ISSN : 0974 - 7532 Volume 6 Issue 8

Trade Science Inc.

Research & Reviews in

BioSciences

Regular Paper

RRBS, 6(8), 2012 [204-209]

Effect of permeability of porous medium on MHD flow of blood in very narrow capillaries

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Received: 14th May, 2012 ; Accepted: 13th September, 2012

ABSTRACT

The present paper deals with a mathematical model for the effect of permeability of porous medium on MHD flow of blood in very narrow capillaries. It is assumed that there is a lubricating layer between red blood cells and tube wall. The analysis of fluid flow between red cell and tube wall, when the cell appears to be at rest and the capillary wall moving backwards, is made. The effect of porous medium is examined. Analytical expressions are shown through graphs to describe the effect of various parameters on velocity profile, leak back flow rate and skin friction. It is found that the velocity profile increases numerically as permeability of porous medium increases and leak-back flow rate decreases numerically as magnetic intensity increases.

KEYWORDS

MHD flow;
Single file flow;
Narrow capillaries;
Lubricating zone;
Magnetic field;
Porous medium;
Red blood cells.

INTRODUCTION

Human blood is a suspension of cells of particles in a complex, continuous, aqueous solution called plasma. The cells consist of variety of blood cells e.g. red blood cells (erythrocytes), white blood cells (leukocytes) and platelets. Normally, the blood cells compose about 50% of the blood volume and they are small and about 5 millions/ $(mm)^3$ in number. Investigation of blood flow dynamics and erythrocyte (i.e red blood cell) rheology in capillaries is of great importance not only because they are the major site of oxygen and nutrient exchange, but also because the proper microcirculatory function is primarily determined by the rheological behaviour of red blood cells (RBCs) in these vessels. In human physiology, we come across mainly two types of circulations, macro circulation and microcirculation. The macro circulation system consists of large arteries and vessels while microcirculation system comprises of the smallest arteries, veins and capillaries whose diameter are equal to or smaller than that of red cell. Conditions are very different for the circulations in narrow capillaries than large vessels. For describing the mechanics of red blood cell motion in narrow capillaries, we distinguish two situations according t the convenience with which the cells fit into the vessels. In the first case when the capillary has diameter larger than that of the cell, the cell can fit into the tube without distortion; this flow situation is called positive clearance. In the second situation called negative clearance, when the diameter of the cell is larger than that of capillary as such the cell will be deformed in order to fit into the capillary. In this case pressure must be generated in thin layer of fluid round the edge of the cell in order to deform it and depends on elastic properties of the cell.

When red cell is severely deformed then in blood flow the red cell seems to plug the capillary of blood vessel and the motion of the plasma in capillary between successive red cells is called bolus flow. Ahmadi and Manvi[1] described equation of motion for viscous flow through a rigid porous medium. Aviles et.al.[2] investigated ferromagnetic seeding for the magnetic targeting of drugs and radiation in capillaries beds. Baumgartner et. al.[3] discussed blood flow switching among pulmonary capillaries is decreased during high hematocrit. Bishop et. al.[4] studied rheological effects of red blood cell aggregation in the venous network: A review of recent studies. Dash et. al.[6] discussed Casson fluid flow in a pipe filled with a homogeneous

porous medium. El-Shahed[7] investigated pulsatile flow of blood through a stenosed porous medium under periodic body acceleration. Fitz-Gerald[8] studied mechanics of red cell motion through very narrow capillaries. Haik et. al.[9] analysed apparent viscosity of human blood in a high static magnetic field. Huo and Kassab[10] studied pulsatile blood flow in the entire coronary arterial tree: theory and experiment. Jain et. al.[11] discussed mathematical analysis of MHD flow of blood in very narrow capillaries. Lighthill[12] introduced pressure forcing of tightly fitting pellets along fluid filled elastic tubes. Mittal et. al.[13] discussed analysis of blood flow in the entire coronary arterial tree. Ozkayan[14] studied viscous flow of particles in tubes: Lubrication theory and finite element model.Pries and Secomb[15] discussed rheology of the microcirculation. Pries et. al.[16] investigated blood viscosity in tube flow: Dependence on diameter and hematocrit. Prothero and Burton[17] analysed the physics of blood in capillaries. Sharan and Popel[18] described a two phase model for flow of blood in narrow tubes with increased effective viscosity near the wall. Sharma et. al.[19] discussed performance modeling and analysis of blood flow in elastic artery. Tozern and Skalak[20] studied flow of elastic compressible spheres in tubes.Tozern and Skalak[21] discussed the steady flow of closely fitting in incompressible elastic spheres in a tube. Weiderhielm et. al.[22] analysed pulsatile pressure in microcirculation of the frog's mesentery. Zweifach and Lipowsky[23] described quantitative studies of microcirculatory structure and function, microvascular hemodynamics of cat mesentery and rabbit omentum.

In the present paper we consider the problem Jain et. al.[11] with permeability of porous medium. The purpose of this study is to investigate the effect of permeability of porous medium on MHD flow of blood in very narrow capillaries.

It is hoped that this investigation may help for the further studies in the field of medical research, the application of magnetic field and porous medium for the treatment of certain cardiovascular diseases and also the results of this analysis can be applied to the pathological situations of blood flow in coronary arteries when fatty plagues of cholesterol and artery- clogging blood clots are formed in the lumen of the coronary artery.

MATHEMATICAL MODEL AND GOVERNING EQUATIONS

In this model we consider the axially symmetric and Newtonian flow of blood in a tube of uniform radius. The blood is assumed to be homogeneous fluid, while the red blood cells are assumed to be elastic and incompressible. The single cell is fitted in the tube so as to generate a single file flow. In this investigation, we study fluid flow in lubricating zone i.e. fluid flow between red blood cell and tube wall. The effect of transverse magnetic field on the flow of narrow capillary is taken into account. The induced magnetic field has been neglected. The viscous forces are predominant in the flow of such tubes. The inertial terms are considered negligible. During passing down single red cell in narrow capillary, it deforms due to its elastic property. The shape of red cell is bi-concave disk. The axial velocity is taken zero at the surface of red blood cell and –W at the tube wall. To obtain the axial velocity of the fluid relative to the tube, we add W velocity in the direction of the flow of fluid.

Let us assume the coordinate z in the direction of the axis of the tube; r is transverse distance from the highest point of the surface of the RBC (Figure 1).

The equations governing the motion fluid flow are:

$$\frac{\partial(rv)}{\partial r}+\frac{\partial(rw)}{\partial z}=0 \qquad (1)$$

$$\rho\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\sigma\mu_e^2B_0^2w-\frac{\mu}{K}w \qquad (2)$$

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Boundary conditions are:

$$\begin{aligned} &w = -We^{-int}, \quad v = 0, \quad at\ r = h \quad t > 0 \\ &w = 0, \quad v = 0, \quad at\ r = 0 \quad t > 0 \end{aligned} \tag{3}$$

From equations (1) and (3), we have:

$$\int_0^h rw\,dr = -Q_{lb} \text{ (Constant)} \tag{4}$$

The pressure gradient of blood flow can be taken as:

$$\frac{\partial p}{\partial z} = -Pe^{-int}$$

Also we may take the velocity as $w(r,t) = \overline{w}(r)e^{-int}$ without loss of generality.

From equation (2) using these values of $\frac{\partial p}{\partial z}$ and $w(r,t)$, we get:

$$\frac{d^2\overline{w}}{dr^2} + \frac{1}{r}\frac{d\overline{w}}{dr} - \frac{\alpha}{\mu}\overline{w} = -\frac{p}{\mu} \tag{5}$$

Where $\alpha = -in\rho + H$ and $H = \sigma\mu_e^2 B_o^2 + \frac{\mu}{K}$

The solution of (5) is given by

$$\overline{w}(r) = AJ_o\left(ir\sqrt{(\alpha/\mu)}\right) + BY_o\left(ir\sqrt{\alpha/\mu}\right) + \frac{P}{\alpha} \tag{6}$$

Where J_o and Y_o are Bessel's functions of first kind and second kind respectively. Now $B = 0$ as otherwise at $r = 0$, $\overline{w}(r)$ is not finite.

Then the solution of (6) becomes:

$$\overline{w}(r) = AJ_o\left(ir\sqrt{(\alpha/\mu)}\right) + \frac{P}{\alpha} \tag{7}$$

Now transformed boundary conditions are:

$$\begin{aligned} &\overline{w}(r) = -W, \quad at\ r = h, \quad t > 0 \\ &\overline{w}(r) = 0, \quad at\ r = 0, \quad t > 0 \end{aligned} \tag{8}$$

Using second condition of equation (8) in (7), we have:

$$A = -\frac{P}{\alpha}$$

Thus (7) becomes

$$\overline{w}(r) = \left[1 - J_0\left(ir\sqrt{(\alpha/\mu)}\right)\right]\frac{P}{\alpha} \tag{9}$$

By using first condition of equation (8) in (7), we get:

$$p = -\frac{\alpha W}{\left(1 - J_o(ih\sqrt{(\alpha/\mu)})\right)}$$

Now equation (7) yields:

$$\overline{w}(r) = -W\left[\frac{\left(1 - J_0(ir\sqrt{(\alpha/\mu)})\right)}{\left(1 - J_0(ih\sqrt{(\alpha/\mu)})\right)}\right] \tag{10}$$

Thus

$$w = -W\left[\frac{\left(1 - J_0(ir\sqrt{(\alpha/\mu)})\right)}{\left(1 - J_0(ih\sqrt{(\alpha/\mu)})\right)}\right]e^{-int}$$

Expanding the Bessel function in a series and retaining only up to bi-quadratic terms,

$$w = -W\left[\frac{r^2\left(16\mu + \alpha r^2\right)}{h^2\left(16\mu + \alpha h^2\right)}\right]e^{-int} \tag{11}$$

The real part of w is given by:

$$\mathrm{Re}(w) = -W\left[\frac{r^2\left(\left\{(16\mu + Hr^2)(16\mu + Hh^2) + (nh\rho r)^2\right\}\cos nt - \left\{n\rho h^2(16\mu + Hr^2) - n\rho r^2(16\mu + Hh^2)\right\}\sin nt\right)}{h^2\left\{(16\mu + Hh^2)^2 + (n\rho h^2)^2\right\}}\right] \tag{12}$$

The velocity Relative to tube wall is given by:

$$w_1 = w + W\cos nt \tag{13}$$

THE LEAK-BACK FLOW RATE AND SKIN FRICTION

With the help of equation (4), the leak-back flow rate Q_{lb} is given by:

$$= W\left[\frac{h^2\left(\left\{64\mu + \frac{20H\mu h^2}{3} + \frac{h^4}{6}\left(H^2 + (n\rho)^2\right)\right\}\cos nt - \frac{4}{3}n\rho\mu h^2\sin nt\right)}{\left\{(16\mu + Hh^2)^2 + (n\rho h^2)^2\right\}}\right] \tag{14}$$

The skin friction at the RBC surface is given by:

$$\tau_{rbc} = -\mu\left(\frac{1}{r}\frac{\partial w}{\partial r}\right)_{r=0}$$

$$= 32\mu^2 W\left[\frac{\left((16\mu + Hh^2)\cos nt - n\rho h^2\sin nt\right)}{h^2\left\{(16\mu + Hh^2)^2 + (n\rho h^2)^2\right\}}\right] \tag{15}$$

NUMERICAL RESULTS AND DISCUSSION

The expression for relative velocity profile w_1 obtained in equation (13) has been depicted in figures 1-4 by plotting t versus w_1 for different values of magnetic intensity B_0, viscosity μ, transverse distance r and permeability of porous medium K. Figures 5-7 are constructed for t versus leak-back flow rate for different values of magnetic intensity B_0, permeability of porous medium K and thickness h. Figures 8-10 are constructed for t versus skin friction τ for different values of magnetic intensity B_0, permeability of porous medium K and thickness h. Figure 11 and 12 are constructed for r versus relative velocity profile w_1 for different values of B_0 and K. Figure 1 show the velocity profile for different values of B_0 (magnetic intensity). It is noticed that the flow is pulsatile where velocity changes periodically and there is no significant of magnetic intensity B_0 on relative flow velocity. Figure 2 show the velocity profile for different values of μ. It is observed the flow is pulsatile where velocity changes periodically and there is no notable effect of μ on flow velocity relative to the tube wall, as all the graph are overlapping. Figure 3 show the velocity profile for different values of r. It is noticed that in lower half cycles velocity relative to tube is increases as r increases and in upper half cycles velocity relative to tube is decreases as r increases and at the wall it is zero. Figurethe result of earlier known work is found 4 shows that velocity profile for different values of K. It is noticed that in lower half cycles velocity relative to tube increases as K increases and in upper half cycles velocity relative to tube decreases as K increases and as K’! “, the result of earlier known work is found. Figure 5 shows the leak-back flow rate for different values of B_0. It is observed that in lower half cycles leak-back flow rate decreases as B_0 increases and in upper half cycles leak-back flow rate increases as B_0 increases. For the value of B_0 from 30 to 300, the leak-back flow rate increases slightly but for the value of B_0 from 3 to 30, the leak-back flow rate gives a big change as comparatively 30 to 300. Figure 6 shows that the leak-back flow rate for different values of K. It is observed that in lower half cycles leak-back flow rate increases as K increases and in upper half cycles leak-back flow rate decreases as K increases and as K’!”, the result of earlier known work is found. Figure 7 display the effect of thickness (gap between RBC and tube wall) of fluid on leak-back flow. It shows that in lower half cycles leak-back flow rate decreases as h increases and in upper half cycles leak-back flow rate increases as h increases. Figure 8 show the effect of skin friction vs. time for different values of magnetic intensity. It is shown that in lower half cycles skin friction increases as B_0 increases and in upper half cycles skin friction decreases as B_0 increases, skin friction zero about B_0 = 300. Figure 9 displays that skin friction vs. time for different values of K. It is shown that in lower half cycles skin friction slightly decreases with increase of K and in upper half cycles skin friction slightly increases with increase of K. Figure 10 displays that skin friction vs. time for different values of thickness h. It is shown that in lower half cycles skin friction increases as h increases and in upper half cycles skin friction decreases as h increases. Figure 11 shows the effect of velocity profile vs. r for different values of B_0. It is noticed that velocity relative to tube increases as B_0 increases except highest point of surface of RBC and tube wall. Figure 12 shows the effect of velocity profile vs. r for different values of K. It is noticed that velocity relative tube decreases as K increases except highest point of surface of RBC and tube wall.

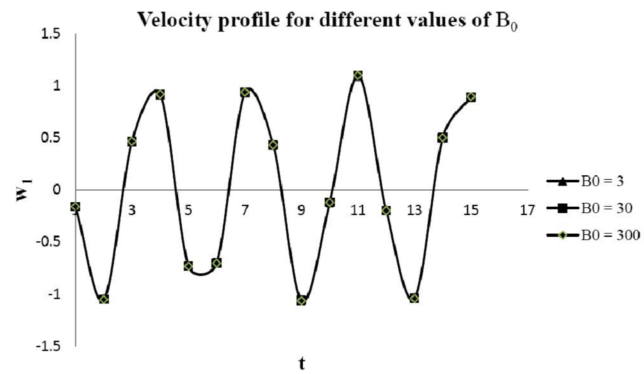

Figure 1

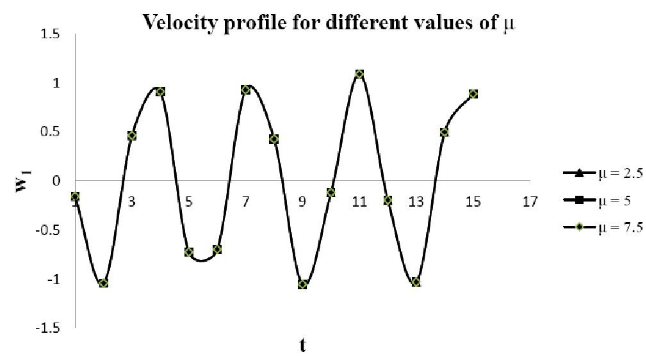

Figure 2

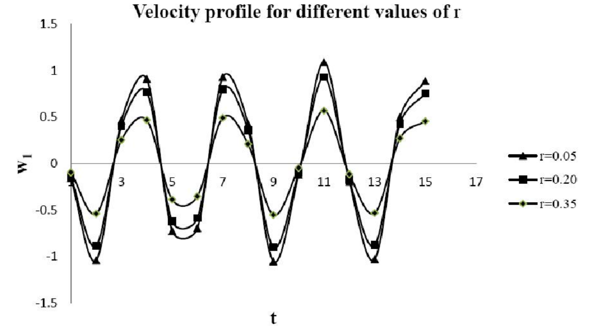

Figure 3

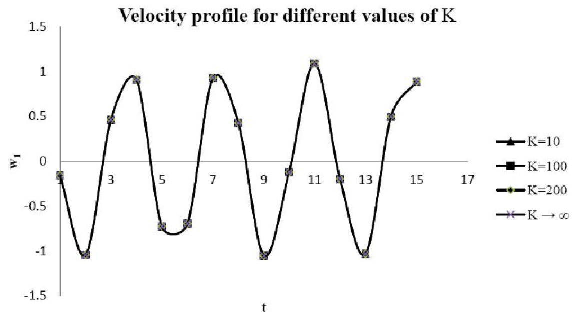

Figure 4

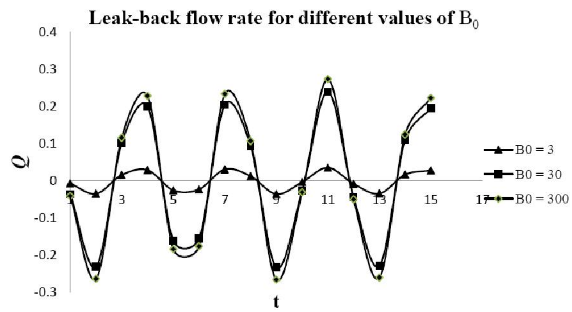

Figure 5

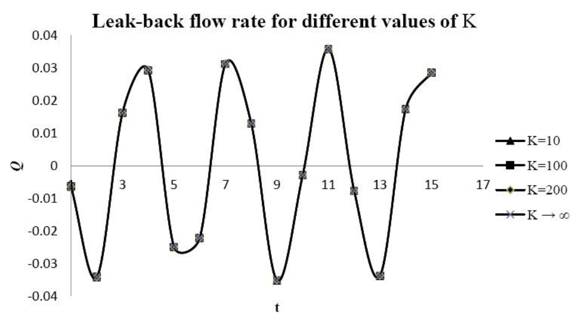

Figure 6

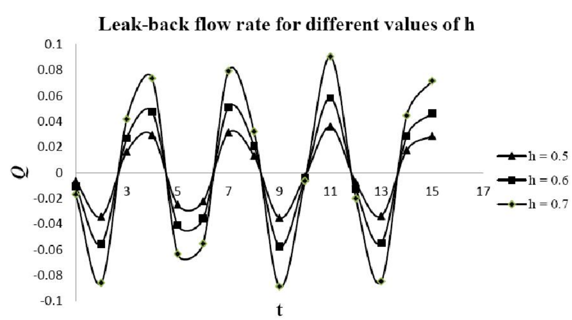

Figure 7

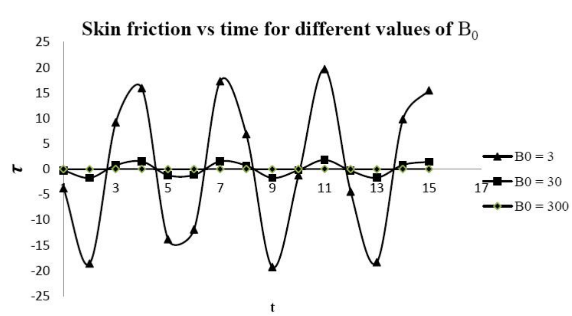

Figure 8

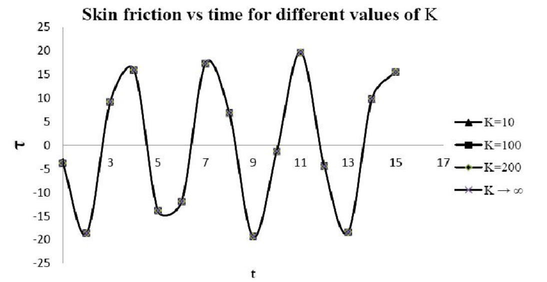

Figure 9

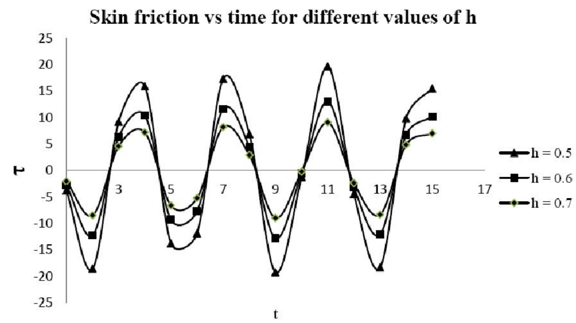

Figure 10

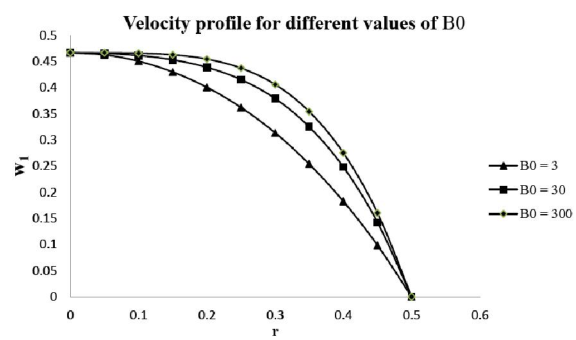

Figure 11

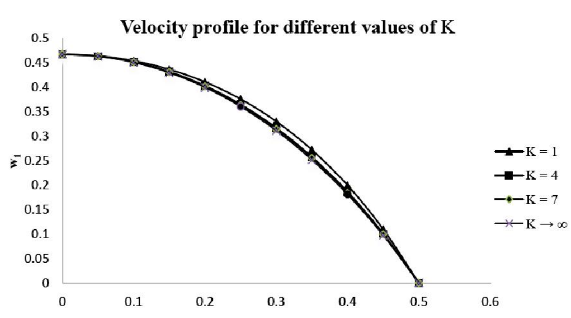

Figure 12

CONCLUTIONS

It is clear from above discussions that magnetic field has no effect on velocity flow in lubricating zone relative to the tube wall and it is also independent viscosity. The effect of magnetic field on leak-back flow rate and skin friction seems to be significant. The effect of permeability of porous medium is no notable for relative velocity. The viscosity effect is very much dependent on the thickness of the lubricating zone (thickness between RBC and tube wall). The magnet and thickness of lubricating zone affect the skin friction at RBC surface remarkably.

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