(x + y \pm \sqrt{0.75m^2 + (0.5m^2 - 1)^2}t))^{-2}, \quad (26)$$

$$w_{11} = v_{11} = u_{11} = \frac{1}{3(1 - 0.5m^2 + \sqrt{0.75m^4 + (0.5m^2 - 1)^2})} - \frac{m^2}{4}(sn(x + y \pm \sqrt{0.75m^4 + (0.5m^2 - 1)^2}t) + ics(x + y \pm \sqrt{0.75m^4 + (0.5m^2 - 1)^2}t))^2 - \frac{m^2}{4}(sn(x + y \pm \sqrt{0.75m^4 + (0.5m^2 - 1)^2}t) + ics(x + y \pm \sqrt{0.75m^4 + (0.5m^2 - 1)^2}t))^{-2}, \quad (27)$$

$$w_{12} = v_{12} = u_{12} = \frac{1}{3(1 + m^2 + \sqrt{(1 + m^2)^2 - 3m^2})} - ns^2(x + y \pm \sqrt{(1 + m^2)^2 - 3m^2}t), \quad (28)$$

$$w_{13} = v_{13} = u_{13} = \frac{1}{3(1 + m^2 + \sqrt{(1 + m^2)^2 - 3m^2})} - dc^2(x + y \pm \sqrt{(1 + m^2)^2 - 3m^2}t), \quad (29)$$

$$w_{14} = v_{14} = u_{14} = \frac{1}{3(1 - 2m^2 + \sqrt{(2m^2 - 1)^2 + 3m^2(1 - m^2)})} - (1 - m^2)nc^2(x + y \pm \sqrt{(2m^2 - 1)^2 + 3m^2(1 - m^2)}t), \quad (30)$$

$$w_{15} = v_{15} = u_{15} = \frac{1}{3(m^2 - 2 + \sqrt{(2 - m^2)^2 + 3(m^2 - 1)})} - (m^2 - 1)nd^2(x + y \pm \sqrt{(2 - m^2)^2 + 3(m^2 - 1)}t), \quad (31)$$

$$w_{16} = v_{16} = u_{16} = \frac{1}{3(m^2 - 2 + \sqrt{3(m^2 - 1) + (2 - m^2)^2})} - (1 - m^2)sc^2(x + y \pm \sqrt{3(m^2 - 1) + (2 - m^2)^2}t), \quad (32)$$

$$w_{17} = v_{17} = u_{17} = \frac{1}{3(1 - 2m^2 + \sqrt{3m^2(1 - m^2) + (1 - 2m^2)^2})} + m^2(1 + m^2)sd^2(x + y \pm \sqrt{3m^2(1 - m^2) + (1 - 2m^2)^2})t), \quad (33)$$

$$w_{18} = v_{18} = u_{18} = \frac{1}{3(m^2 - 0.5 + \sqrt{(0.5 - m^2)^2 - \frac{3}{16}})} - \frac{1}{4}(ns(x + y \pm \sqrt{(0.5 - m^2)^2 - \frac{3}{16}}t) + cs(x + y \pm \sqrt{(0.5 - m^2)^2 - \frac{3}{16}}t))^{-2}, \quad (34)$$

$$w_{19}=v_{19}=u_{19}=\frac{1}{3(\sqrt{3(0.5m^2-0.5)(0.25-0.25m^2)+(0.5+0.5m^2)^2}-0.5-0.5m^2)} -(0.25-0.25m^2)(nc(x+y\pm\sqrt{3(0.5m^2-0.5)(0.25-0.25m^2)+(0.5+0.5m^2)^2}t) +sc(x+y\pm\sqrt{3(0.5m^2-0.5)(0.25-0.25m^2)+(0.5+0.5m^2)^2}t))^{-2}, \quad (35)$$

$$w_{20}=v_{20}=u_{20}=\frac{1}{3(1-0.5m^2+\sqrt{(0.5m^2-1)^2-\frac{3m^2}{16}})} -\frac{1}{4}(ns(x+y\pm\sqrt{(0.5m^2-1)^2-\frac{3m^2}{16}}t)+ds(x+y\pm\sqrt{(0.5m^2-1)^2-\frac{3m^2}{16}}t))^{-2}, \quad (36)$$

$$w_{21}=v_{21}=u_{21}=\frac{1}{3(1-0.5m^2+\sqrt{(0.5m^2-1)^2-\frac{3m^4}{16}})} -\frac{m^2}{4}(sn(x+y\pm\sqrt{(0.5m^2-1)^2-\frac{3m^4}{16}}t)+ics(x+y\pm\sqrt{(0.5m^2-1)^2-\frac{3m^4}{16}}t))^{-2}, \quad (37)$$

$$w_{22}=v_{22}=u_{22}=\frac{1}{3(1+m^2+\sqrt{(1+m^2)^2-3m^2})}-m^2sn^2(x+y\pm\sqrt{(1+m^2)^2-3m^2}t), \quad (38)$$

$$w_{23}=v_{23}=u_{23}=\frac{1}{3(1+m^2+\sqrt{(1+m^2)^2-3m^2})}-m^2cd^2(x+y\pm\sqrt{(1+m^2)^2-3m^2}t), \quad (39)$$

$$w_{24}=v_{24}=u_{24}=\frac{1}{3(1-2m^2+\sqrt{(2m^2-1)^2+3m^2(1-m^2)})} +m^2cn^2(x+y\pm\sqrt{(2m^2-1)^2+3m^2(1-m^2)}t), \quad (40)$$

$$w_{25}=v_{25}=u_{25}=\frac{1}{3(m^2-2+\sqrt{(2-m^2)^2+3(-1+m^2)})} +dn^2(x+y\pm\sqrt{(2-m^2)^2+3(-1+m^2)}t), \quad (41)$$

$$w_{26}=v_{26}=u_{26}=\frac{1}{3(-2+m^2+\sqrt{3(m^2-1)+(2-m^2)^2})} -cs^2(x+y\pm\sqrt{3(m^2-1)+(2-m^2)^2}t), \quad (42)$$

$$w_{27}=v_{27}=u_{27}=\frac{1}{3(1-2m^2+\sqrt{3m^2(1-m^2)+(1-2m^2)^2})} -ds^2(x+y\mp\sqrt{3m^2(1-m^2)+(1-2m^2)^2}t), \quad (43)$$

$$w_{28}=v_{28}=u_{28}=\frac{1}{3(m^2-0.5+\sqrt{(0.5-m^2)^2-\frac{3}{16}})} -\frac{1}{4}(ns(x+y\pm\sqrt{(0.5-m^2)^2-\frac{3}{16}}t)+cs(x+y\pm\sqrt{(0.5-m^2)^2-\frac{3}{16}}t))^2, \quad (44)$$

$$w_{29}=v_{29}=u_{29}=\frac{1}{3(\sqrt{3(0.5m^2-0.5)(0.25-0.25m^2)+(0.5+0.5m^2)^2}-0.5-0.5m^2)} -(0.5-0.5m^2)(nc(x+y\pm\sqrt{3(0.5m^2-0.5)(0.25-0.25m^2)+(0.5+0.5m^2)^2}t) +sc(x+y\pm\sqrt{3(0.5m^2-0.5)(0.25-0.25m^2)+(0.5+0.5m^2)^2}t))^2, \quad (45)$$

$$w_{30}=v_{30}=u_{30}=\frac{1}{3(1-0.5m^2+\sqrt{(-1+0.5m^2)^2-\frac{3m^2}{16}})} -\frac{m^2}{4}(ns(x+y\pm\sqrt{(-1+0.5m^2)^2-\frac{3m^2}{16}}t)+ds(x+y\pm\sqrt{(-1+0.5m^2)^2-\frac{3m^2}{16}}t))^2. \quad (46)$$

Soliton solutions

Some solitary wave solutions can be obtained, if the modulus m approaches to 1 in Eqs. (17)-(46)

$$w_{31}=v_{31}=u_{31}=\frac{1}{18}-\tanh^2(x+y\pm 4t)-\coth^2(x+y\pm 4t), \quad (47)$$

$$w_{32}=v_{32}=u_{32}=\frac{2}{9}-\frac{1}{4}(\tanh(x+y\pm\frac{1}{4}t)+i\,\mathrm{csch}(x+y\pm\frac{1}{4}t))^2, \quad (48)$$

$$w_{33}=v_{33}=u_{33}=\frac{2}{9}-\frac{1}{4}(\coth(x+y\pm t)+\mathrm{csch}(x+y\pm t))^2 -\frac{1}{4}(\coth(x+y\pm t)+\mathrm{csch}(x+y\pm t))^{-2}, \quad (49)$$

$$w_{34}=v_{34}=u_{34}=\frac{2}{9}-\frac{1}{4}(\tanh(x+y\pm t)+i\,\mathrm{csch}(x+y\pm t))^2 -\frac{1}{4}(\tanh(x+y\pm t)+i\,\mathrm{csch}(x+y\pm t))^{-2}, \quad (50)$$

$$w_{35}=v_{12}=u_{12}=\frac{1}{9}-\coth^2(x+y\pm t), \quad (51)$$

$$w_{36}=v_{18}=u_{18}=\frac{1}{3}-\frac{1}{4}(\coth(x+y\pm\frac{1}{2}t)+\mathrm{csch}(x+y\pm\frac{1}{2}t))^{-2}, \quad (52)$$

$$w_{37}=v_{37}=u_{37}=\frac{1}{3}-\frac{1}{4}(\tanh(x+y\pm\frac{1}{2}t)+i\,\mathrm{csch}(x+y\pm\frac{1}{2}t))^{-2}, \quad (53)$$

$$w_{38}=v_{38}=u_{38}=\frac{1}{9}-\tanh^2(x+y\pm t), \quad (54)$$

$$w_{39}=v_{39}=u_{39}=\frac{1}{3}-\frac{1}{4}(\coth(x+y\pm\frac{1}{2}t)+\mathrm{csch}(x+y\pm\frac{1}{2}t))^2. \quad (55)$$

Triangular periodic solutions

Some trigonometric function solutions can be obtained, if the modulus m approaches to zero in Eqs. (17)-(46)

$$w_{40}=v_{40}=u_{40}=\frac{1}{6}-\sin^2(x+y\pm t), \quad (56)$$

$$w_{41}=v_{41}=u_{41}=-\frac{1}{3}-\tan^2(x+y\pm t), \quad (57)$$

$$w_{42}=v_{42}=u_{42}=\frac{1}{6}-\csc^2(x+y\pm t), \quad (58)$$

$$w_{43}=v_{43}=u_{43}=\frac{1}{6}-\sec^2(x+y\pm t), \quad (59)$$

$$w_{44}=v_{44}=u_{44}=\frac{1}{6}-\cot^2(x+y\pm 4t)-\tan^2(x+y\pm 4t), \quad (60)$$

$$w_{45}=v_{45}=u_{45}=-\frac{4}{3}-\frac{1}{4}(\csc(x+y\pm\frac{1}{4}t)+\cot(x+y\pm\frac{1}{4}t))^{-2}, \quad (61)$$

$$w_{46}=v_{46}=u_{46}=-\frac{1}{3}-\cot^2(x+y\pm t), \quad (62)$$

$$w_{47}=v_{47}=u_{47}=-\frac{4}{3}-\frac{1}{4}(\csc(x+y\pm\frac{1}{4}t)+\cot(x+y\pm\frac{1}{4}t))^2, \quad (63)$$

$$w_{48}=v_{48}=u_{48}=\frac{2}{3}-\frac{1}{4}(\csc(x+y\pm\frac{1}{2}t)+\cot(x+y\pm\frac{1}{2}t))^2 -\frac{1}{4}(\csc(x+y\pm\frac{1}{2}t)+\cot(x+y\pm\frac{1}{2}t))^{-2}, \quad (64)$$

$$w_{49}=v_{49}=u_{49}=\frac{1}{3(\sqrt{\frac{14}{8}}-0.5)}-0.5(\sec(x+y\pm\sqrt{\frac{14}{8}}t)+\tan(x+y\pm\sqrt{\frac{14}{8}}t))^2 -0.25(\sec(x+y\pm\sqrt{\frac{14}{8}}t)+\tan(x+y\pm\sqrt{\frac{14}{8}}t))^{-2}. \quad (65)$$

The modulus of solitary wave solutions u_1, u_2, u_{21} and u_{23} are displayed in figures 1, 2, 3 and 4 respectively, with values of parameters listed in their captions.

TABLE 1 : Relation between values of (A,B,C) and corresponding F

A	B	C	F(ζ)
1	-1-m^2	m^2	sn(ζ) or cd(ζ)=$\frac{cn(\zeta)}{dn(\zeta)}$
1-m^2	2m^2-1	-m^2	cn(ζ)
m^2-1	2-m^2	-1	dn(ζ)
m^2	-1-m^2	1	ns(ζ)$\frac{1}{sn(\zeta)}$ or dc(ζ)=$\frac{dn(\zeta)}{cn(\zeta)}$
-m^2	2m^2-1	1-m^2	nc(ζ)=$\frac{1}{cn(\zeta)}$
-1	2-m^2	m^2-1	nd (ζ)=$\frac{1}{dn(\zeta)}$
1	2-m^2	1-m^2	sc (ζ)=$\frac{sn(\zeta)}{cn(\zeta)}$
1	2m^2-1	-m^2 (-1-m^2)	sd(ζ)=$\frac{sn(\zeta)}{dn(\zeta)}$
1-m^2	2-m^2	1	cs(ζ)=$\frac{cn(\zeta)}{sn(\zeta)}$
-m^2 (1-m^2)	2m^2-1	1	ds(ζ)=$\frac{dn(\zeta)}{sn(\zeta)}$
$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	ns(ζ)+cs(ζ)
$\frac{1-m^2}{4}$	$\frac{1+m^2}{2}$	$\frac{1-m^2}{2}$	nc(ζ)+sc(ζ)
$\frac{1}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	ns(ζ)+ds(ζ)
$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	sn(ζ)+$ics(\zeta)$

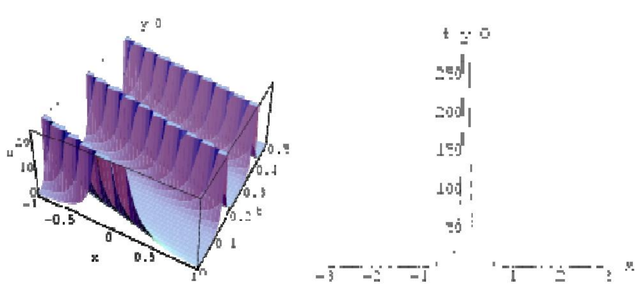

Figure 1 : The modulus of solitary wave solution u_1 (Eq. 17) where m=0.5

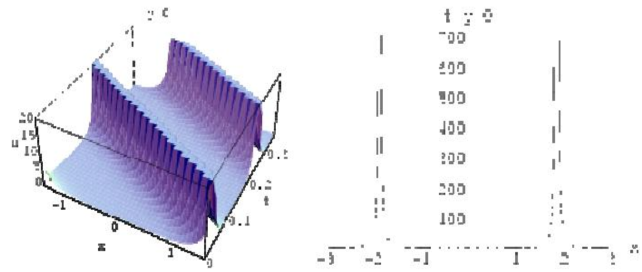

Figure 2 : The modulus of solitary wave solution u_2 (Eq. 18) where m=0.5

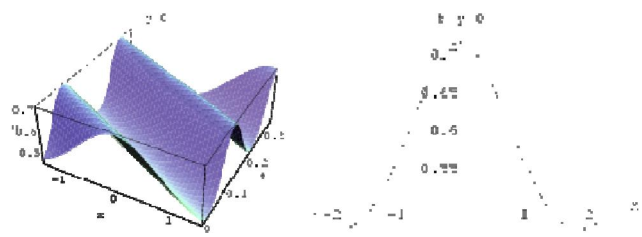

Figure 3 : The modulus of solitary wave solution u_{22} (Eq. 38) where m=0.5

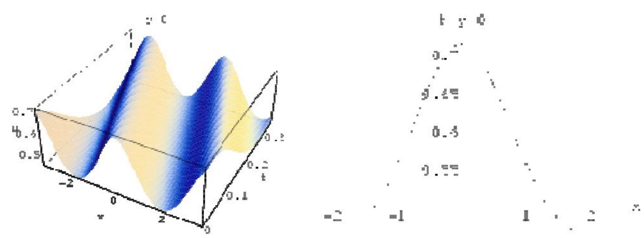

Figure 4 : The modulus of solitary wave solution u_{24} (Eq. 40) where m=0.5

CONCLUSION

By introducing appropriate transformations and using extended F-expansion method, we have been able to obtain in a unified way with the aid of symbolic computation system-mathematica, a series of solutions including single and the combined Jacobi elliptic function. Also, extended F-expansion method is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. When $m \to 1$, the Jacobi functions degenerate to the hyperbolic functions and given the solutions by the extended hyperbolic functions methods. When $m \to 0$, the Jacobi functions degenerate to the triangular functions and given the solutions by extended triangular functions methods.

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